Non-Cooperative Coexistence of Co-located Independent Wireless Mesh Networks*

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Abstract

As more wireless networks are being deployed in a given geographic area, the problem of interference and coexistence of these independently operated networks is becoming an increasing problem. This paper looks at the coexistence of independent multihop Wireless Mesh Networks (WMNs). We argue that cooperation is difficult in such scenarios. We define a coexistence game model and apply it to study channel assignment in co-located WMNs. In addition, we propose using no-regret learning algorithms that allow WMNs to iteratively arrive at Nash Equilibrium outcomes. Simulation results show that the informed no-regret learning algorithms we have tested converge to a set of Nash Equilibrium strategy profiles. We also show that network information is not critical for games with large numbers of players.

1. Introduction

Wireless Mesh Networks (WMNs) consist of multiple wireless routers forwarding data packets to and from a small set of gateways. The gateway is usually connected via a physical wired interface to the Internet. Wireless clients, e.g. laptops and PDAs, could potentially communicate with each other and the Internet over multiple hops via this infrastructure.

The merits of WMNs as a means of extending coverage and improving performance over existing single-hop Wireless LAN (WLAN) access points (APs) have prompted much activity in the research as well as the standardization communities. Community wireless mesh networks are being set up in neighborhoods where residents pool together wireless networking resources to enable connectivity to one another and the Internet. Municipal and city councils have also shown interest in setting up city-wide wireless mesh deployments that can serve both the government agencies and residents. In the home, individual users will soon be able to connect up their wireless devices to form a mesh network by using the IEEE 802.11s standard.

Despite the advances made in WMNs, several key technical challenges still remain. One such challenge relates to the broadcast nature of the wireless medium. This creates the potential of interference among communicating nodes that reside within the same locality, known as a *collision domain* [8]. Briefly, if more than a pair of communicating nodes are present in a collision domain, coordination is required to prevent both links from being active at the same time. Otherwise, packet collisions occur.

In this paper, we postulate that with the increasingly widespread deployment of WMNs, the interference among WMNs under different management control is bound to become a critical problem. This *inter-network* interference is different from the interference found among nodes from the same network, an area that researchers have been addressing. Since interfering nodes may belong to different WMNs, there is often no mechanism nor any incentive for them to cooperate. It essentially becomes a competitive environment where networks try to utilize the available resources in a selfish manner, leading to a sub-optimal performance. We term these non-cooperative networks *Independent WMNs*.

In the next section, we will take a closer look at the coexistence problem present among independent WMNs. We also highlight the usefulness of game theoretic tools in analyzing and solving this coexistence problem. In Section 3, we present a generic coexistence game model that we use to analyze the problem. Using this model, we will apply it to study single collision domain problems. Section 4 contains results of simulations conducted to investigate the interaction of WMNs that use game theoretic learning to solve the coexistence problem. Some related works are discussed in Section 5. Finally, we conclude in Section 6.

2. Motivation and Background

In this section, we motivate the need to look at coexistence issues among co-located WMNs. We also discuss why game theory is a suitable tool for analyzing such a problem.

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2.1. Interference among Independent WMNs

We define a *link* as the presence of a communication medium between the wireless interfaces of a pair of nodes, allowing data to be transferred. Due to the broadcast nature of the wireless medium, multiple communication links located within interference range of one another will experience degradation of performance if there is no mechanism to coordinate and manage the communication [7]. In [1], the authors highlight the increasing problem of multiple WLAN deployments located in the same area. Using actual data of hotspot deployments in major cities, they argue that the presence of multiple independently-managed APs may lead to a "chaotic" environment, where networks experience sub-optimal performance. They propose using power-control to manage the APs.

We believe that as communities of residential users cooperate by linking up their APs, and as Wireless Broadband Providers, city councils and even individuals set up their own WMNs, there may exist more than one WMN in a particular geographic area. Like the WLAN hotspots, these WMNs are independently operated and so have limited mechanisms and incentives to cooperate. However, the multihop nature of WMNs makes it a different problem to that found in single-hop WLANs.

The first difference is the fact that a WMN potentially covers a more extensive area compared to a hotspot. A situation may arise when only a subset of the links of one WMN interferes with another part of a second WMN. Secondly, a flow in a WMN typically traverses multiple links (hops). While reducing the interference of the link between an AP and station may increase flow throughput, doing so will have no effect on the end-to-end throughput of a WMN flow if there is interference on a link further upstream or downstream as well.

In wireless multihop networks, the bandwidth available to a flow is a function of not just the interference between independent flows on different links (inter-flow interference), but also the interference of the same flow with itself on subsequent paths along its route (intra-flow interference). This phenomenon has been studied in [8] and [15]. In co-located WMNs, we assert that the inter-flow interference can be subdivided into two categories – *internal* inter-flow interference and *external* inter-flow interference.

While internal inter-flow interference occurs among independent flows on different links within a *single* WMN, external inter-flow interference relates to the interference experienced by flows from links belonging to *different* WMNs. A distinction between them is needed because the former is usually managed *cooperatively* using schemes implemented within a network, e.g. rate control [15], channel assignment [13] and routing [14]. In the case of the external inter-flow interference, high level cooperation is not available, as the links belong to different independent WMNs. In reality, networks may adopt a selfish approach of trying to get as much resource (e.g. bandwidth) as possible, thereby creating a competitive environment.

