COMP1927 17x1
Computing 2

Complexity

Problems, Algorithms, Programs and Processes

- Problem: A problem that needs to be solved
- Algorithm: Well defined instructions for completing the problem
- Program: Implementation of the algorithm in a particular programming language
- Process: An instance of the program as it is being executed on a particular machine

Analysis of software

What makes “good” software?

- **Correct:** returns expected result for all valid inputs

  guaranteed through formal specification
- **Reliable:** behaves "sensibly" for non-valid inputs/errors and handled gracefully

  Correctness/Reliability ensured through robust testing
- **Maintainable:** clear, well-structure code

  Coding style, recommended conventions
- **Efficient:** produces results quickly (even for large inputs)

  Efficiency determined through algorithm efficiency

We may sometimes also be interested in other measures

- memory/disk space, network traffic, disk IO etc

Algorithm Efficiency

- The algorithm is by far the most important **determinant** of the efficiency of a program
- Algorithm efficiency determined through **algorithm analysis,** can save factors of thousands or millions in the running time
- Small speed ups in terms of operating systems, compilers, computers and implementation details are irrelevant. They may give small speed ups but usually only by a small constant factor

Algorithm Analysis

Branch of computer science to determine choice of the best algorithm for a particular task.

- Mathematical Analysis
  - Analyse asymptotic time complexity – the limiting behaviour of the execution time of an algorithm when the size of the problem goes to infinity
  - Usually denoted in big-O notation.
  - Can be done at design-stage (pseudo-code)
- Empirical Analysis
  - Post-implementation stage
  - Once it is implemented and correct, evaluate which algorithm takes longer e.g., using the time command

Timing

- Note we are not interested in the absolute time it takes to run.
- We are interested in the relative time it takes as the problem increases
- Absolute times differ on different machines and with different languages

Time Complexity Analysis

- Enables us to understand the performance of **algorithms**
- Define a function to characterize execution cost (≅time)

  - Identify the core operation in the algorithm

  - Identify the value to measure the size of the input (***N***) (e.g. #items in data structure, length of input file, no of chars in string etc)

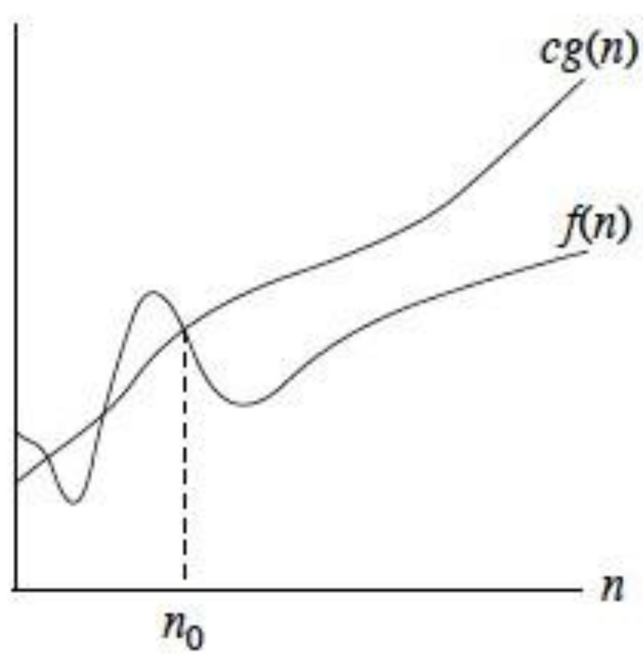
  - Express cost in terms of #operations = *f(n)*, which is the time-complexity as a function of input size

- Shows how the cost increases with increase in input size
- Is the algorithm feasible for 100, 10000, 100000 ?

Big O-notation Formal Definition

The big O-notation is used to classify the work complexity of algorithms

Definition: A function $f(n)$ is said to be in (the set) $O(g(n))$ if there exist constants c and N_o such that $f(n) < c * g(n)$ for all $n > N_o$



Informal Definition of Big-O Notation

- **Big-O notation r**epresents the asymptotic **worst case** (unless stated otherwise) time complexity
- Big-O expressions do not have constants or low-order terms as when n gets larger these do not matter
- For example: For a problem of size n, if the cost of the worst case is

  $1.5n^2 + 3n + 10$

  in Big-O notation would be $O(n^2)$

Exercise: Time Complexity

Example: finding max value in an unsorted array

```
int findMax(int a[], int N) {
    int i, max = a[0];
     for (i = 1; i < N; i++)
            if (a[i] > max) max = a[i]; return max;
     }
```

Core operation? ... compare **`a[i]`** to **`max`**

How many times? ... clearly **`N-1`** ... $O(n)$

Execution cost grows linearly (i.e. 2 × #elements ⇒ 2 × cost)

Exercise: Time Complexity

Example: finding max value in a sorted array

```
int findMax(int a[], int N)  {
        return a[N-1];
}
```

No iteration needed; max is always last.
Core operation? ... index into array
How many times? ... once ... *O(1)*
Execution cost is constant (same regardless of #elements)

Predicting Time

- If I know my algorithm is quadratic and I know that it takes 1.2 seconds to run on a data set of size 1000
- Approximately how long would you expect to wait for a data set of size 2000?
- What about 10000?
- What about 100000?

Empirical Analysis

- Use the ‘time’ command in linux.

