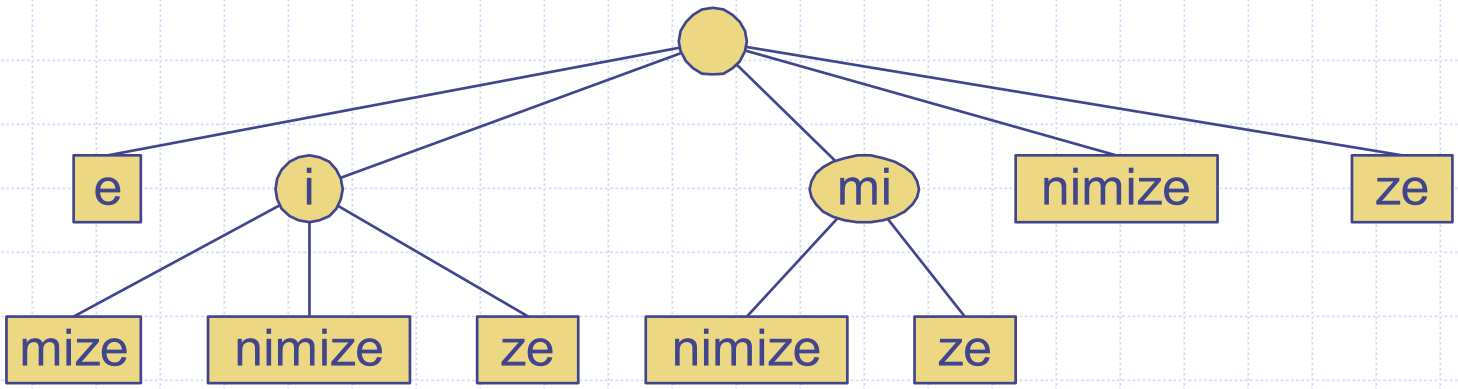
Tries



Outline and Reading

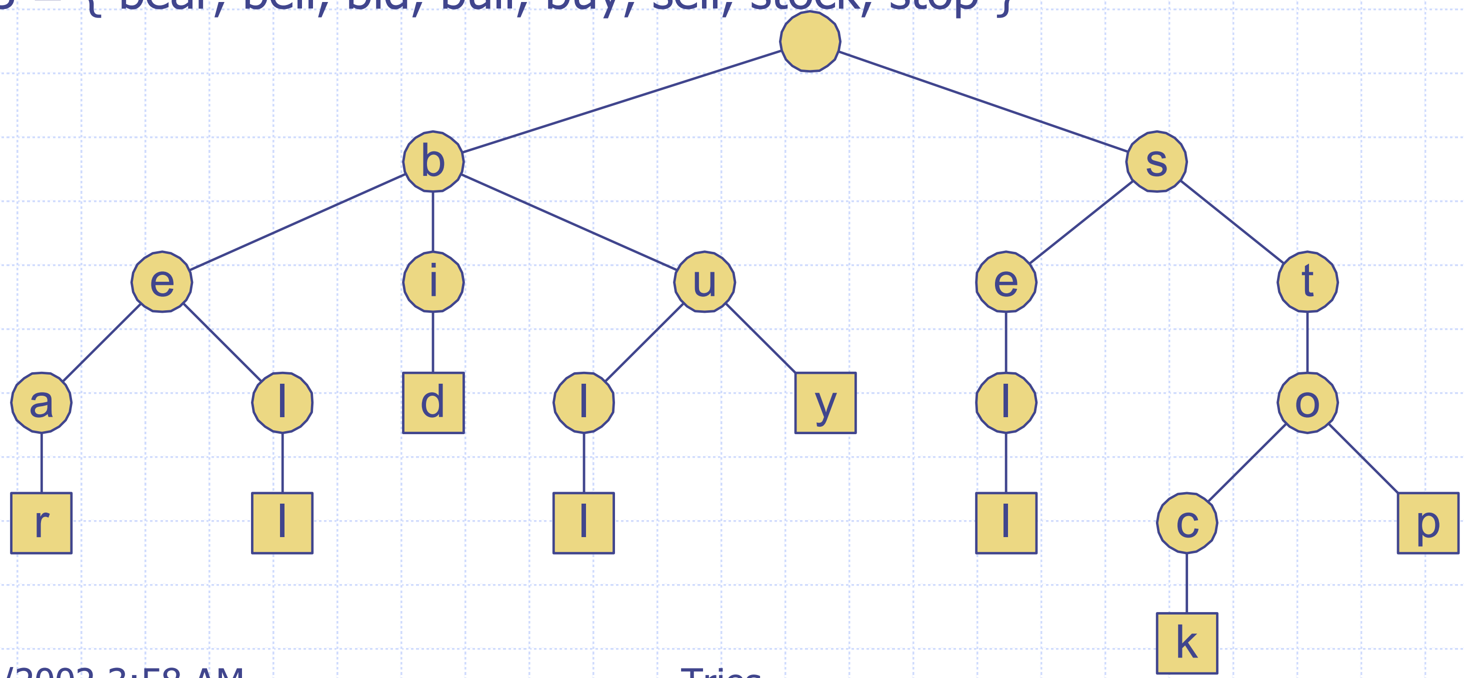
- Standard tries (§9.2.1)
- Compressed tries (§9.2.2)
- Suffix tries (§9.2.3)
- Huffman encoding tries (§9.3.1)