In addition, the cooperative schemes used to optimize network resources in the presence of internal interferences assume a certain degree of network information available, e.g. size, topology and traffic patterns. A WMN would have less information about other co-located WMNs. For example, a WMN is unable to know the size, topology or even number of co-located WMNs.

We have thus far motivated the need to study the problem of interference among co-located independent WMNs, which we term as the *Coexistence Problem*. We have also illustrated why it is a different and more challenging problem than interference among single-hop networks or within a single multihop network. We propose game theory as a suitable tool for studying and managing the Coexistence Problem.

2.2. Game Theory Basics

Game theory [5] is a branch of applied mathematics that describes and studies the interactions of decision processes. The following description will be limited to the scope that is needed to understand and analyze the Coexistence Problem.

Normal Form Game Model
A normal form non-cooperative game is defined by $\Gamma = \langle \mathcal{N}, \mathcal{S}, \{U_i\}_{i\in\mathcal{N}} \rangle$, where $\mathcal{N}$ is a finite set of players, and $\mathcal{S}$ is the Cartesian product of the set of strategies available to each player in $\mathcal{N}$; i.e., $\mathcal{S} = \times_{i\in\mathcal{N}}\mathcal{S}_i$ where $\mathcal{S}_i$ is the set of strategies available to player i. $S = [s_1, s_2, \ldots, s_{|\mathcal{N}|}] \in \mathcal{S}$ is a strategy profile consisting of a strategy each from every player in $\mathcal{N}$. $U_i \colon \mathcal{S} \rightarrow \mathbb{R}$ is defined as a utility function of player i representing the value of the outcome resulting from a strategy profile S. For a particular strategy profile S, if the strategy used by player i is $s_i \in \mathcal{S}_i$, we collectively term the strategies of the other players as s_{-i}.

Nash Equilibrium
A Nash Equilibrium (NE) is a strategy profile where no player has any incentive to unilaterally use a different strategy s_i'. Mathematically, a strategy profile $S = [s_i, s_{-i}]$ is a pure strategy NE if and only if $U_i(s_i, s_{-i}) \geq U_i(s_i', s_{-i}), \forall i \in \mathcal{N}, s_i' \in \mathcal{S}_i$. It should be noted that a NE may involve mixed strategies.

Pareto Efficiency
Pareto Efficiency (PE) or Optimality is sometimes used as a measure of the efficiency of an outcome. We say that a strategy profile is PE when a player cannot further increase his utility without decreasing the utility of another player. Therefore, S is PE if and only if there exists no other strategy profile S' where $U_i(S') > U_i(S)$, for some $i \in \mathcal{N}$ when $U_j(S') \geq U_j(S), \forall j \in \mathcal{N}, j \neq i$.

Extending from this definition, we further define that a strategy profile is more *efficient* than another if the utility of at least one of the player is higher while the utilities of the rest of the players are not worse off, i.e., S is more efficient than S' if $U_i(S) > U_i(S')$, for some $i \in \mathcal{N}$ when $U_j(S) \geq U_j(S'), \forall j \in \mathcal{N}, j \neq i$.

2.3. No-Regret Learning

Even though classical non-cooperative game theory can provide insights into the interactions among co-located WMNs, it assumes common knowledge of, among other things, the sets $\mathcal{N}$ and $\mathcal{S}$. This is highly unlikely in reality as a WMN's knowledge of other interfering networks is restricted to what it experiences on its affected links. In this section, we introduce the concept of learning in game theory that we believe can provide practical solutions to the Coexistence Problem.

Learning in game allows initially uninformed players to acquire knowledge about the state of the world they are in as the game is played repeatedly. Learning has been applied to networking research [6], where they study what strategies players will play in the long run. In this paper, we apply a class of learning, known as *No-Regret Learning*, to solve the Coexistence Problem. The attributes of no-regret learning is that information like the number of players in the game and their utility functions is not required by a player to play efficient strategies in the long run.

In no-regret learning, time dependency is introduced in the form of t, where $S^t = (s_1^t, s_2^t, \ldots, s_{|\mathcal{N}|}^t)$ denotes the strategy profile of the players at time t and $q^t = (q_1^t, q_2^t, \ldots, q_{|\mathcal{N}|}^t)$ represents the weights players placed on their strategies at time t. The weight q_i^t can be seen as a vector containing the probabilities of playing each of player i's strategies or as a function $q_i^t(s_i)$ returning the probability of playing strategy $s_i \in \mathcal{S}_i$, at time t.

No-regret learning allows a player to play his strategies with certain probabilities. The concept of *regret* involves the benefits a player feels after playing a particular strategy, compared to his other strategies. Those strategies that produce lower regrets will be updated with higher probabilities in the long run. Ultimately, strategies that are more successful will be played more often. There are different algorithms relating to different regret measures and updating methods. In Section 4, we will simulate how WMNs use two such algorithms, described in [6] and attributed to Freund and Schapire [4] and Foster and Vohra [3], to solve the Coexistence Problem. Due to space constraints, the algorithms will be described briefly. Readers are directed to [6], [4] and [3] for more details.

Freund and Schapire
This algorithm makes use of the cumulative utility obtained by player i over time t if he plays s_i given that the other players had played s_{-i}^t, for every $s_i \in \mathcal{S}_i$. We denote this as $U_i^t(s_i) = \sum_{x=1}^{t} U_i(s_i, s_{-i}^x)$. The weights updating algorithm is such that at time $t+1$, the probability of playing strategy s_i is updated using:

$$q_i^{t+1}(s_i) = \frac{(1+\alpha)^{U_i^t(s_i)}}{\sum_{s_i' \in \mathcal{S}_i} (1+\alpha)^{U_i^t(s_i')}}$$

for some $\alpha > 0$.

