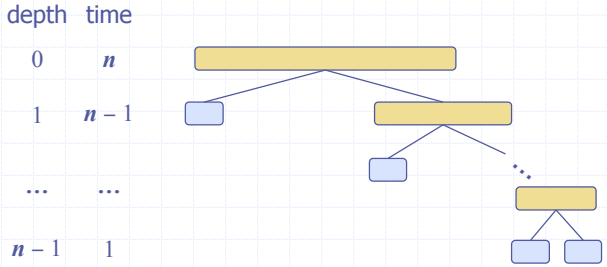
Worst-case Running Time

- The worst case for quick-sort occurs when the pivot is the unique minimum or maximum element
- One of L and G has size $n - 1$ and the other has size 0
- The running time is proportional to the sum

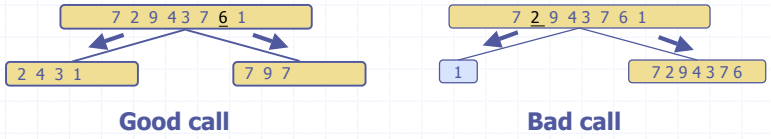
$$n + (n - 1) + \ldots + 2 + 1$$

- Thus, the worst-case running time of quick-sort is $O(n^2)$



Expected Running Time

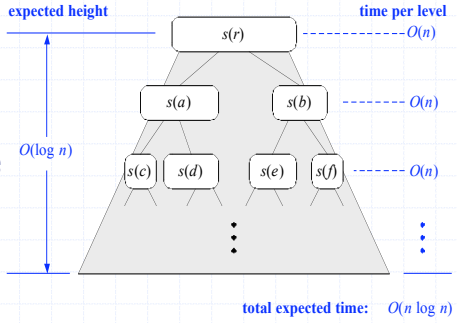
- Consider a recursive call of quick-sort on a sequence of size s
  - **Good call:** the sizes of L and G are each less than $3s/4$
  - **Bad call:** one of L and G has size greater than $3s/4$



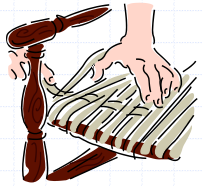
- A call is good with probability $1/2$
  - 1/2 of the possible pivots cause good calls:

Expected Running Time, Part 2

- Probabilistic Fact: The expected number of coin tosses required in order to get k heads is $2k$
- For a node of depth i, we expect
  - $i/2$ ancestors are good calls
  - The size of the input sequence for the current call is at most $(3/4)^{i/2}n$
- Therefore, we have
  - For a node of depth $2\log_{4/3} n$, the expected input size is one
  - The expected height of the quick-sort tree is $O(\log n)$
- The amount or work done at the nodes of the same depth is $O(n)$
- Thus, the expected running time of quick-sort is $O(n \log n)$



In-Place Quick-Sort



- Quick-sort can be implemented to run in-place
- In the partition step, we divide into two non-empty parts (less than or equal to pivot and greater than or equal to pivot).
- Recursively divide each part into two smaller parts, etc.

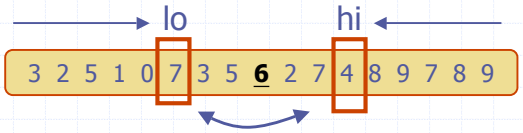
In-Place Partitioning

- Perform the partition using indices lo & hi to split S into two non-empty subsets ≤ pivot and ≥ pivot.

lo hi

3 2 5 1 0 7 3 5 **6** 2 7 4 8 9 7 8 9 (pivot = 6)

- Repeat until j and k cross:
  - Scan lo to the right until find an element ≥ x.
  - Scan hi to the left until find an element ≤ x.
  - Swap elements at indices lo and hi



Java Code

```
static void qSort( int a[], int lo0, int hi0 )  {
    int lo  = lo0;
    int hi  = hi0;
    int mid  = ( lo + hi ) / 2 ;
    int pivot = a[mid];

    // partition array in-place into low and high parts
    while( lo <= hi ) {
       while(( lo < hi0 )&&( a[lo] < pivot )) {
          lo++;
       }
       while(( hi > lo0 )&&( a[hi] > pivot )) {
          hi--;
       }
       if( lo <= hi ) {
          swap( a, lo, hi );
          lo++;
          hi --;
       }
    }
    // sort the left and right parts recursively
    if( hi > lo0 ) {
       qSort( a, lo0, hi );
    }
    if( lo < hi0 ) {
       qSort( a, lo, hi0 );
    }
}
```