Growable Array-based Stack

- In a push operation, when the array is full, instead of throwing an exception, we can replace the array with a larger one
- How large should the new array be?
  - incremental strategy: increase the size by a constant c
  - doubling strategy: double the size

Algorithm *push*(o)
  if $t = S.length - 1$ **then**
    $A \leftarrow$ new array of size …
    for $i \leftarrow 0$ **to** t **do**
      $A[i] \leftarrow S[i]$
    $S \leftarrow A$
  $t \leftarrow t + 1$
  $S[t] \leftarrow o$

Comparison of the Strategies

- We compare the incremental strategy and the doubling strategy by analyzing the total time $T(n)$ needed to perform a series of n push operations
- We assume that we start with an empty stack represented by an array of size 1
- We call amortized time of a push operation the average time taken by a push over the series of operations, i.e., $T(n)/n$

Incremental Strategy Analysis

- We replace the array $k = n/c$ times
- The total time $T(n)$ of a series of n push operations is proportional to

$$n + c + 2c + 3c + 4c + \ldots + kc =$$
$$n + c(1 + 2 + 3 + \ldots + k) =$$
$$n + ck(k + 1)/2$$

- Since c is a constant, $T(n)$ is $O(n + k^2)$, i.e., $O(n^2)$
- The amortized time of a push operation is $O(n)$

Doubling Strategy Analysis

- We replace the array $k = \log_2 n$ times
- The total time $T(n)$ of a series of n push operations is proportional to

$$n + 1 + 2 + 4 + 8 + \ldots + 2^k =$$

$$n + 2^{k+1} - 1 = 2n - 1$$

- $T(n)$ is $O(n)$
- The amortized time of a push operation is $O(1)$

geometric series