Preprocessing Strings

- Preprocessing the pattern speeds up pattern matching queries
  - After preprocessing the pattern, KMP's algorithm performs pattern matching in time proportional to the text size
- If the text is large, immutable and searched for often (e.g., works by Shakespeare), we may want to preprocess the text instead of the pattern
- A trie is a compact data structure for representing a set of strings, such as all the words in a text
  - A tries supports pattern matching queries in time proportional to the pattern size

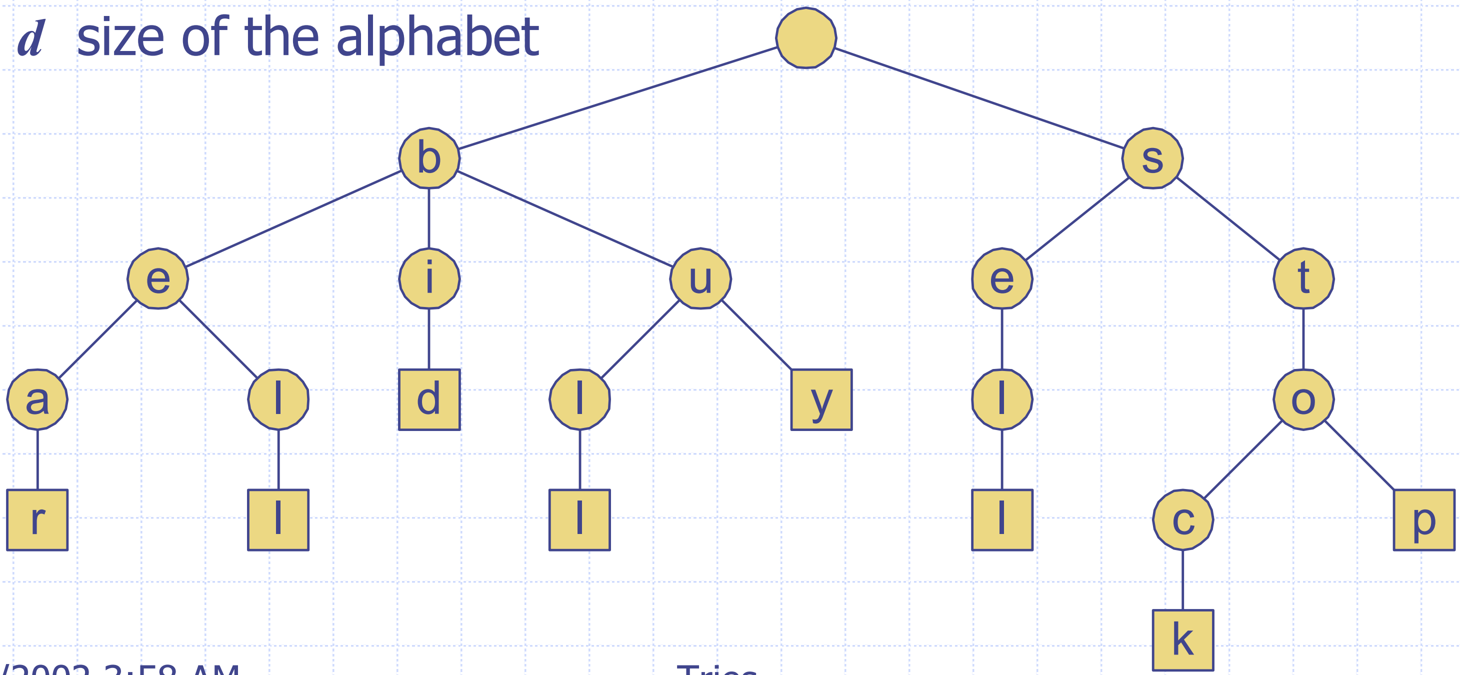
Standard Trie (1)