Run on different sized inputs

```
time ./prog < input > /dev/null
```

not interested in real-time

interested in user-time

What is the relationship between

- input size
- time

Size of input(n)	Time
100000	
1000000	
10000000	
100000000	

Big-O Notation

- All constant functions are in O(1)
- All linear functions are in O(n)
- All logarithmic function are in the same class O(*log(n)*)
  - O($log_2(n)$) = O($log_3(n)$)= ....
    - (since $log_b(a) * log_a(n) = log_b(n)$)
- We say an algorithm is O($g(n)$) if, for an input of size n, the algorithm requires $T(n)$ steps, with $T(n)$ in O($g(n)$), and O($g(n)$) minimal
  - binary search is $O(log(n))$
  - linear search is *O(N)*
  - finding maxium in an unsorted sequence is *O(N)*

Common Categories

- $O(1)$: constant - instructions in the program are executed a fixed number of times, independent of the size of the input
- $O(\log N)$: logarithmic - some divide & conquer algorithms with trivial splitting and combining operations
- $O(N)$: linear - every element of the input has to be processed, usually in a straight forward way
- $O(N * \log N)$: Divide &Conquer algorithms where splitting or combining operation is proportional to the input
- $O(N^2)$: quadratic. Algorithms which have to compare each input value with every other input value. Problematic for large input
- $O(N^3)$: cubic, only feasible for very small problem sizes
- $O(2^N)$: exponential, of almost no practical use

Complexity Matters

n	log n	nlogn	n^2	2^n
10	4	40	100	1024
100	7	700	10000	1.3E+30
1000	10	10000	1000000	REALLY BIG
10000	14	140000	100000000	
100000	17	1700000	10000000000	
1000000	20	20000000	1000000000000	

Exercise

What would be the time complexity of inserting an element at the beginning of

- a linked list
- an array

What would be the time complexity of inserting an element at the end of

- a linked list
- an array

Searching

An extremely common application in computing

- given a (large) collection of **items** and a **key** value
- find the item(s) in the collection containing that key
- an item is defined as {key, val_1, val_2} (i.e. a struct)
- key = value used to distinguish items (e.g. student ID)

Keys may be ...

- primary ... key value uniquely identifies one item
- secondary ... many items may have the same key value

Applications: Google, databases, ...

Searching (cont)

If we are dealing largely with primary keys, then search problem can be encapsulated as:

```
Item search(Collection c, Key k) { ... }
```

Possible return values are:

- an **Item**
- “NOT FOUND” value

For secondary keys return an array (possibly empty) of matching items

```
Item *search(Collection c, key k, int *nmatches) {...}
```

Searching in Linear Structures

Search in an unsorted array or list

```
Item searchArray(Key k, Item a[], int n)  {
          int i;
           for (i = 0; i < n; i++)
               { if (a[i].key == k)
                        return a[i];
               }
               return NOT_FOUND;
   }
Item searchList(Key k, List L)  {
    List n;
    for (n = L; n != NULL; n = n->next)
        { if (n->key) == k)
               return n->data;
        }
        return NOT_FOUND;
}
```

Linear Search Cost

- Core operation? … compare **`a[i]`**.key to k
- What is the worst cast cost?
- How many comparisons between data instances were made?
- How many times does each line run in the worst case?

  C0: line 2: For loop n+1 times

  C1: line 3: n comparisons

  C2: line 4: 0 times (worst case)

  C3: line 5: 1 time (worst case)

  Total: C0(n+1) + C1(n) + C3 = O(n)
- For an unsorted sequence that is the best we can do

Searching in a Sorted Array

Given an array a of N elements, with a[i] <= a[j] for any pair of indices i,j, with i <= j < N,

- search for a key k in the array

```
Item searchSortedArray(Key k,  Item a[], int n) {
      int found = 0;
      int finished = 0;
      int i = 0;
      while ((i < N) && (!found) && (!finished)){
            found = (a[i].key == k);
            finished = (k < a[i].key);    exploit the fact that a is sorte
             i++;
      }
```

Searching in a Sorted Array

How many steps are required to search an array of N elements

- Best case: $T_N = 1$
- Worst case: $T_N = N$
- Average: $T_N = N/2$

Still a linear algorithm, like searching in a unsorted array

Binary Search in a sorted array

- We start in the middle of the array:
- if a[N/2] == e, we found the element and we’re done
- and, if necessary, `split’ array in half to continue search
- if a[N/2] < e, continue search on a[0] to a[N/2 -1]
- if a[N/2] > e, continue search on a[N/2+1] to a[N-1]
- This algorithm is called binary search.

Binary Search (cont)

- We maintain two indices, l and r, to denote leftmost and rightmost array index of current part of the array
  - initially l=0 and r=N-1
- iteration stops when:
  - left and right index define an empty array, element not found
  - Eg l > r
  - a[(l+r)/2] holds the element we're looking for
- if: a[(l+r)/2] is larger than element, continue search on left

  a[l]..a[(l+r)/2-1]

  else continue search on right

  a[(l+r)/2+1]..a[r]

Binary Search

- How many comparisons do we need for
- an array of size N?
  - Best case:
    - $T_N = 1$
  - Worst case:
    - $T_1 = 1$
    - $T_N = 1 + T_{N/2}$
    - $T_N = \log_2 N + 1$
    - $O(\log n)$
- Binary search is a
  - logarithmic algorithm