Foster and Vohra
At time t, we denote the regret r_i^t that player i feels for playing strategy s_i^t rather than s_i as the difference in the utilities obtained from playing the strategies, given that the other players play the strategy profile, s_{-i}^t; i.e.,

$$r_i^t(s_i, s_i^t \mid s_{-i}^t) = U_i(s_i, s_{-i}^t) - U_i(s_i^t, s_{-i}^t)$$

This algorithm makes use of the cumulative regret felt by a player i over time t, given by $R_i^t(s_i) = \sum_{x=1}^{t} r_i^x(s_i, s_i^x \mid s_{-i}^x)$ for playing s_i^x rather than s_i. In this case, the probability of playing strategy s_i is updated using:

$$q_i^{t+1}(s_i) = \frac{[R_i^t(s_i)]^+}{\sum_{s_i' \in \mathcal{S}_i} [R_i^t(s_i')]^+}$$

where $[R]^+ = \max(\{R, 0\})$.

2.3.1 Informed vs. Naive No-Regret

The above learning algorithms are known as informed algorithms. This is because they assume that a player i is able to evaluate how s_{-i}^t, the strategies played by the other players at time t, could affect the utilities of all his strategies, $s_i \in \mathcal{S}_i$, even those that are not being played at that time. When the player is only able to know the utility of the strategy that he has played, a modification to the learning algorithms is needed.

In [6], the authors provide a way to convert an informed algorithm to a naive one. It involves converting the utility function, U_i to $\hat{U}_i$, where

$$\hat{U}_i(s_i, s_{-i}^t) = \begin{cases} \frac{U_i(s_i, s_{-i}^t)}{\hat{q}_i^t(s_i)}, & \text{if } s_i^t = s_i; \\ 0, & \text{otherwise.} \end{cases}$$

The same algorithms could then be used with $\hat{U}_i$ replacing U_i, and modifying the resulting probabilities q_i^t with $\hat{q}_i^t = (1-\epsilon)q_i^t + \frac{\epsilon}{|\mathcal{S}_i|}$, for some $\epsilon > 0$.

3. The Coexistence Game

In this section, we will define a general model of the Coexistence Game. The model is made as general as possible in order to encompass the different interactions of co-located independent WMNs. It could easily be adapted to more specific interaction scenarios. Subsequently, we will describe a channel assignment coexistence game using this model to study and solve a specific coexistence problem.

3.1. The General Coexistence Game

We believe many of the schemes proposed to minimize internal and external interferences can be studied as a game. We define the general Coexistence Game as Γ_g.

In Γ_g, the set of players $\mathcal{N}$ represents the *decision makers* in the independent WMNs. Each WMN consists of a set of nodes or links whose collective actions affect network performance. There are two possibilities of defining $\mathcal{N}$, depending on the entities taking part in the decision making process. In the first case, each WMN has a centralized decision making process, where a single entity within each WMN collects information (e.g. network conditions) from the nodes/links, makes the decisions and directs the nodes/links to act on them. In this case, we represent each WMN as a player. Alternatively, the decision making can be distributed, where the nodes/links within each WMN make decentralized decisions that collectively (as a coalition) seek to optimize the performance of their respective network. In this case, we represent each WMN as a coalition of players where each player is a node/link from the WMN. In this paper, we will use the centralized process to explain the concepts. Henceforth, the terms network and player will be used interchangably.

Each player has at his disposal a set of strategies $\mathcal{S}_i$, which may be different for different schemes. For example, when routing is used to direct traffic flows to paths with low interference [14], $\mathcal{S}_i$ represents the set of routes available. If transmit power control is used to limit the interference of the links, $\mathcal{S}_i$ consists of the power levels a player can assign to each of his links.

The utility function U_i denotes the value player i places on the outcome of a strategy profile S. Player i can be seen as trying to optimize U_i through its choice of s_i in the light of s_{-i} that are played by the other players. A possible way to express the utility of a WMN is the sum of the utilities of all its individual flows. Let us assume a WMN has flows with rates $r_1, \ldots, r_k$. The utility of this WMN can be expressed as $\sum_{i=1}^{k} u(r_i)$, where u is some concave function. Note that this type of utility definition is commonly used in network utility maximization [11, 15].

3.2. Channel Assignment Coexistence Game

In this section, we apply the general coexistence game, Γ_g to a more specific scenario — channel assignment. We define this as a channel assignment coexistence game, Γ_{ca}. In Γ_{ca}, the player set $\mathcal{N}$ contains the independent WMNs. We define $\mathcal{C}$ as the set of channels available, with $c = |\mathcal{C}|$.

In Γ_{ca}, we will consider multi-radio WMNs [13, 12] where each node contains multiple wireless interfaces. We focus on channel assignment in this game. In order to simplify the explanation and analysis, we assume that each link within a WMN is bound to a pair of dedicated interfaces, giving us a fixed topology of nodes and links per WMN. This can be accomplished by schemes found in multiradio WMN architectures like [13]. These links are represented by $\mathcal{L}_i = \{1, 2, \ldots, l_i\}$ for a player i. A strategy, s_i of player i assigns a channel $j \in \mathcal{C}$ to each of the link $k \in \mathcal{L}_i$. For now, we assume that the links are unidirectional, i.e., each link has a predetermined transmitter and receiver.