- The standard trie for a set of strings S is an ordered tree such that:
  - Each node but the root is labeled with a character
  - The children of a node are alphabetically ordered
  - The paths from the external nodes to the root yield the strings of S
- Example: standard trie for the set of strings
  S = { bear, bell, bid, bull, buy, sell, stock, stop }



Standard Trie (2)

- A standard trie uses $O(n)$ space and supports searches, insertions and deletions in time $O(dm)$, where:
  - n total size of the strings in S
  - m size of the string parameter of the operation
  - d size of the alphabet



Word Matching with a Trie

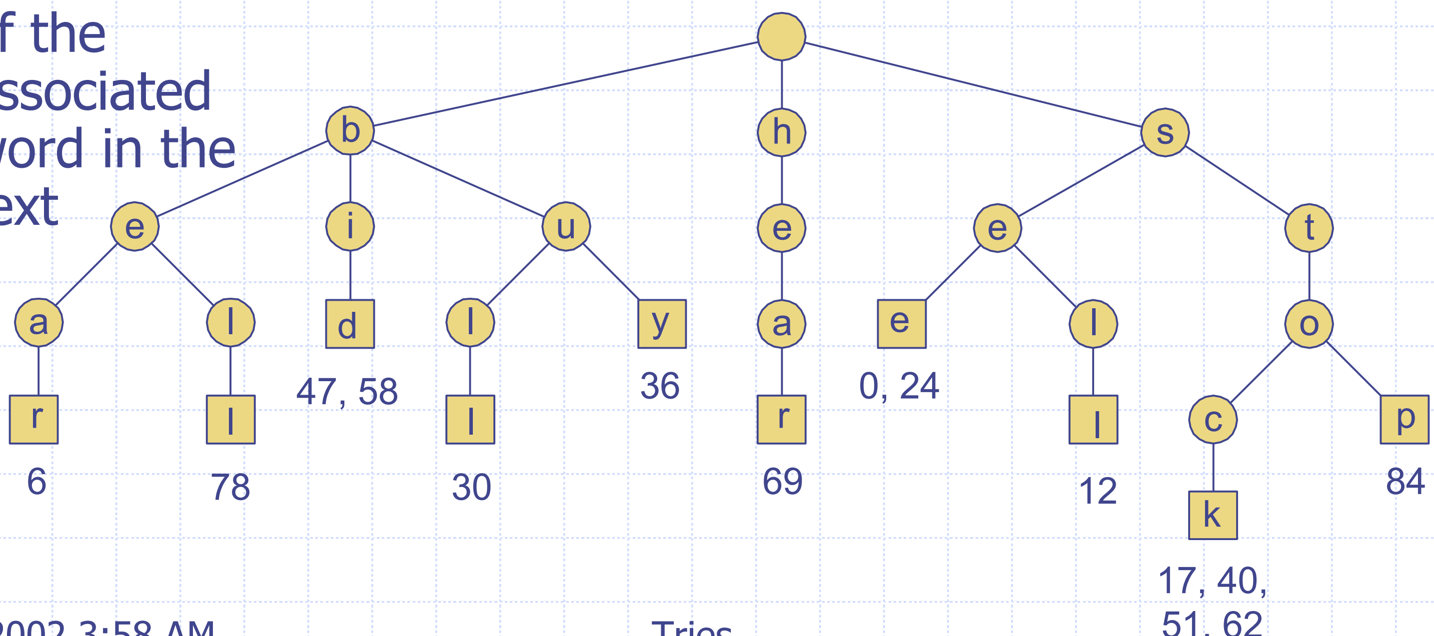
- We insert the words of the text into a trie
- Each leaf stores the occurrences of the associated word in the text

