Where are we?

Last Lecture
We saw how to treat the semantics of concurrent programs and the properties they should satisfy.

This Lecture
We will give a syntactic way to specify properties (Temporal Logic) and introduce one of two methods we will cover to show properties hold (Model Checking) using the famous Critical Section problem.
We typically state our requirements with a logic.
Logic

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**Definition**

A logic is a formal language designed to express logical reasoning. Like any formal language, logics have a syntax and semantics.
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**Example (Propositional Logic Syntax)**

- A set of atomic propositions $\mathcal{P} = \{a, b, c, \ldots \}$
- An inductively defined set of formulae:
  - Each $p \in \mathcal{P}$ is a formula.
  - If $P$ and $Q$ are formulae, then $P \land Q$ is a formula.
  - If $P$ is a formula, then $\neg P$ is a formula.

(Other connectives are just sugar for these, so we omit them)
Semantics

Semantics are a mathematical representation of the meaning of a piece of syntax. We will use models to give semantics to logic.

Example (Propositional Logic Semantics)

A model for propositional logic is a valuation $V \subseteq P$, a set of "true" atomic propositions. We can extend a valuation over an entire formula, giving us a satisfaction relation:

- $V \models p \iff p \in V$
- $V \models \phi \land \psi \iff V \models \phi$ and $V \models \psi$
- $V \models \neg \phi \iff \neg V \models \phi$

We read $V \models \phi$ as $V$ "satisfies" $\phi$. 
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Example (Propositional Logic Semantics)

A model for propositional logic is a valuation $\mathcal{V} \subseteq \mathcal{P}$, a set of “true” atomic propositions. We can extend a valuation over an entire formula, giving us a satisfaction relation:

\[
\begin{align*}
\mathcal{V} \models p & \iff p \in \mathcal{V} \\
\mathcal{V} \models \varphi \land \psi & \iff \mathcal{V} \models \varphi \text{ and } \mathcal{V} \models \psi \\
\mathcal{V} \models \neg \varphi & \iff \mathcal{V} \not\models \varphi
\end{align*}
\]

We read $\mathcal{V} \models \varphi$ as $\mathcal{V}$ “satisfies” $\varphi$. 
Linear temporal logic (LTL) is a logic designed to describe linear time properties.

**Linear temporal logic syntax**

We have normal *propositional operators*:

- $p \in P$ is an LTL formula.
- If $\varphi, \psi$ are LTL formulae, then $\varphi \land \psi$ is an LTL formula.
- If $\varphi$ is an LTL formula, $\neg \varphi$ is an LTL formula.
*Linear temporal logic* (LTL) is a *logic* designed to describe linear time properties.

**Linear temporal logic syntax**

We have normal **propositional operators**:

- $p \in \mathcal{P}$ is an LTL formula.
- If $\varphi, \psi$ are LTL formulae, then $\varphi \land \psi$ is an LTL formula.
- If $\varphi$ is an LTL formula, $\neg \varphi$ is an LTL formula.

We also have **modal** or **temporal** operators:

- If $\varphi$ is an LTL formula, then $\Diamond \varphi$ is an LTL formula.
- If $\varphi, \psi$ are LTL formulae, then $\varphi \mathcal{U} \psi$ is an LTL formula.
LTL Semantics in Pictures
LTL Semantics in Pictures
LTL Semantics in Pictures
LTL Semantics in Pictures

\[
\sigma \\
\emptyset \rightarrow \{\spadesuit\} \rightarrow \{\heartsuit\} \rightarrow \{\heartsuit\} \rightarrow \{\heartsuit\} \rightarrow \{\spadesuit\}
\]


\[
\neg \heartsuit, \neg \spadesuit \\
\neg (\heartsuit \cup \spadesuit) \\
\heartsuit \cup \spadesuit \\
\heartsuit \cup \spadesuit \\
\heartsuit \cup \spadesuit \\
\heartsuit \cup \spadesuit \\
\heartsuit \cup \spadesuit \\
\heartsuit \cup \spadesuit
\]
LTL Semantics in Pictures
LTL Semantics in Pictures

\[
\sigma \quad \emptyset \quad \{\spadesuit\} \quad \{\heartsuit\} \quad \{\heartsuit\} \quad \{\heartsuit\} \quad \{\spadesuit\}
\]

\[
\neg \heartsuit, \neg \spadesuit \quad \neg \heartsuit, \neg \spadesuit \quad \heartsuit, \neg \spadesuit \quad \heartsuit, \neg \spadesuit \quad \heartsuit, \neg \spadesuit \quad \neg \heartsuit, \neg \spadesuit
\]

\[
\neg (\heartsuit U \spadesuit) \quad \heartsuit U \spadesuit \quad \heartsuit U \quad \heartsuit U \quad \heartsuit U
\]
LTL Semantics in Pictures

\[ \sigma \]

