Relcon: A Tensor Model of Relational Categorization

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Abstract

The Relcon algorithm models category formation using relational storage and retrieval mechanisms with a Tensor memory. Relcon settles on a category structure that matches prototypicality characteristics of human categories. A tensor intersection operation simulates the influence of context on category structure and on similarity. The results have implications for models of binding and representation, and provide a framework for answering the current call for relation-based representations for categories.

Introduction

While various theories have been formulated to describe how categorical structure is formed in memory (e.g., Medin 1989; Markman & Stilwell, 2001, Gentner & Kurtz, 2005), these have not been implemented and tested via modeling. We present Relcon, (Relational Concept) a model of relational representation used for forming categories, able to simulate sophisticated characteristics such as context sensitivity and context dependent similarity.

The Tensor Memory Model

If the relational-based descriptions of categories are valid, then models that can represent hierarchical relational structure should also be able to represent categories. The model is based on the tensor memory model (Humphreys, Bain & Pike, 1989), and the role-filler binding tensor model of Smolensky (1990). These have been utilized by Halford, Wilson and Phillips (1998) in a tensor model of working memory processing limitations and analogy.

The Relcon Procedure

Assume a 3-dimensionsal memory **T** contains the following propositions about furniture:

T = *Made_of(chair, wood)* + *Stands_on(chair, floor)* + *Lives_in(chair, living room)* + *Made_of(desk, wood)* + *Stands_on(desk, floor)* + *Lives_in(desk, study)* + *Made_of(vase, glass)* + *Lives_in(vase, living room)* + *Stands_on(vase, table)* + *Large(Table)* + *Small(Vase).*

Each proposition in the tensor memory can be viewed as a relation, or as being made of items (e.g. *chair*) bound to relational features (e.g. *Made_of wood*). Thus the tensor containing the relations *Made_of(chair, wood)* and *Stands_on(chair, floor)* and *Small(vase)*, would be: **T** = $v_{Made\_of} \otimes v_{chair} \otimes v_{wood} + v_{Stands\_on} \otimes v_{chair} \otimes v_{floor} + v_{Small} \otimes v_{Vase} \otimes v_{neutral}$. The Relcon procedure uses the constructed memory for extracting category information, by applying two main operations that probe the memory, (Wiles, Humphreys, Bain and Dennis, 1990). The first operation, called 'probing with an item' is where **T** is probed with items (e.g. *chair*) that form part of the relations already stored, using a form of the dot-product. This bundle of relational features (attributes) can then be input to the memory, to find out what other objects in the memory share these same attributes. This is called 'probing with a tensor'. These two operations are used by the Recon model (Gray, 2003) to describe how an object that is used to probe memory can be expanded to the category to which it belongs. In Step 1, memory **T** is probed with items (e.g., *chair*) to retrieve a set of relational features describing those items ($\mathbf{v} \bullet \mathbf{T} = \mathbf{U}$; e.g. **U** = *Made_of(wood)* + *Stands_on(floor)* + *Lives_in(chair, living room)*). In step 2, this relational feature set, is used to probe memory to retrieve a set of items that share these features ($\mathbf{T} \bullet \mathbf{U} = \mathbf{v}$; e.g. **v** = *3×chair, 2× desk, vase*; each item being weighted by how many relational features it has in common with *chair*). In step 3, **v** is normalized, and then steps 1 to 3 are repeated until convergence. The resultant item bundle is said to represent the category; i.e., a set of items sharing relational features. This basic procedure can be modified by using a dot product operation to simulate Rosch & Mervis' (1975) prototypicality results, and by using a pruning mechanism to simulate category separation and hierarchical category structure (see Gray 2003).

Context Sensitivity Context has been shown to change the goodness of example (GOE) distribution on category members (Roth & Shoben, 1983). Using Relcon, we have developed an 'implied feature' explanation, which assumes that context indicates a number of weighted features. For example, in the sentence: 'The furniture was unfolded in the backyard', the implied relational features for furniture, due to being in the context of 'backyard', might be *Lives_in(____, backyard),* and *Unfolds(_____).* We propose that relations that share these relational features become more heavily weighted than others in the category, causing a restructuring of the GOE distribution. This process is modeled by using an additional tensor, **C**, to represent the context. **C** has the same structure as **T**, but each unit in **C** represents a single relational feature, taking a value greater-than 1 if context implies that feature, and less-than 1 if context implies the lack of that feature, and 0 if no context is implied. When a tensor intersection operation ($\mathbf{U} = \mathbf{U} \cap \mathbf{C}$; Wiles, Humphreys, Bain & Dennis, 1991) is inserted between Step 1 and Step 2 of the Relcon procedure, this causes the relational features being retrieved from **T** in Step 1 to be weighted by a corresponding increase/decrease, depending on their value in **C**. This adjusted set of relational features is then used to probe **T** to produce an object bundle sharing those features. At the end of the Relcon procedure, the items that emerge from the expanded set are now more likely to share context features with one another as well as common features. For both the *vehicl*e and *furniture* categories, two relational features were randomly chosen to be the context implied features for the category, and were given different weights in the context-implied tensor, **C.** Within the resultant stable categories, items associated with more features or more highly weighted context features received the largest increase in prototypicality.

Context Dependent Similarity: To determine the similarity between 2 objects, say *table* and *chair,* the tensor memory can be probed with each item, producing two relational feature bundles, **U** and **S**, which are then compared using a normalized dot product. Thus the similarity between any two elements is based on the common relational features owned by those items within the memory. Relcon results showed that the average similarity between items in the *furniture* category was 0.22, in the *vehicles* category it was 0.23, and within the sub-categories it was 0.49 (since these items shared more relational features).

A contextualization operation can be added between Steps 1 and 2 of Relcon and the difference in similarity measures that are produced with and without the context can be calculated. The results showed that if two items possessed a context feature, their similarity increased by 0.22, but when only one item of a pair had had a context feature, their similarity decreased by 0.01 (reflecting that the items were becoming more different due to the context).

Summary

The Relcon procedure shows how the apparent existence of categories can emerge as a result of the representational topology that occurs in a knowledge domain when items are represented as relational item-attribute bindings. Relcon is able to simulate properties of human categories, including context sensitivity and context dependent similarity.

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