s	e	e		a		b	e	a	r	?		s	e	l	l		s	t	o	c	k	!	
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23

s	e	e		a		b	u	l	l	?		b	u	y		s	t	o	c	k	!	
24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46

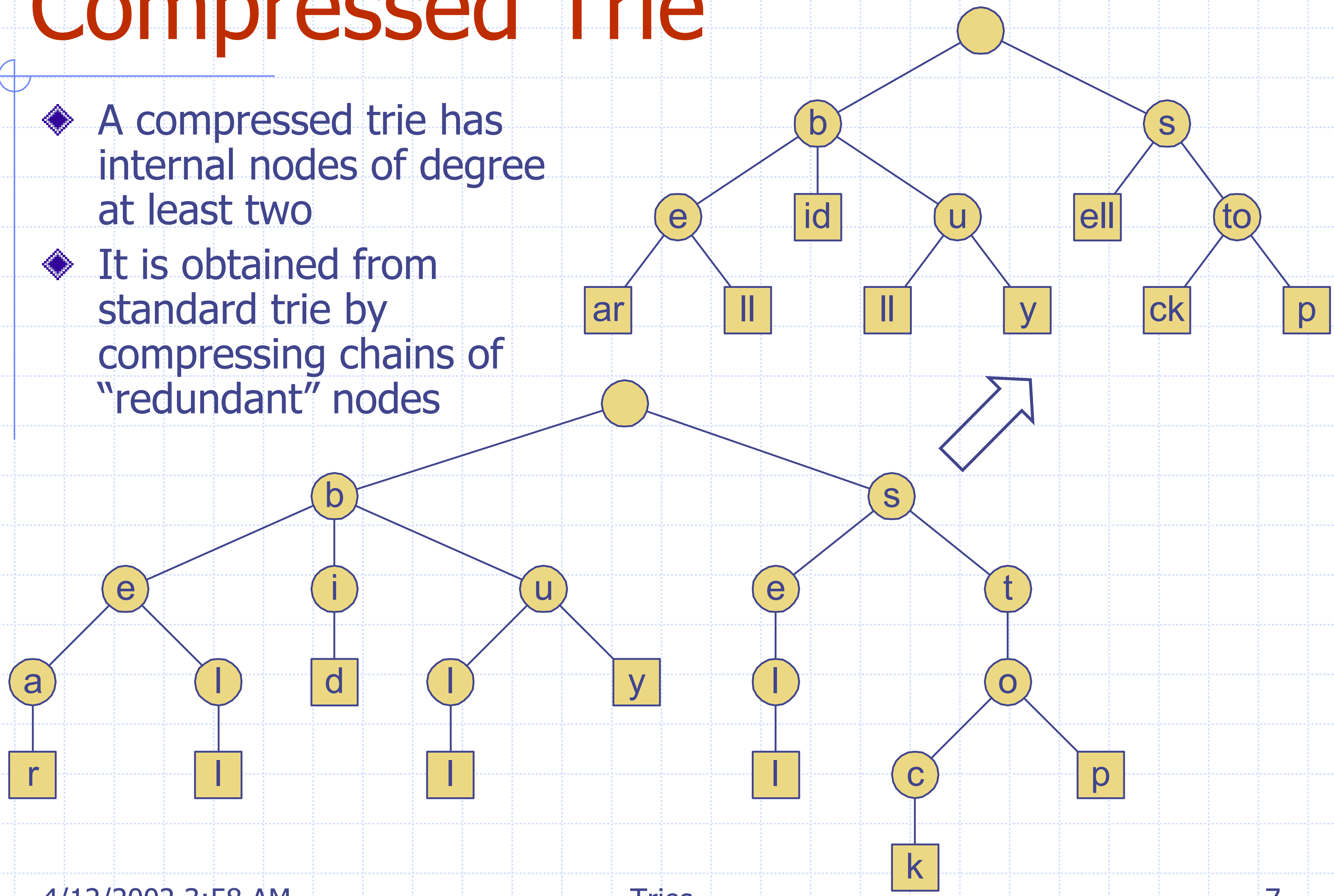
b	i	d		s	t	o	c	k	!		b	i	d		s	t	o	c	k	!	
47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68

h	e	a	r		t	h	e		b	e	l	l	?		s	t	o	p	!
69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88