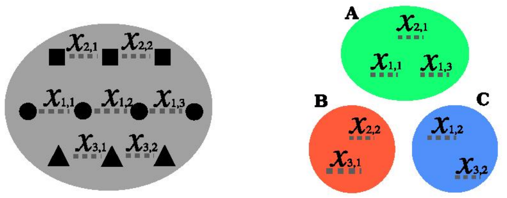
Let $\mathcal{F}_i$ represent the set of flows in player i, where $f_{i,k} \in \mathcal{F}_i$ is the flow originating on link k. In other words, the source of flow $f_{i,k}$ is the transmitter of link k. We restrict each source to one unicast flow. For convenience, we will identify each flow $f_{i,k}$ with its flow rate. Let $x_{i,k}$ be the aggregate flow rate of all the flows that pass through link k, i.e., $x_{i,k} = \sum f_{i,r}$, where $f_{i,r}$ is every flow that has to go through link k to reach its destination.

In graph theory, a complete subgraph of a graph is known as a clique. If we represent the set of links $\mathcal{L}_i$ as the vertices of a graph, with an edge drawn between a pair of vertices if the links they represent interferes with each other when they are on the same channel, we can represent a collision domain as a clique [7]. Let $\mathcal{D}$ denote the set of cliques where $d \in \mathcal{D}$ represents the set of links in a game that are in the same collision domain. For a strategy profile S that allocates every link in a game to a channel $j \in \mathcal{C}$, we define $d_j \subseteq d$ where $\bigcup_{j \in \mathcal{C}} d_j = d$. In other words, S can be seen as breaking up each clique d, into smaller cliques $d_j, \forall j \in \mathcal{C}$. We will use the term collision domain and clique interchangably in this paper.

Finally, we assume all the nodes use a common MAC protocol that allocates the rates to the links in each channel collision domain, $d_j, j \in \mathcal{C}$ in a max-min fair manner. We also assume there is a transport layer or flow control protocol [15] in each WMN that ensures each link does not transmit more than the aggregate end-to-end flow rates.

Example 1. *To illustrate the notations and concepts described so far, consider 3 WMNs within a single collision domain as shown in Figure 1(a).* $\mathcal{N} = \{1, 2, 3\}$ *and the collision domain* d *contains the links of all the players. There are 3 channels available,* $\mathcal{C} = \{A, B, C\}$*. Except for the first and last node, each node has two interfaces, one for each link. Suppose flows* $f_{1,1}$, $f_{1,2}$ *and* $f_{1,3}$ *flow through all of player 1's, 2's and 3's links respectively. We have* $x_{1,1} = x_{1,2} = x_{1,3} = f_{1,1}$, $x_{2,1} = x_{2,2} = f_{2,1}$ *and* $x_{3,1} = x_{3,2} = f_{3,1}$.

If a strategy profile involves player 1 assigning his links 1 and 3 to channel A, and link 2 to channel C, player 2 assigning his link 1 to channel A and link 2 to channel B, and player 3 assigning link 1 to channel B and link 2 to channel C, d *would be broken up into* d_A, d_B *and* d_C*, as shown in Figure 1(b). The rates of flows* $f_{1,1}$ *and* $f_{2,1}$ *are* $\frac{1}{3}$ *each, since* $x_{1,1} + x_{1,3} + x_{2,1} = 1$ *in* d_A*. Even though links* $x_{1,2}$ *and* $x_{2,2}$ *can use* $\frac{1}{2}$ *of the bandwidth in channels C and B respectively, a flow control protocol limits them to* $\frac{1}{3}$*, as the flows* $f_{1,1}$ *and* $f_{2,1}$ *are limited by the links in channel*



(a) (b)

Figure 1. Example 1 showing three WMNs in a collision domain. (a) Network topology. (b) The 3 channel collision domains after channel assignment.

A to that rate. A max-min fair MAC allows $x_{3,1}$ and $x_{3,2}$ to get the remaining $\frac{2}{3}$ of the bandwidth. Hence, $f_{3,1} = \frac{2}{3}$.

From Example 1, we can see that the channel assignment choices of the players affect each other's flow rates. If a player's objective is to maximize the rate of his flows, the utility can be defined as $U_i = \sum_k f_{i,k}$. Hence, $U_1 = U_2 = \frac{1}{3}$ and $U_3 = \frac{2}{3}$ in the example.

In the next section, we will use the model to analyze a restricted case of this channel assignment coexistence game.

3.3. Single Collision Domain

In this game, we assume that the WMNs have links that are all within a single collision domain, i.e., $|\mathcal{D}| = 1$. Each player i has $l_i + 1$ mesh nodes with each node containing 2 interfaces. There is a single unicast flow ($|\mathcal{F}_i| = 1$) from a source node to a destination, going through l_i links. The flow is always saturated, i.e., the source tries to send as much traffic as possible. Note that Example 1 described above can be classified as such a game. We call this 2 interface per node, single collision domain, 1 flow per WMN game, $\Gamma_{\text{ca-2i-1d-1f}}$.

Since all the links belong to the same collision domain, each link of player i is indistinguishable from another. A strategy s_i can be simplified to $[l_{i1}, l_{i2}, \ldots, l_{ic}]$ where l_{ij} is the number of links player i has on channel $j, \forall j \in \mathcal{C}$. Hence, $\sum_j l_{ij} = l_i$. Let $L_j = \sum_i l_{ij}$ denote the total number of links on channel j. The total number of links in the game is $L = \sum_j L_j = \sum_i l_i$. We will look at the non-trivial case when $L > c$ and $c > 1$.

