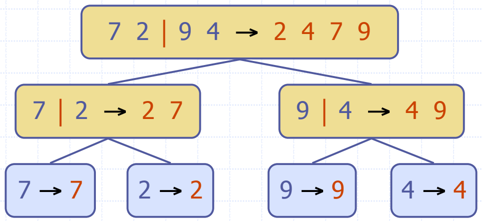
Merge Sort



Divide-and-Conquer (§ 10.1.1)

- Divide-and conquer is a general algorithm design paradigm:
  - Divide: divide the input data S in two disjoint subsets S_1 and S_2
  - Recur: solve the subproblems associated with S_1 and S_2
  - Conquer: combine the solutions for S_1 and S_2 into a solution for S
- The base case for the recursion are subproblems of size 0 or 1 (sometimes 1 or 2)
- Merge-sort is a sorting algorithm based on the divide-and-conquer paradigm
- Like heap-sort
  - It uses a comparator
  - It has $O(n \log n)$ running time
- Unlike heap-sort
  - It does not use an auxiliary priority queue
  - It accesses data in a sequential manner (suitable to sort data on a disk)

Merge-Sort (§ 10.1)

- Merge-sort on an input sequence S with n elements consists of three steps:
  - Divide: partition S into two sequences S_1 and S_2 of about $n/2$ elements each
  - Recur: recursively sort S_1 and S_2
  - Conquer: merge S_1 and S_2 into a unique sorted sequence

Algorithm *mergeSort(S, C)*
  Input sequence S with n elements, comparator C
  Output sequence S sorted according to C
  if $S.size() > 1$
    $(S_1, S_2) \leftarrow partition(S, n/2)$
    $mergeSort(S_1, C)$
    $mergeSort(S_2, C)$
    $S \leftarrow merge(S_1, S_2)$

Merging Two Sorted Sequences

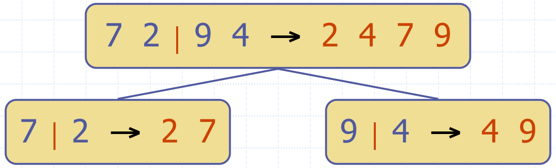
- The conquer step of merge-sort consists of merging two sorted sequences A and B into a sorted sequence S containing the union of the elements of A and B
- Merging two sorted sequences, each with $n/2$ elements and implemented by means of a doubly linked list, takes $O(n)$ time

Algorithm *merge(A, B)*
  Input sequences A and B with $n/2$ elements each
  Output sorted sequence of $A \cup B$

  $S \leftarrow$ empty sequence
  while $\neg A.isEmpty() \wedge \neg B.isEmpty()$
    if $A.first().element() < B.first().element()$
      $S.insertLast(A.remove(A.first()))$
    else
      $S.insertLast(B.remove(B.first()))$
  while $\neg A.isEmpty()$
  $S.insertLast(A.remove(A.first()))$
  while $\neg B.isEmpty()$
  $S.insertLast(B.remove(B.first()))$
  return S

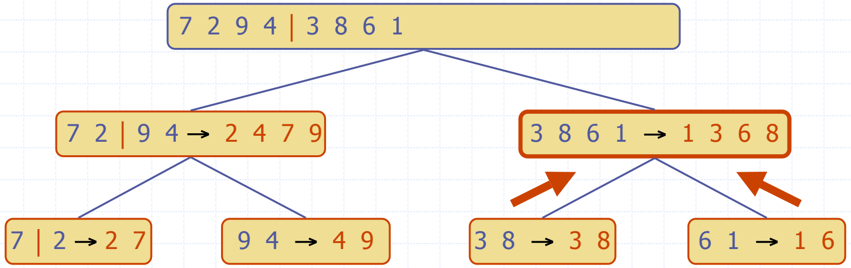
Merge-Sort Tree

- An execution of merge-sort is depicted by a binary tree
  - each node represents a recursive call of merge-sort and stores
    - unsorted sequence before the execution and its partition
    - sorted sequence at the end of the execution
  - the root is the initial call
  - the leaves are calls on subsequences of size 1 or 2



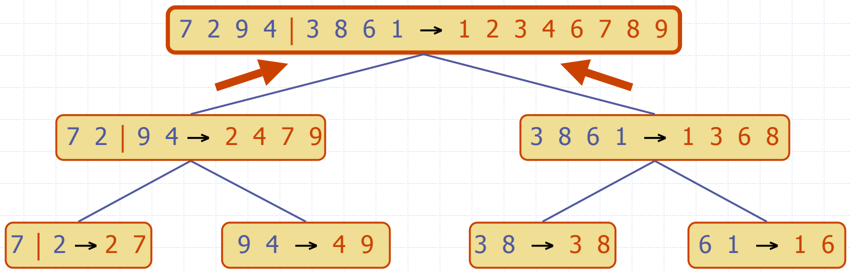
Execution Example (cont.)

- Recursive call, …, merge, merge



Execution Example (cont.)

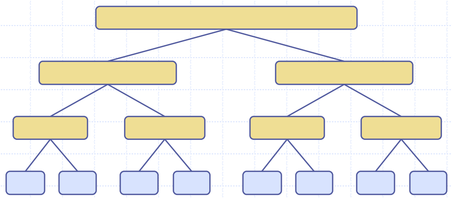
- Merge



Analysis of Merge-Sort

- The height h of the merge-sort tree is $O(\log n)$
  - at each recursive call we divide in half the sequence,
- The overall amount or work done at the nodes of depth i is $O(n)$
  - we partition and merge 2^i sequences of size $n/2^i$
  - we make 2^{i+1} recursive calls
- Thus, the total running time of merge-sort is $O(n \log n)$

depth	#seqs	size
0	1	n
1	2	$n/2$
i	2^i	$n/2^i$
…	…	…



Nonrecursive Merge-Sort

merge runs of length 2, then 4, then 8, and so on

merge two runs in the in array to the out array

```
public static void mergeSort(Object[] orig, Comparator c) { // nonrecursive
  Object[] in = new Object[orig.length]; // make a new temporary array
  System.arraycopy(orig,0,in,0,in.length); // copy the input
  Object[] out = new Object[in.length]; // output array
  Object[] temp; // temp array reference used for swapping
  int n = in.length;
  for (int i=1; i < n; i*=2) { // each iteration sorts all length-2*i runs
    for (int j=0; j < n; j+=2*i)  // each iteration merges two length-i pairs
      merge(in,out,c,j,i); // merge from in to out two length-i runs at j
    temp = in; in = out; out = temp; // swap arrays for next iteration
  }
  // the "in" array contains the sorted array, so re-copy it
  System.arraycopy(in,0,orig,0,in.length);
}
protected static void merge(Object[] in, Object[] out, Comparator c, int start,
    int inc) { // merge in[start..start+inc-1] and in[start+inc..start+2*inc-1]
  int x = start; // index into run #1
  int end1 = Math.min(start+inc, in.length); // boundary for run #1
  int end2 = Math.min(start+2*inc, in.length); // boundary for run #2
  int y = start+inc; // index into run #2 (could be beyond array boundary)
  int z = start; // index into the out array
  while ((x < end1) && (y < end2))
    if (c.compare(in[x],in[y]) <= 0) out[z++] = in[x++];
    else out[z++] = in[y++];
  if (x < end1) // first run didn't finish
    System.arraycopy(in, x, out, z, end1 - x);
  else if (y < end2) // second run didn't finish
    System.arraycopy(in, y, out, z, end2 - y);
}
```