

8. Lambda abstraction:

$(\lambda p. P \mid E)$ (%p.P|E)
 P must *constrain* the variables in p .
 $(\lambda p. P \mid E) = \{z.P \mid p \mapsto E\}$, where z is a list of variables that appear in the pattern p .

9. Function application: $f(E)$

$E \mapsto y \in f \Rightarrow E \in \text{dom}(f) \wedge f \in X \rightarrow Y$, where $\text{type}(f) = \mathbb{P}(X \times Y)$.
Note: in Event-B, relations and functions only ever have one argument, but that argument may be a pair or tuple, hence $f(E \mapsto F)$ f(E ↦ F)
 $f(E, F)$ is never valid.

6 Models

1. Contexts: contain sets and constants used by other contexts or machines.

CONTEXT	Identifier
EXTENDS	Machine_Identifiers
SETS	Identifiers
CONSTANTS	Identifiers
AXIOMS	Predicates
END	

Note: *theorems* can be presented in the AXIOMS part of a context.

2. Machines: contain events.

MACHINE	Identifier
REFINES	Machine_Identifiers
SEES	Context_Identifiers
VARIABLES	Identifiers
INVARIANT	Predicates
VARIANT	Expression
EVENTS	Events
END	

Note: *theorems* can be presented in the INVARIANT section of a machine and the WHERE part of an event.

Acknowledgement: Jean-Raymond Abrial, Laurent Voisin and Ian Hayes have all given valuable feedback and corrections at various stages of the evolution of this summary.

6.1 Events

Event_name	
REFINES	Event_identifiers
ANY	Identifiers
WHERE	Predicates
WITH	Witnesses
THEN	Actions
END	

There is one distinguished event named *INITIALISATION* used to initialise the variables of a machine, thus establishing the invariant.

6.2 Actions

Actions are used to change the state of a machine. There may be multiple actions, but they take effect concurrently, that is, in parallel. The semantics of events are defined in terms of *substitutions*. The substitution $[G]P$ defines a predicate obtained by replacing the values of the variables in P according to the action G . General substitutions are not available in the Event-B language.

Note on concurrency: any single variable can be modified in at most one action, otherwise the effect of the actions would, in general, be inconsistent.

1. *skip*, the null action:
 skip denotes the empty set of actions for an event.
2. Simple assignment action: $z := E$ x := E
 $:=$ = “*becomes equal to*”: replace free occurrences of x by E .
3. Choice from set: $x \in S$ x :: S
 \in = “*becomes in*”: arbitrarily choose a value from the set S .
4. Choice by predicate: $z :| P$ z :| P
 $:|$ = “*becomes such that*”: arbitrarily choose values for the variable in z that satisfy the predicate P . Within P , x refers to the value of the variable x before the action and x' refers to the value of the variable after the action.
5. Functional override: $f(x) := E$ f(x) := E
Substitute the value E for the function/relation f at the point x .
This is a shorthand:
 $f(x) := E = f := f \Leftarrow \{x \mapsto E\}$.

Each construct will be given in its presentation form, as displayed in the Rodin toolkit, followed by the ASCII form that is used for input to Rodin.

In the following: P, Q and R denote *predicates*;
 x and y denote single variables;
 z denotes a single or comma-separated list of variables;
 p denotes a pattern of variables, possibly including \mapsto and parentheses;
 S and T denote set expressions;
 U denotes a set of sets;
 m and n denote integer expressions;
 f and g denote functions;
 r denotes a relation;
 E and F denote expressions;
 E, F is a recursive pattern, *ie* it matches e_1, e_2 and also $e_1, e_2, e_3 \dots$; similarly for x, y ;

Freeness: The meta-predicate $\neg \text{free}(z, E)$ means that none of the variables in z occur free in E . This meta-predicate is defined recursively on the structure of E , but that will not be done here explicitly. The base cases are: $\neg \text{free}(z, \forall z. P \Rightarrow Q)$, $\neg \text{free}(z, \exists z. P \wedge Q)$, $\neg \text{free}(z, \{z \cdot P \mid F\})$, $\neg \text{free}(z, \lambda z. P|E)$, and $\text{free}(z, z)$.

In the following the statement that P *must constrain* z means that the type of z must be at least inferrable from P .

In the following, parentheses are used to show syntactic structure; they may of course be omitted when there is no confusion.

Note: Event-B has a formal syntax and this summary does not attempt to describe that syntax. What it attempts to do is to *explain* Event-B *constructs*. Some words like *expression* collide with the formal syntax. Where a syntactical entity is intended the word will appear in *italics*, e.g. *expression*, *predicate*.