We define $\mathcal{C}_{max} = \{j \in \mathcal{C} : L_j = \max_j L_j\}$. In other words, $\mathcal{C}_{max}$ contains the set of channels with the maximum number of links among all the channels. Moreover, we define $\mathcal{N}_{max} \subseteq \mathcal{N}$, where $i \in \mathcal{N}_{max}$ iff $l_{ij} \neq 0$ for some $j \in \mathcal{C}_{max}$. That is, $\mathcal{N}_{max}$ contains the set of players with at least one link in any channel belonging to $\mathcal{C}_{max}$.

We define the utility of a player i to be $U_i = f_i$, the rate of his flow. As f_i flows through all of player i's links, this also happens to be the minimum share of the bandwidth a

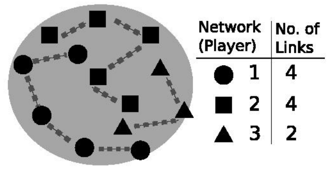


Figure 2. Example 2: Three WMNs in a single collision domain.

player can get from any of his links across all the channels. Obviously, if $i \in \mathcal{N}_{max}$, $U_i = \frac{1}{L_j}$, where $j \in \mathcal{C}_{max}$.

Nash Equilibrium

We will now look at what constitutes a pure strategy Nash Equilibrium (NE) in the game $\Gamma_{\text{ca-2i-1d-1f}}$.

Proposition 1. *In the single collision domain channel assignment game, $\Gamma_{\text{ca-2i-1d-1f}}$, a strategy profile that results in every channel having either r or $(r-1)$ links, where $r = \lceil \frac{L}{c} \rceil$ is a pure strategy Nash Equilibrium.*

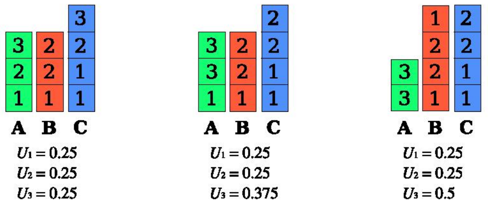
Example 2. *Consider the networks shown in Figure 2, where $L = 4 + 4 + 2 = 10$ and $\mathcal{C} = \{A, B, C\}$ (i.e., $c = 3$). In addition to $\mathcal{C}_{max}$, we define $\mathcal{C}_{min} = \{j \in \mathcal{C} : L_j = \min_j L_j\}$, and $\mathcal{N}_{min} \subseteq \mathcal{N}$ where $i \in \mathcal{N}_{min}$ iff $l_{ij} = 0, \forall j \in \mathcal{C}_{max}$. That is, $\mathcal{C}_{min}$ is the set of channels with the minimum number of links among all the channels, and $\mathcal{N}_{min}$ is the set of all players with no link in the channels in $\mathcal{C}_{max}$.*

With the strategy profiles shown in Figures 3(a) and 3(b), the number of links in $\mathcal{C}_{max} = \{C\}$ is $r = \lceil \frac{10}{3} \rceil = 4$ and the number of links in $\mathcal{C}_{min} = \{A, B\}$ is $r - 1 = 3$. Therefore, these two strategy profiles meet the condition in Proposition 1. Clearly, if the condition in Proposition 1 holds, $i \in \mathcal{N}_{max}$ or $i \in \mathcal{N}_{min}$, $\forall i \in \mathcal{N}$. As we can see from the way the links are distributed in Figures 3(a) and 3(b), this also results in all the links in the game being spread evenly across the channels. We will call this type of strategy profile a global spreading of links.

We note here that if $i \in \mathcal{N}_{max}$, player i's utility $U_i = \frac{1}{r}$, since he is restricted by his links in some channel in $\mathcal{C}_{max}$. However, if $i \in \mathcal{N}_{min}$, player i's utility $U_i \geq \frac{1}{r-1}$. This is because if all the links in the channels with player i's links belong to players in $\mathcal{N}_{min}$, he can get a utility of $\frac{1}{r-1}$. If in all the channels with player i's links, there exist links belonging to players in $\mathcal{N}_{max}$, player i can get additional bandwidth not used by those players and his utility becomes larger than $\frac{1}{r-1}$.

To prove proposition 1, we will show that for all $i \in \mathcal{N}$, moving player i's links to another channel will not improve his utility.

Proof. The proof is divided into 2 cases – Case 1: player i belongs to $\mathcal{N}_{max}$; and Case 2: player i belongs to $\mathcal{N}_{min}$.



(a) NE1 (b) NE2 (c) NE3

Figure 3. Different possible NE channel assignments for networks in Example 2. The letters represent the channels, each box represents a link and the number in the box represents the player the link belongs to.

In Case 1, player i's current utility is $U_i = \frac{1}{r}$. If he moves any of his link from a channel $j \in \mathcal{C}$ to a different channel, he can choose to move this link to a channel $j' \in \mathcal{C}_{max}$ or $j' \in \mathcal{C}_{min}$, where $j' \neq j$. If he moves the link to channel $j' \in \mathcal{C}_{max}$, his new utility will be $U_i' = \frac{1}{r+1}$ which is less than the original U_i. If he moves the link to $j' \in \mathcal{C}_{min}$, his utility becomes $U_i' = \frac{1}{(r-1)+1} = \frac{1}{r}$, which is the same as the original. Either way, player i has no incentive to move his links.