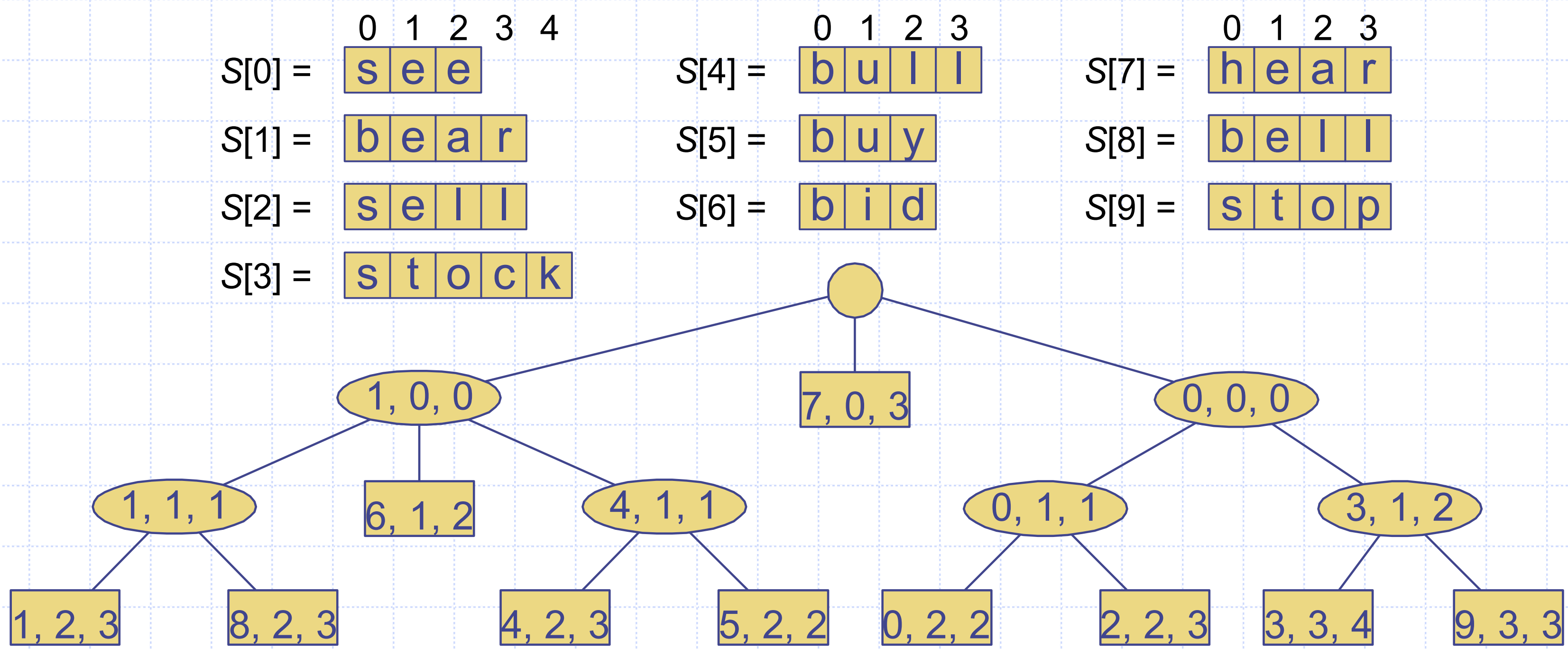
Compressed Trie

- A compressed trie has internal nodes of degree at least two
- It is obtained from standard trie by compressing chains of "redundant" nodes



