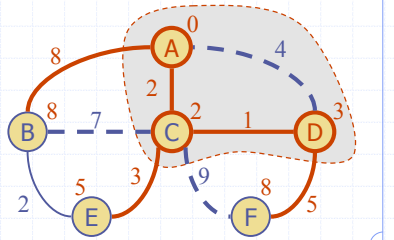
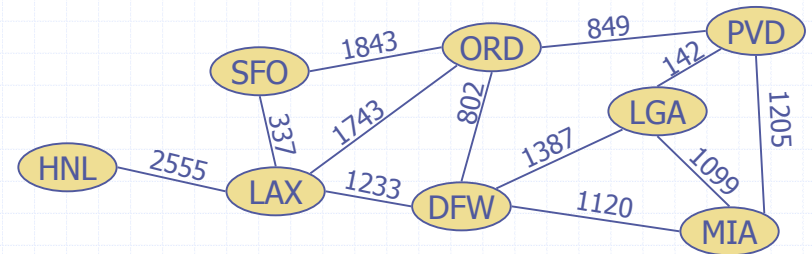


Shortest Paths



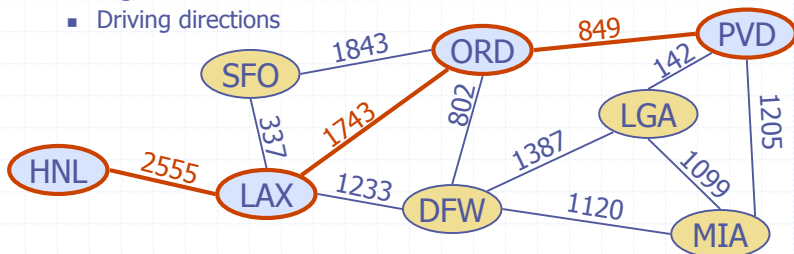
Weighted Graphs (§ 12.5)

- ◆ In a weighted graph, each edge has an associated numerical value, called the weight of the edge
- ◆ Edge weights may represent, distances, costs, etc.
- ◆ Example:
 - In a flight route graph, the weight of an edge represents the distance in miles between the endpoint airports



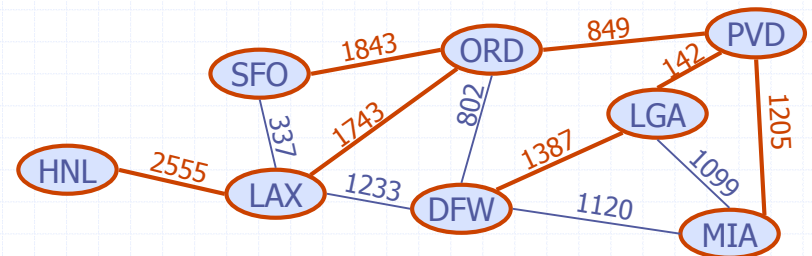
Shortest Paths (§ 12.6)

- ◆ Given a weighted graph and two vertices u and v , we want to find a path of minimum total weight between u and v .
 - Length of a path is the sum of the weights of its edges.
- ◆ Example:
 - Shortest path between Providence and Honolulu
- ◆ Applications
 - Internet packet routing
 - Flight reservations
 - Driving directions



Shortest Path Properties

- Property 1:
 - A subpath of a shortest path is itself a shortest path
- Property 2:
 - There is a tree of shortest paths from a start vertex to all the other vertices
- Example:
 - Tree of shortest paths from Providence



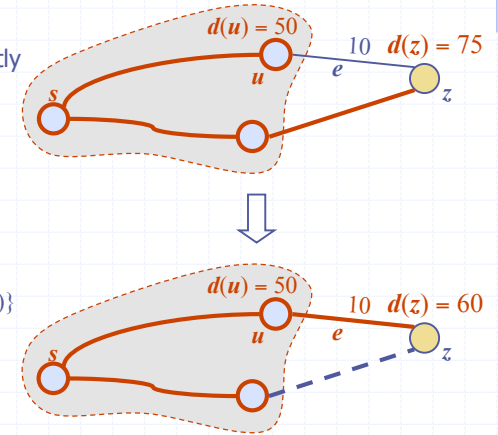
Dijkstra's Algorithm (§ 12.6.1)

- ◆ The distance of a vertex v from a vertex s is the length of a shortest path between s and v
- ◆ Dijkstra's algorithm computes the distances of all the vertices from a given start vertex s
- ◆ Assumptions:
 - the graph is connected
 - the edges are undirected
 - the edge weights are **nonnegative**
- ◆ We grow a "cloud" of vertices, beginning with s and eventually covering all the vertices
- ◆ We store with each vertex v a label $d(v)$ representing the distance of v from s in the subgraph consisting of the cloud and its adjacent vertices
- ◆ At each step
 - We add to the cloud the vertex u outside the cloud with the smallest distance label, $d(u)$
 - We update the labels of the vertices adjacent to u

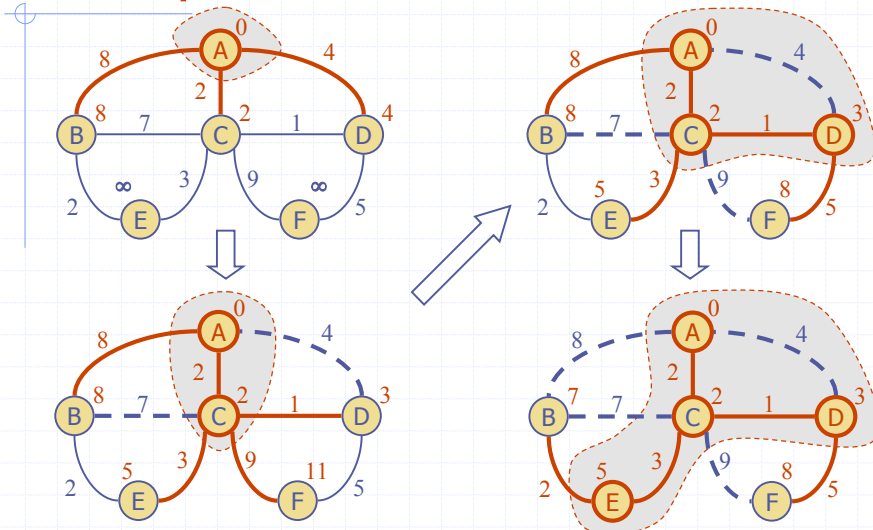
Edge Relaxation

- ◆ Consider an edge $e = (u, z)$ such that
 - u is the vertex most recently added to the cloud
 - z is not in the cloud
- ◆ The relaxation of edge e updates distance $d(z)$ as follows:

$$d(z) \leftarrow \min\{d(z), d(u) + \text{weight}(e)\}$$



Example



Example (cont.)