1 Predicates

1. False \perp false
2. True \top true
3. Conjunction: $P \wedge Q$ P & Q
Left associative.
4. Disjunction: $P \vee Q$ P or Q
Left associative.
5. Implication: $P \Rightarrow Q$ P => Q
Non-associative: this means that $P \Rightarrow Q \Rightarrow R$ must be parenthesised or an error will be diagnosed.
6. Equivalence: $P \Leftrightarrow Q$ P <=> Q
 $P \Leftrightarrow Q = P \Rightarrow Q \wedge Q \Rightarrow P$
Non-associative: this means that $P \Leftrightarrow Q \Leftrightarrow R$ must be parenthesised or an error will be diagnosed.
7. Negation: $\neg P$ not P
8. Universal quantification:
 $\forall z. P \Rightarrow Q$!z.P => Q
Strictly, $\forall z.P$, but usually an implication.
For all values of z , satisfying P , Q is satisfied.
The types of z must be inferrable from the *predicate* P .
9. Existential quantification:
 $\exists z. P \wedge Q$ #z.P & Q
Strictly, $\exists z.P$, but usually a conjunction.
There exist values of z , satisfying P , that satisfy Q .
The type of z must be inferrable from the *predicate* P .

10. Equality: $E = F$ E = F

11. Inequality: $E \neq F$ E /= F

2 Sets

1. Singleton set: $\{E\}$ {E}
2. Set enumeration: $\{E, F\}$ {E, F}
See note on the pattern E, F at top of summary.
3. Empty set: \emptyset { }
4. Set comprehension: $\{z.P \mid F\}$ { z . P | F }
General form: the set of all values of F for all values of z that satisfy the *predicate* P . P must *constrain* the variables in z .
5. Set comprehension: $\{F \mid P\}$ { F | P }
Special form: the set of all values of F that satisfy the *predicate* P . In this case the set of bound variables z are all the free variables in F .
 $\{F \mid P\} = \{z.P \mid F\}$, where z is all the variables in F .
6. Set comprehension: $\{x \mid P\}$ { x | P }
A special case of item 5: the set of all values of x that satisfy the *predicate* P .
 $\{x \mid P\} = \{x.P \mid x\}$
7. Union: $S \cup T$ S \vee T
8. Intersection: $S \cap T$ S /\ T

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9. Difference: $S \setminus T$
 $S \setminus T = \{x \mid x \in S \wedge x \notin T\}$

10. Ordered pair: $E \mapsto F$
 $E \mapsto F \neq (E, F)$
 Left associative.

In all places where an ordered pair is required, $E \mapsto F$ must be used. E, F will not be accepted as an ordered pair, it is always a list. $\{x, y, P \mid x \mapsto y\}$ illustrates the different usage.

11. Cartesian product: $S \times T$
 $S \times T = \{x \mapsto y \mid x \in S \wedge y \in T\}$
 Left-associative.

12. Powerset: $\mathbb{P}(S)$
 $\mathbb{P}(S) = \{s \mid s \subseteq S\}$

13. Non-empty subsets: $\mathbb{P}_1(S)$
 $\mathbb{P}_1(S) = \mathbb{P}(S) \setminus \{\emptyset\}$

14. Cardinality: $\text{card}(S)$
 Defined only for *finite*(S).

15. Generalized union: $\text{union}(U)$
 The union of all the elements of U .
 $\text{union}(U) = \{x \mid x \in S \vee \exists s \cdot s \in U \wedge x \in s\}$
 where $\neg \text{free}(x, s, U)$

16. Generalized intersection: $\text{inter}(U)$
 The intersection of all the elements of U .
 $\text{inter}(U) = \{x \mid x \in S \wedge \forall s \cdot s \in U \Rightarrow x \in s\}$
 where $\neg \text{free}(x, s, U)$

17. Quantified union:
 $U \cdot z \cdot P \mid S$
 $\text{UNION } z \cdot P \mid S$
 $U \cdot z \cdot P \mid S \Rightarrow \forall z \cdot P \Rightarrow S \subseteq T \Leftrightarrow \forall z \cdot P \Rightarrow S \subseteq T \Leftrightarrow \forall z \cdot P \Rightarrow S \subseteq T$
 where $\neg \text{free}(x, z, J)$, $\neg \text{free}(x, P)$, $\neg \text{free}(x, S)$,
 $\text{where } \neg \text{free}(x, z)$