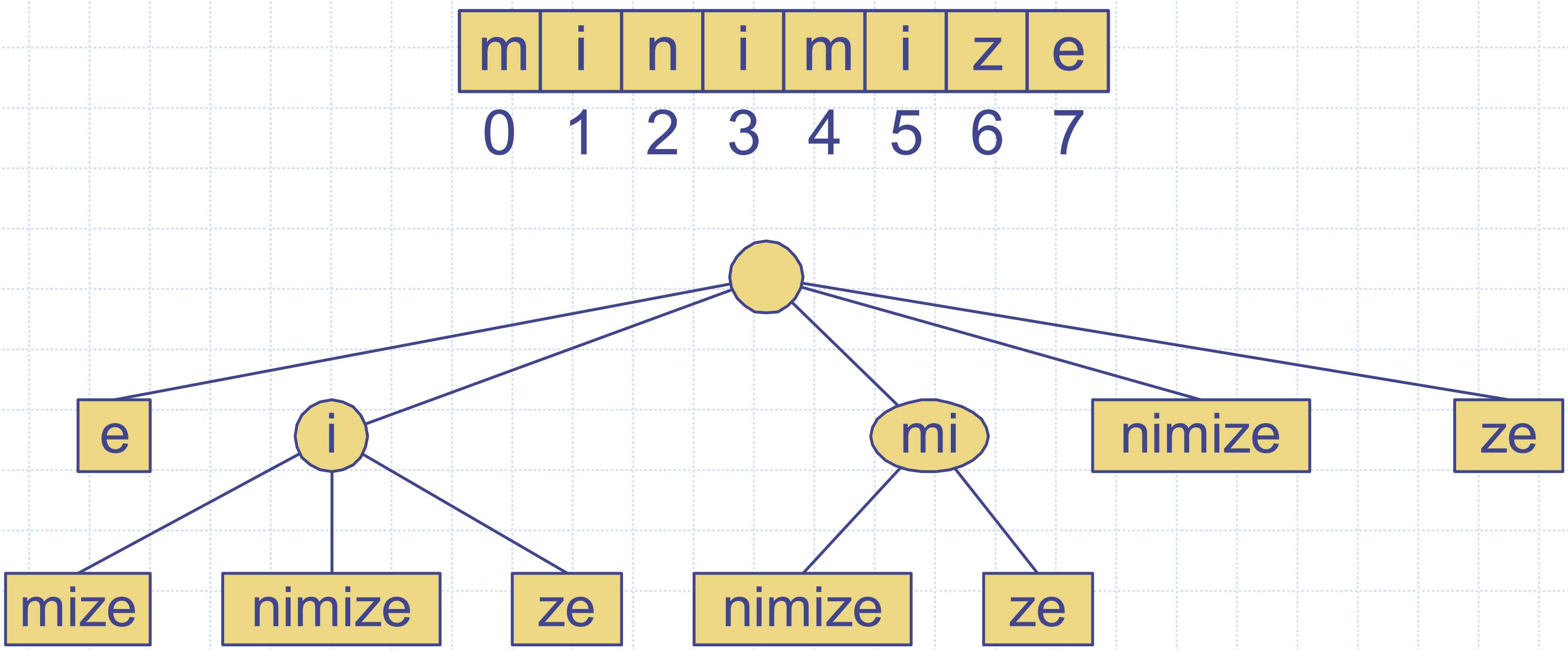
Compact Representation

- Compact representation of a compressed trie for an array of strings:
  - Stores at the nodes ranges of indices instead of substrings
  - Uses $O(s)$ space, where s is the number of strings in the array
  - Serves as an auxiliary index structure