For Case 2, player i's current utility is $U_i \geq \frac{1}{r-1}$. He can choose to move his link in channel $j \in \mathcal{C}_{min}$ to a channel $j' \in \mathcal{C}_{max}$ or $j' \in \mathcal{C}_{min} \setminus \{j\}$. If he chooses channel $j' \in \mathcal{C}_{max}$, his new utility will be $U_i' = \frac{1}{r+1}$. If he chooses $j' \in \mathcal{C}_{min} \setminus \{j\}$, his new utility becomes $U_i' = \frac{1}{r}$. Both are less than his current utility. Therefore, player i has no incentive to move his links.

Since player i does not benefit from changing his strategy, this is a pure strategy NE. □

Proposition 1 is a sufficient condition for the existence of NE. There may exist other NE outcomes that are not global spreading. To describe a necessary condition for the existence of NE, we state the following lemma:

Lemma 1. *For the game $\Gamma_{ca\text{-}2i\text{-}1d\text{-}1f}$ with L links and c channels, if there exists a channel j with more than r links, where $r = \lceil \frac{L}{c} \rceil$, then there always exists a channel $j' \neq j$ such that $L_j - L_{j'} > 1$.*

Proof. We will proof by contradiction that Lemma 1 holds.

The condition in Lemma 1 means that the total number of links, $L = (r-1)c + k$ where $0 < k \leq c$. Hence, $(r-1)c < L \leq rc$. Assuming the lemma does not hold. Then, there exists a channel $j \in \mathcal{C}_{max}$ where $L_j \geq r+1$; and for all other channel $j' \neq j$, $L_j - L_{j'} \leq 1$; i.e., $L_{j'} \geq r$. Summing up the links in all the channels, $L_j + \sum_{j' \neq j} L_{j'} \geq (r+1) + (c-1)r = rc+1$. Since this is greater than the maximum possible L, it is a contradiction. Therefore, Lemma 1 is true. □

We now state the following necessary condition for the existence of pure strategy NE:

Proposition 2. *In the channel assignment game $\Gamma_{ca\text{-}2i\text{-}1d\text{-}1f}$, a strategy profile that results in at least one channel with more than r links, where $r = \lceil \frac{L}{c} \rceil$, is not a pure strategy NE.*

In other words, a necessary condition for a NE outcome is that all the channels can have *at most* r links. We can see that the NE outcome of Proposition 1 satisfies this condition. To prove this proposition, we will show that when a strategy profile results in a channel having more than r links, at least one player can increase his utility by changing his strategy.

Proof. Suppose a strategy profile results in $\mathcal{C}_{max}$ channels such that $L_j > r, \forall j \in \mathcal{C}_{max}$. We consider an arbitrary player i with at least a link in any channel in $\mathcal{C}_{max}$. We will refer to those channels in $\mathcal{C}_{max}$ that player i has a link in as $j_1, j_2, \ldots, j_x$. Let us consider the channel j_1. We know from Lemma 1 that there always exists a channel $j' \neq j_1$ such that $L_{j_1} - L_{j'} > 1$, we move a link of player i from channel j_1 to channel j'. If $x = 1$, then the utility of player i has increased by this operation. If $x > 1$, we can repeat the above operation by another $(x-1)$ times. This is possible because Lemma 1 guarantees the existence of a channel which has at least 2 links fewer than those in $\mathcal{C}_{max}$.

Therefore, player i is able to increase his utility, which means that this cannot be a NE strategy profile. □

Pareto Efficiency

We note that depending on where a player's links are found, a more efficient NE may be possible.

Consider Example 2 shown in Figure 2 with possible channel assignments shown in Figure 3. Figure 3(a) shows a possible strategy profile (NE1) that results in global spreading, and hence a NE. Each player has a utility of 0.25 since they all have links in the $\mathcal{C}_{max} = \{C\}$ channel. A different NE strategy profile (NE2) allows player 3 to get a utility of 0.375, as shown in Figure 3(b). We say that NE2 is more efficient than NE1 as it allows player 3 to get a higher utility without lowering the other players' utilities. Incidentally, Figure 3(c) shows an even more efficient NE outcome that is not global spreading. It can be easily verified that NE3 is also a NE since no player can improve his utility by deviating. Notice that NE3 satisfies the necessary condition in Proposition 2.

From studying this single collision domain example as a game, we know that a way to achieve an equilibrium point is for a network to monitor all the channels to ensure that its channel assignment does not cause any channel to contain more than $\lceil \frac{L}{c} \rceil$ links. Ensuring that there is a global spreading of the links across all channels will also guarantee a NE. Short of using explicit communication, trying to do so is extremely difficult. In Section 4, we describe simulations done to explore the possibility of co-located WMNs arriving at NE outcomes without explicit communication, by using no-regret learning algorithms.

In addition, we learn that there may exist more efficient NE outcomes and it is desirable for networks to reach such outcomes. Again, this is not easy to achieve without explicit communication. We also find that at times, a game can have a social optimal outcome that is not a NE. Briefly, a *social optimal* outcome is one that maximizes the total utility of *all* the players in the game. Consider the game $\Gamma_{ca\text{-2i-1d-1f}}$ with 2 players and 3 channels $\{A, B, C\}$. Player 1 has 3 links and player 2 has 2 links. If the utility of a player is given by his flow rate, it can be shown that a social optimal outcome is realized by player 1 putting all his links in channel A and player 2 putting one link in each of channels B and C. However, this channel allocation is not a NE, because it does not satisfy the condition in Proposition 2. As part of our future work, we plan to characterize pareto efficient NE outcomes, social optimal outcomes and explore ways to achieve them.

