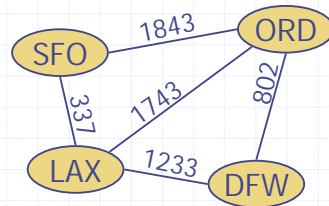


Graphs

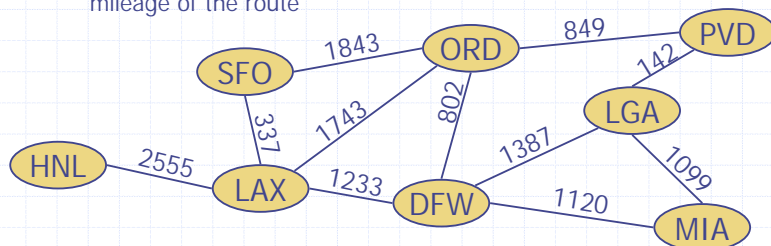


Outline and Reading

- ◆ Graphs (§6.1)
 - Definition
 - Applications
 - Terminology
 - Properties
 - ADT
- ◆ Data structures for graphs (§6.2)
 - Edge list structure
 - Adjacency list structure
 - Adjacency matrix structure

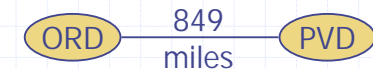
Graph

- ◆ A graph is a pair (V, E) , where
 - V is a set of nodes, called **vertices**
 - E is a collection of pairs of vertices, called **edges**
 - Vertices and edges are positions and store elements
- ◆ Example:
 - A vertex represents an airport and stores the three-letter airport code
 - An edge represents a flight route between two airports and stores the mileage of the route



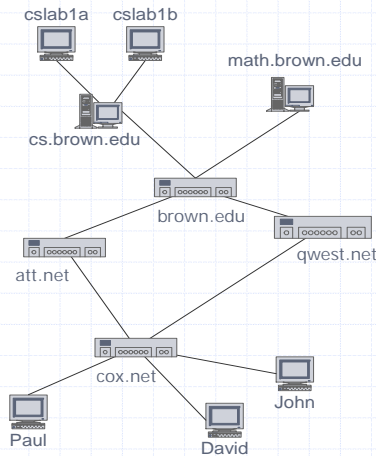
Edge Types

- ◆ Directed edge
 - ordered pair of vertices (u,v)
 - first vertex u is the origin
 - second vertex v is the destination
 - e.g., a flight
- ◆ Undirected edge
 - unordered pair of vertices (u,v)
 - e.g., a flight route
- ◆ Directed graph
 - all the edges are directed
 - e.g., route network
- ◆ Undirected graph
 - all the edges are undirected
 - e.g., flight network



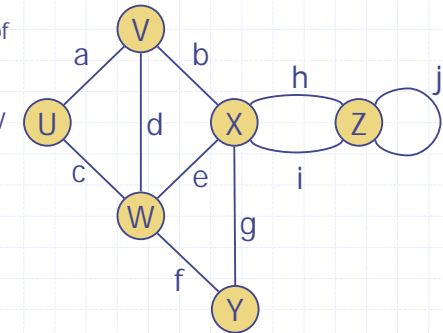
Applications

- ◆ Electronic circuits
 - Printed circuit board
 - Integrated circuit
- ◆ Transportation networks
 - Highway network
 - Flight network
- ◆ Computer networks
 - Local area network
 - Internet
 - Web
- ◆ Databases
 - Entity-relationship diagram



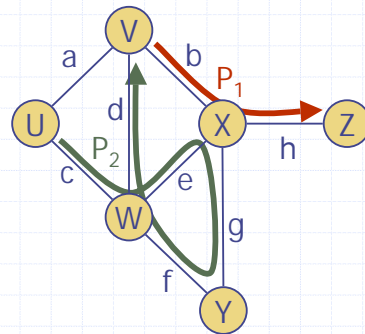
Terminology

- ◆ End vertices (or endpoints) of an edge
 - U and V are the endpoints of a
- ◆ Edges incident on a vertex
 - a, d, and b are incident on V
- ◆ Adjacent vertices
 - U and V are adjacent
- ◆ Degree of a vertex
 - X has degree 5
- ◆ Parallel edges
 - h and i are parallel edges
- ◆ Self-loop
 - j is a self-loop



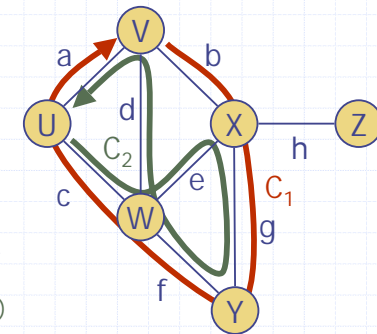
Terminology (cont.)

- ◆ Path
 - sequence of alternating vertices and edges
 - begins with a vertex
 - ends with a vertex
 - each edge is preceded and followed by its endpoints
- ◆ Simple path
 - path such that all its vertices and edges are distinct
- ◆ Examples
 - $P_1 = (V, b, X, h, Z)$ is a simple path
 - $P_2 = (U, c, W, e, X, g, Y, f, W, d, V)$ is a path that is not simple



Terminology (cont.)