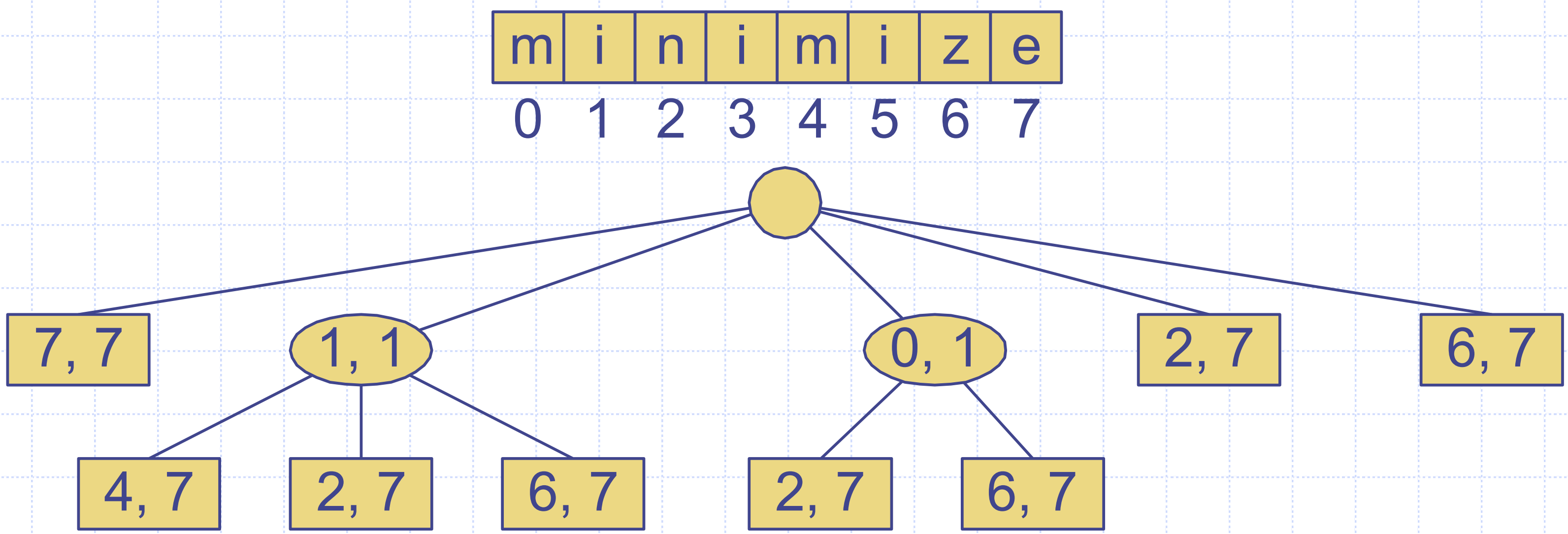
Suffix Trie (1)

- The suffix trie of a string X is the compressed trie of all the suffixes of X



Suffix Trie (2)

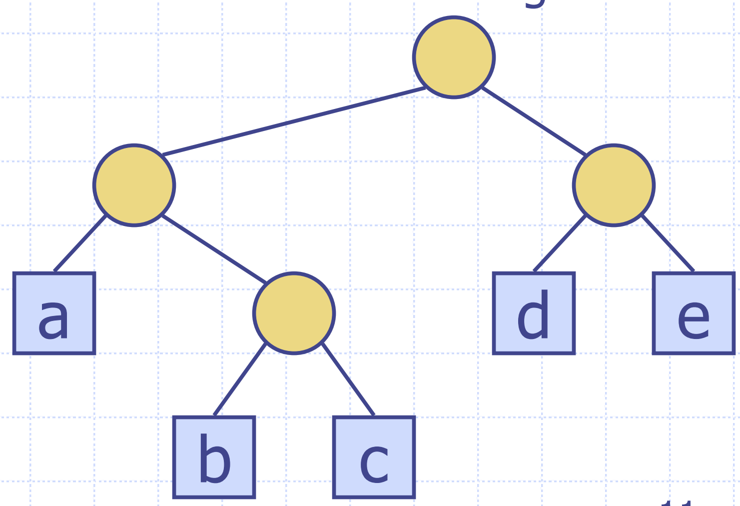
- Compact representation of the suffix trie for a string X of size n from an alphabet of size d
  - Uses $O(n)$ space
  - Supports arbitrary pattern matching queries in X in $O(dm)$ time, where m is the size of the pattern



Encoding Trie (1)