18. Quantified intersection:
 $U \cdot z \cdot P \mid S$
 $\text{INTER } z \cdot P \mid S$
 $U \cdot z \cdot P \mid S \Rightarrow \forall z \cdot P \Rightarrow S \subseteq T \Leftrightarrow \forall z \cdot P \Rightarrow S \subseteq T \Leftrightarrow \forall z \cdot P \Rightarrow S \subseteq T$
 where $\neg \text{free}(x, z)$, $\neg \text{free}(x, J)$, $\neg \text{free}(x, P)$, $\neg \text{free}(x, S)$,
 $\text{where } \neg \text{free}(x, z)$

19. Difference: $S \setminus T$
 $S \setminus T = \{x \mid x \in S \wedge x \notin T\}$

20. Projection: prj_1
 prj_1 is generic.
 $(S \times T) \triangleright \text{prj}_1 = \{x \mid x \mapsto y \mid x \in S \times T\}$

21. Projection: prj_2
 prj_2 is generic.
 $(S \times T) \triangleright \text{prj}_2 = \{x \mapsto y \mid x \mapsto y \in S \times T\}$

22. Relations: $S \leftrightarrow T$
 $S \leftrightarrow T = \mathbb{P}(S \times T)$
 Associativity: relations are *right associative*:
 $r \in X \leftrightarrow Y \leftrightarrow Z = r \in X \leftrightarrow (Y \leftrightarrow Z)$.

23. Domain: $\text{dom}(r)$
 $\forall r \cdot r \in S \leftrightarrow T \Rightarrow \text{dom}(r) = S$

24. Total relation: $S \leftrightarrow T$
 If $r \in S \leftrightarrow T$ then $\text{ran}(r) = T$

25. Satisfactive relation: $S \leftrightarrow T$
 If $r \in S \leftrightarrow T$ then $\text{ran}(r) = T$

26. Total satisfactive relation: $S \leftrightarrow T$
 If $r \in S \leftrightarrow T$ then $\text{dom}(r) = S$ and $\text{ran}(r) = T$

27. Iteration: r^n
 $r \in S \leftrightarrow S \Rightarrow r^0 = S \triangleright \text{Id} \wedge r^{n+1} = r \cdot r^n$
 Note: to avoid inconsistency S should be the finite base set for r , ie the smallest set for which all $r \in S \leftrightarrow S$.

28. Reflexive Closure: r^*
 $r^* = \cup \{n \cdot (n \mid r^n)\}$
 Could be defined as a function *rclosure*(r).
 Note: $r^0 \subseteq r^*$.

29. Irreflexive Closure: r^+
 $r^+ = \cup \{n \cdot (n \mid r^n) \mid r^n \neq \emptyset\}$
 Could be defined as a function *rclosure*(r).
 Note: $r^0 \subseteq r^+$ depending on r .

30. Identity: id
 $S \triangleright \text{id} = \{x \mapsto x \mid x \in S\}$.
 id is generic and the set S is inferred from the context.

31. Minimum: $\text{min}(S)$
 $S \subseteq \mathbb{Z}$ and *finite*(S) or S must have a lower bound.

32. Maximum: $\text{max}(S)$
 $S \subseteq \mathbb{Z}$ and *finite*(S) or S must have an upper bound.

33. Sum: $m + n$

34. Difference: $m - n$

35. Product: $m \times n$

36. Quotient: m / n

37. Remainder: $m \bmod n$

38. Interval: $m \dots n$

39. Set membership: $E \in S$

40. Set non-membership: $E \notin S$

41. Subset: $S \subseteq T$

42. Not a subset: $S \not\subseteq T$

43. Proper subset: $S \subset T$

44. Not a proper subset: $s \not\subset t$

45. Finite set: *finite*(S)

46. Partition: *partition*(S, x, y)
 x and y partition the set S , ie $S = x \cup y \wedge x \cap y = \emptyset$

47. Finite set: *finite*(S) $\Leftrightarrow S$ is *finite*.

48. Partition: *partition*(S, x, y)
 $S = \{A, B, C\} \wedge A \neq B \wedge B \neq C \wedge C \neq A$

49. Specialised use for enumerated sets:
 $\text{partition}(S, \{A\}, \{B\}, \{C\})$

50. The following is based on the set of integers, the set of natural numbers (non-negative integers), and the set of positive (non-zero) natural numbers.

51. The set of integer numbers: \mathbb{Z}

52. The set of natural numbers: \mathbb{N}

53. The set of positive natural numbers: \mathbb{N}_1

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