4. Simulation

From Section 3.3, we know that there exist NE outcomes in a single collision domain channel assignment game. In this section, we look at whether players can arrive at these equilibrium outcomes by using learning.

We implement the Freund and Schapire algorithm [4], for both the informed (FSI) and naive (FSN) cases, and the Foster and Vohra informed algorithm (FVI) [3], described in Section 2.3. At every iteration, each player evaluates his utility gained during the previous iteration and uses the algorithms to update the weights associated to his strategies. We compare the two different no-regret learning algorithms (FSI and FVI) to evaluate their respective merits and drawbacks. We also compare an informed version of the algorithm (FSI) with its naive counterpart (FSN). In addition, we compare how these no-regret learning algorithms compare against a purely random strategy, where each player simply chooses a strategy randomly at each iteration.

In all our simulations, we have chosen appropriate values of $\alpha = 0.2$ in FSI and FSN, and $\epsilon = 0.1$ in FSN. This paper does not aim to evaluate the performance of different values of α and ϵ. Essentially, α and ϵ determine how much and how fast an algorithm reduces the probability of playing a strategy when it gives a bad utility. A high α and low ϵ causes larger and faster reduction. This may mean faster convergence but also increases the chance of players dropping those strategies that could have formed an efficient strategy profile.

4.1. Simulation Results

In this simulation, we have $n = |\mathcal{N}|$ number of players in a single collision domain. Each player has $l_i = 3$ links and there are $c = 4$ available channels. At each iteration,

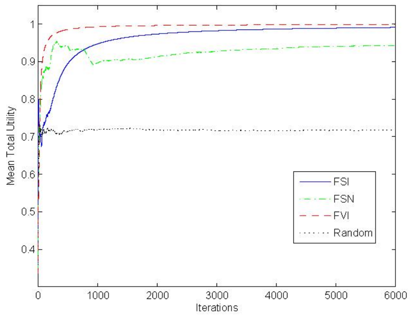


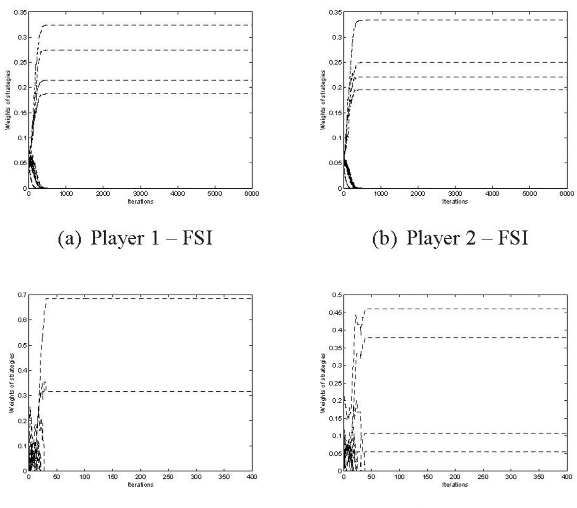
Figure 4. Total mean utilities acquired by two players during a typical simulation run.

as a player updates his weights, we also record the mean utility that the player has acquired so far. This is done by normalizing the total utility the player has acquired since the start of the simulation with the number of iterations that has gone by.

Figure 4 shows a typical simulation run with $n = 2$ players. We present the total mean utility acquired by both players over 6000 iterations. We see that all the algorithms are able to converge to a fix mean utility. Though not shown, we have also collected the mean utilities for individual players and note that each player is able to get a fair share of the total utility over time.

Comparing FSI and FVI, the two informed no-regret learning algorithms, we see that the players are able to get similar utilities in the long run. FVI tends to converge faster, i.e., seemingly unsuccessful strategies are dropped faster in FVI, resulting in stable, long-term utilities. This is confirmed by Figure 5, which shows how the weights associated with each of the player's strategies evolve over time. We see that in FSI (Figures 5(a) and 5(b)), both players converge to playing a fix set of strategies after about 500 iterations. When using FVI, the players' choice of strategies converges within 50 iterations (Figures 5(c) and 5(d)).

Figure 6 shows the proportion of time the NE outcomes occur during the duration of the simulation, computed by normalizing the number of times NE outcomes have occurred with the number of iterations so far. We notice that both the informed algorithms are able to learn to play NE outcomes over time. In all the simulations for multiple players, we find that the set of strategies that each player plays in the long run results in a global spreading of the links across the channels, a NE outcome as described by Proposition 1. With FSN, the players generally are not able to converge to a fix set of strategies to play, resulting in NE strategies only played a certain proportion of the time. Nonetheless, FSN learns to eliminate the strategies that gives low utility for one player whatever strategies the other player play (known as dominated strategies in game the-



(a) Player 1 – FSI (b) Player 2 – FSI

(c) Player 1 – FVI (d) Player 2 – FVI

Figure 5. Weights associated to strategies over time for two players in a collision domain.

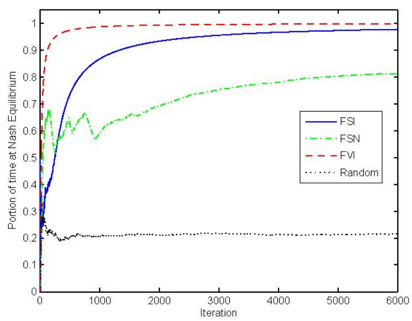


Figure 6. Proportion of time a NE strategy profile is played during a typical simulation with two players.

ory). In all cases, learning outperforms random choosing of channel assignments in terms of utilities and NE outcomes.