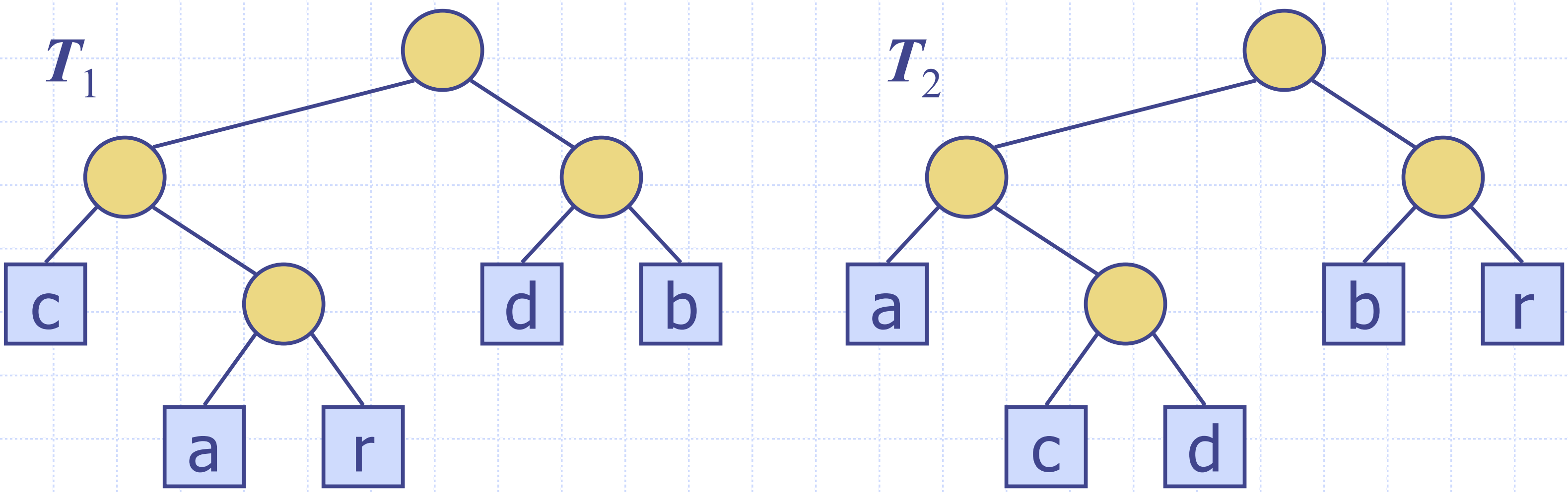
- A code is a mapping of each character of an alphabet to a binary code-word
- A prefix code is a binary code such that no code-word is the prefix of another code-word
- An encoding trie represents a prefix code
  - Each leaf stores a character
  - The code word of a character is given by the path from the root to the leaf storing the character (0 for a left child and 1 for a right child

00	010	011	10	11
a	b	c	d	e



Encoding Trie (2)

- Given a text string X, we want to find a prefix code for the characters of X that yields a small encoding for X
  - Frequent characters should have long code-words
  - Rare characters should have short code-words
- Example
  - $X =$ abracadabra
  - T_1 encodes X into 29 bits
  - T_2 encodes X into 24 bits



Huffman's Algorithm

- Given a string X, Huffman's algorithm construct a prefix code the minimizes the size of the encoding of X
- It runs in time $O(n + d \log d)$, where n is the size of X and d is the number of distinct characters of X
- A heap-based priority queue is used as an auxiliary structure

Algorithm *HuffmanEncoding*(X)
  Input string X of size n
  Output optimal encoding trie for X
  $C \leftarrow$ *distinctCharacters*(X)
  computeFrequencies(C, X)
  $Q \leftarrow$ new empty heap
  for all $c \in C$
    $T \leftarrow$ new single-node tree storing c
    Q.insert(*getFrequency*(c), T)
  while *Q.size*() > 1
    $f_1 \leftarrow$ *Q.minKey*()
    $T_1 \leftarrow$ *Q.removeMin*()
    $f_2 \leftarrow$ *Q.minKey*()
    $T_2 \leftarrow$ *Q.removeMin*()
    $T \leftarrow$ *join*(T_1, T_2)
    Q.insert($f_1 + f_2$, T)
  return *Q.removeMin*()

Example

X = abracadabra

Frequencies

a	b	c	d	r
5	2	1	1	2