Formal Methods for Probabilistic Systems

Annabelle McIver
Carroll Morgan

- Source-level program logic
- Meta-theorems for loops
- Examples
- Relational operational model
- Almost-certain termination
- Mu-calculus, temporal logic and games

British *EPSRC* (Oxford), then Australian *ARC* (Macquarie/UNSW), 1994-continuing.

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Abstraction, refinement and proof for probabilistic systems

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- Source-level program logic
  - Introduction to probabilistic-program logic
  - Systematic presentation via structural induction
  - Layout of calculations in practice
  - Random variables and expected values
  - The impact of demonic choice

Abstraction, refinement and proof for probabilistic systems

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- Source-level program logic
- Meta-theorems for loops
  - Introduction and example
  - Review of rules for standard loops
  - Rules for probabilistic loops
  - Analysis of an example
  - Probability-one termination
  - The Zero-One Law

Abstraction, refinement and proof for probabilistic systems

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- Source-level program logic
- Meta-theorems for loops
- Examples
  - Probabilistic amplification
  - Uniform selection

Abstraction, refinement and proof for probabilistic systems

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- Source-level program logic
- Meta-theorems for loops
- Examples
- Relational operational model
  - Standard, deterministic, terminating ..... *functions*
  - Standard, deterministic, non-terminating ..... *functions with* $\perp$
  - Standard, demonic, non-terminating ..... *relations with* $\perp$
  - Standard powerdomains ..... *closure*
  - Probabilistic powerdomains ..... *sub-distributions*
  - Demonic, probabilistic powerdomains ..... *sets of...*
  - Examples ..... *program geometry*

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- Source-level program logic
- Meta-theorems for loops
- Examples
- Relational operational model
- Almost-certain termination
  - "Herman's Graph"
  - Probabilistic variant rule
  - Termination of Herman's Graph
  - Herman's Ring
  - Termination of Herman's Ring
  - Expected time to stability; comparison with model-checking (*PRISM*)

Abstraction, refinement and proof for probabilistic systems

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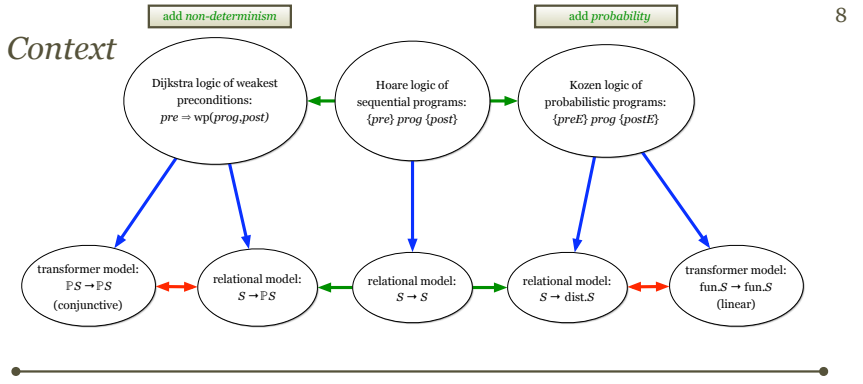
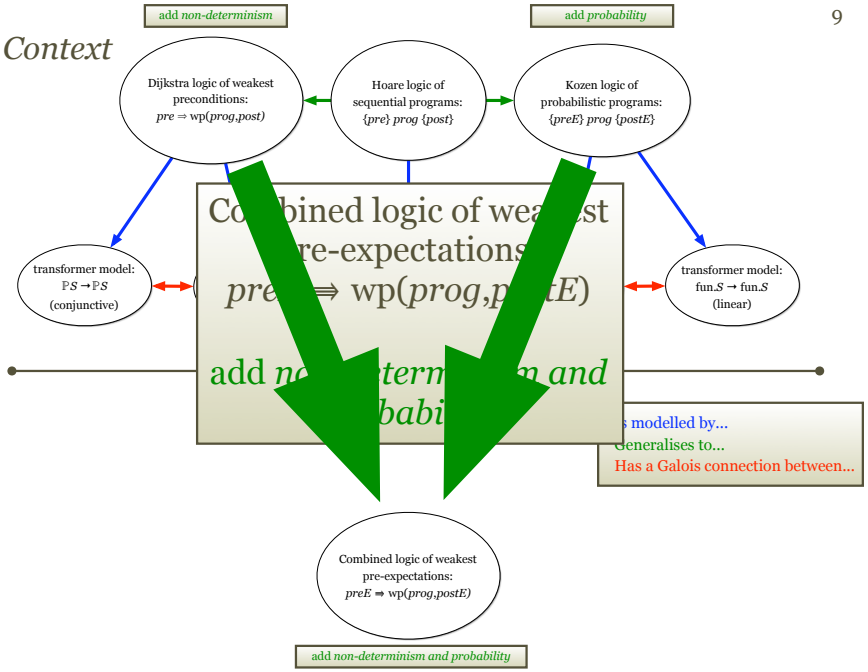
- Source-level program logic
- Meta-theorems for loops
- Examples
- Relational operational model
- Almost-certain termination
- Mu-calculus, temporal logic and games
  - Two-player probabilistic games, and their value
  - The *qM*μ and its game interpretation
  - *Minimax* and *maximin* for games
  - The denotational interpretation of *qM*μ
  - *Theorem*: the equivalence of games and denotations
  - Example: solution via *Mathematica* and *PRISM*



Is modelled by...
Generalises to...
Has a Galois connection between...

Context



Context