Figure 7 shows the total mean utilities of all the players in the game, at the end of 6000 iterations, averaged over 100 independent simulation runs. We investigate the results for $n = 2$ to 5 players. We see that the total mean utilities acquired through FSI and FVI are almost similar, especially when the size of the players is small. When there are 5 players in the game, FVI performs better, but with a larger standard deviation. This is because FVI eliminates seemingly inefficient strategies faster. While it decreases convergence time, efficient strategy profiles may also be missed, leading to lower total utility. A player's utility does not just depend on his strategy, but also the corresponding strategies used by

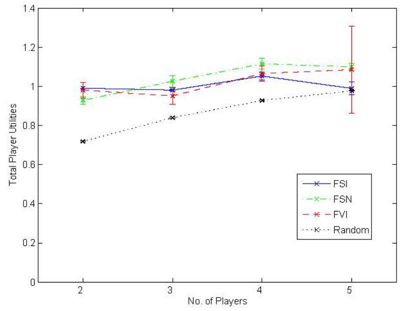


Figure 7. Total mean utilities acquired by the players at the end of 6000 iterations, for different number of players.

other players. The role of learning is to find efficient strategy profiles. If a strategy is dropped before it has a chance to be played against many other strategies, the chance of finding efficient strategy profiles is reduced. Of course, when FVI happens to get a highly efficient strategy profile, it will be played consistently, leading to much higher utility. This accounts for the higher standard deviation.

In comparing FSI with FSN, we notice that as we increase the number of players in the game, the naive scheme actually performs better. This counter-intuitive observation can be attributed to the fact that in the informed algorithms, the players' strategies converge to a small set of strategies, as demonstrated by the results in Figure 5. In FSN, the set of strategies played do not generally converge, though the most inefficient ones (dominated strategies) are eliminated. Therefore, certain players are able to get much higher utilities that are not NE outcomes in some iterations. This accounts for the slightly higher utility in cases where there are larger number of players.

4.2. Discussion

We make the following observations based on our simulation results:

1. No-regret learning algorithms allow players in our channel assignment coexistence game to learn to play NE outcomes. Hence, there is a potential for using them to solve the Coexistence Problem.
2. A learning algorithm that converges faster to playing NE strategies is useful in dynamic scenarios, e.g. when the traffic patterns of the networks changes constantly. However, the fast convergence may cause the networks to miss out on some optimal (pareto efficient) outcomes.
3. A naive learning algorithm performs worse than informed learning when the set of players is small, as the players do not have enough information to converge to efficient strategies. However, this lack of information may be advantageous when the size of the player set is large.
4. Our simulation assumes that all the players update their

weights at the same time. This is unlikely in practical scenario. We will look into the effects of asynchronicity in updates as part of our future work.

5. The game we have studied in this paper relates to channel assignment with one decision maker within each WMN. Practically, this can introduce delays into the learning and decision process. We plan to investigate the effects of such delays and look into the possibility of a distributed approach, where each link learns and makes the decision.

5. Related Work

In [1] and [9], the authors motivated the problem of coexistence in wireless networks of the same technology (WiFi) but under different management control. Unlike these works, which focus on single hop networks, we study the Coexistence Problem among independent multihop WMNs. Our general model could also be applied to different technologies and to inter-technology coexistence.

There exists work that looks at using channel assignment in WMNs with multiple interfaces to reduce network interference. See for example, [13] and the references cited within. They focus on channel assignment schemes implemented within a single multi-radio WMN. In contrast, the channel assignment approach described in this paper applies to multiple independent WMNs. In [12], Ramachandran *et. al.* propose a centralized channel assignment solution that reduces both the intra- and inter-network interferences. They consider only interference from single-hop links, while we have looked at co-located multihop WMNs. In addition, we have proposed a general co-existence framework that encompasses more than just channel assignment.

Game theory has been used to study many aspects of networks. Félegyházi *et. al.* [2] use game theory to study the coexistence strategies of competitive single-hop networks using channel allocation. The distinction in our work is that we are looking at competitive multihop networks, which is a different problem altogether. Also, Nie and Comaniciu [10] apply game theory to dynamically allocate channels in networks with cognitive radios and propose using no-regret learning algorithms to solve the problem. Again, the problem they look at is confined to only single-hop links while we are looking at multihop networks.

6. Conclusion

In this paper, we motivate the need to study and manage the coexistence of co-located independent WMNs. Reducing the interference caused by links from other WMNs requires non-cooperative approaches. We propose a non-cooperative game theoretic approach to solve this Coexistence Problem, consisting of a general framework. As an example, we have used the framework to describe a channel assignment game and study a special case of multiple WMNs in a single collision domain. We apply no-regret learning algorithms as a practical means to solve the problem. Simulation results show that no-regret learning allows multiple networks to learn to play strategies that arrive to Nash Equilibrium outcomes.

This work represents just the first step in our look at the Coexistence Problem in WMNs. As part of our future work, we plan to look at more complex network topologies, traffic patterns and investigate the effects of asynchronicity and delay on the performance of the no-regret algorithms. By using the general framework, we also hope to look at other approaches to solve the Coexistence Problem.

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