- ◆ Cycle
 - circular sequence of alternating vertices and edges
 - each edge is preceded and followed by its endpoints
- ◆ Simple cycle
 - cycle such that all its vertices and edges are distinct
- ◆ Examples
 - $C_1 = (V, b, X, g, Y, f, W, c, U, a, \rightarrow)$ is a simple cycle
 - $C_2 = (U, c, W, e, X, g, Y, f, W, d, V, a, \rightarrow)$ is a cycle that is not simple



Properties

Property 1

$$\sum_v \text{deg}(v) = 2m$$

Proof: each endpoint is counted twice

Property 2

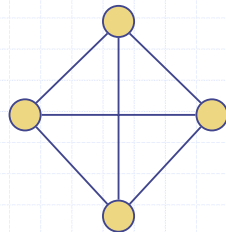
In an undirected graph with no self-loops and no multiple edges

$$m \leq n(n-1)/2$$

Proof: each vertex has degree at most $(n-1)$

Notation

- n number of vertices
- m number of edges
- $\text{deg}(v)$ degree of vertex v



Example

- $n = 4$
- $m = 6$
- $\text{deg}(v) = 3$

Main Methods of the Graph ADT

Vertices and edges

- are positions
- store elements

Accessor methods

- `aVertex()`
- `incidentEdges(v)`
- `endVertices(e)`
- `isDirected(e)`
- `origin(e)`
- `destination(e)`
- `opposite(v, e)`
- `areAdjacent(v, w)`

Update methods

- `insertVertex(o)`
- `insertEdge(v, w, o)`
- `insertDirectedEdge(v, w, o)`
- `removeVertex(v)`
- `removeEdge(e)`

Generic methods

- `numVertices()`
- `numEdges()`
- `vertices()`
- `edges()`

Edge List Structure

Vertex object

- element
- reference to position in vertex sequence

Edge object

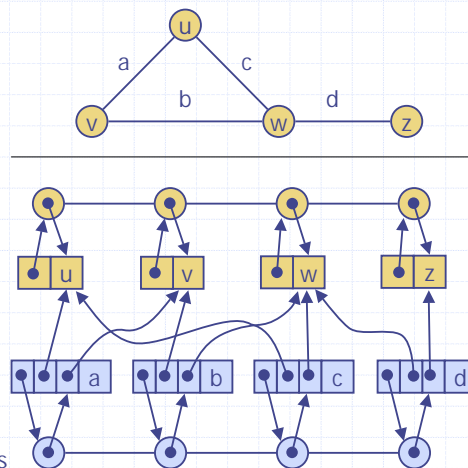
- element
- origin vertex object
- destination vertex object
- reference to position in edge sequence

Vertex sequence

- sequence of vertex objects

Edge sequence

- sequence of edge objects



Adjacency List Structure

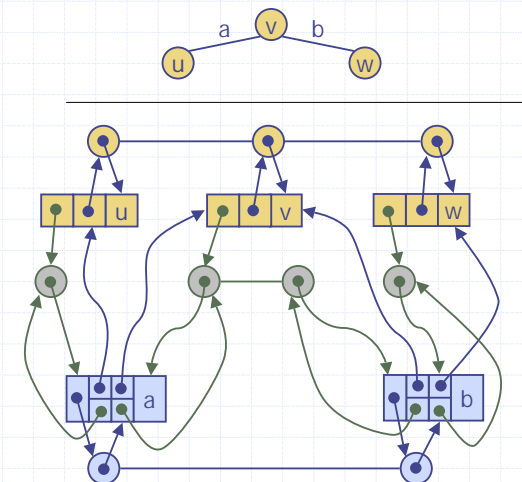
Edge list structure

Incidence sequence for each vertex

- sequence of references to edge objects of incident edges

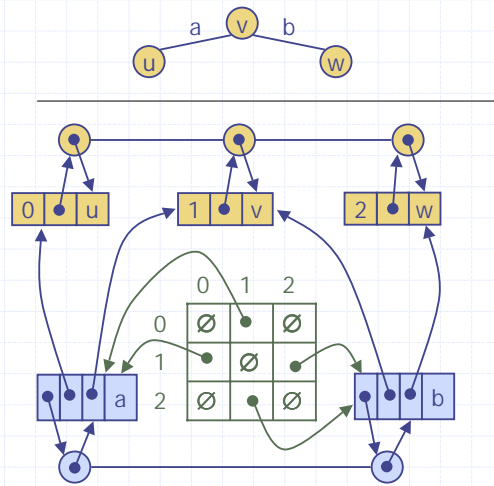
Augmented edge objects

- references to associated positions in incidence sequences of end vertices



Adjacency Matrix Structure

- ◆ Edge list structure
- ◆ Augmented vertex objects
 - Integer key (index) associated with vertex
- ◆ 2D-array adjacency array
 - Reference to edge object for adjacent vertices
 - Null for non adjacent vertices



Performance

	Edge List	Adjacency List	Adjacency Matrix
◆ n vertices			
◆ m edges			
◆ no parallel edges			
◆ no self-loops			
Space	$n + m$	$n + m$	n^2
incidentEdges (v)	m	$\text{deg}(v)$	n
areAdjacent (v, w)	m	$\min(\text{deg}(v), \text{deg}(w))$	1
insertVertex (o)	1	1	n^2
insertEdge (v, w, o)	1	1	1
removeVertex (v)	m	$\text{deg}(v)$	n^2
removeEdge (e)	1	1	1