\[ \emptyset \rightarrow \{ \spadesuit \} \rightarrow \{ \heartsuit \} \rightarrow \{ \heartsuit \} \rightarrow \{ \heartsuit \} \rightarrow \{ \spadesuit \} \]

\[ \neg \heartsuit, \neg \spadesuit \]

\[ \neg (\heartsuit \cup \spadesuit) \]

\[ \heartsuit \cup \spadesuit \]

\[ \heartsuit \cup \spadesuit \]

\[ \heartsuit \cup \spadesuit \]

\[ \heartsuit \cup \spadesuit \]
LTL Semantics in Pictures

\[ \sigma \]

\[ \emptyset \rightarrow \{ \heartsuit \} \rightarrow \{ \diamondsuit \} \rightarrow \{ \diamondsuit \} \rightarrow \{ \diamondsuit \} \rightarrow \{ \heartsuit \} \]

\[ \neg \heartsuit, \neg \diamondsuit \neg (\heartsuit \cup \diamondsuit) \neg \heartsuit, \neg \diamondsuit \neg \heartsuit, \neg \diamondsuit \neg \heartsuit, \neg \diamondsuit \neg \heartsuit, \neg \diamondsuit \]
LTL Semantics in Pictures

\[ \sigma \]

\[ \emptyset \rightarrow \{ \heartsuit \} \rightarrow \{ \diamondsuit \} \rightarrow \{ \heartsuit \} \rightarrow \{ \heartsuit \} \rightarrow \{ \diamondsuit \} \]

\[ \neg \heartsuit, \neg \diamondsuit \]

\[ \neg \left( \heartsuit \cup \diamondsuit \right) \]

\[ \heartsuit \cup \diamondsuit \]

\[ \neg \heartsuit, \neg \diamondsuit \]

\[ \heartsuit \cup \diamondsuit \]

\[ \neg \heartsuit, \neg \diamondsuit \]

\[ \heartsuit \cup \diamondsuit \]
LTL Semantics in Pictures

- $\sigma$
- $\emptyset$
- $\{\heartsuit\}$
- $\{\heartsuit\}$
- $\{\heartsuit\}$
- $\{\spadesuit\}$

$\neg\heartsuit, \neg\spadesuit$
$\neg\heartsuit, \neg\spadesuit$
$\heartsuit, \neg\spadesuit$
$\heartsuit, \neg\spadesuit$
$\neg\heartsuit, \neg\spadesuit$
$\neg\heartsuit, \neg\spadesuit$
LTL Semantics

Let $\sigma = \sigma_0\sigma_1\sigma_2\sigma_3\sigma_4\sigma_5 \ldots$ be a behaviour. Then define notation:

- $\sigma|_0 = \sigma$
- $\sigma|_1 = \sigma_1\sigma_2\sigma_3\sigma_4\sigma_5 \ldots$
- $\sigma|_{n+1} = (\sigma|_1)|_n$
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Semantics

The models of LTL are behaviours. For atomic propositions, we just look at the first state. We often identify states with the set of atomic propositions they satisfy.

$\sigma \models p \iff p \in \sigma_0$
LTL Semantics

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LTL Semantics

Let $\sigma = \sigma_0\sigma_1\sigma_2\sigma_3\sigma_4\sigma_5 \ldots$ be a behaviour. Then define notation:

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- $\sigma \models p \iff p \in \sigma_0$
- $\sigma \models \varphi \land \psi \iff \sigma \models \varphi$ and $\sigma \models \psi$
- $\sigma \models \neg \varphi \iff \sigma \not\models \varphi$
- $\sigma \models \circ \varphi \iff \sigma|_1 \models \varphi$
- $\sigma \models \varphi \mathcal{U} \psi \iff$ There exists an $i$ such that $\sigma|i \models \psi$
  and for all $j < i$, $\sigma|_j \models \varphi$

We say $P \models \varphi$ iff $\forall \sigma \in \llbracket P \rrbracket. \sigma \models \varphi$. 
Derived Operators

The operator $\Diamond \varphi$ ("finally" or "eventually") says that $\varphi$ will be true at some point.

The operator $\Box \varphi$ ("globally" or "always") says that $\varphi$ is always true from now on.

**Exercise**

- Give the semantics of $\Box$ and $\Diamond$.
- Define $\Box$ and $\Diamond$ in terms of other operators.
More Exercises

Let $\rho$ be this behaviour:

\[
\begin{align*}
\heartsuit & \quad \spadesuit & \quad \heartsuit & \quad \heartsuit \spadesuit & \quad \heartsuit \spadesuit & \quad \heartsuit \spadesuit & \quad \heartsuit \spadesuit & \quad \ldots
\end{align*}
\]

$\rho \models \heartsuit$?
More Exercises

Let $\rho$ be this behaviour:

\[ \rho |\!
\begin{array}{ccccccccc}
\heartsuit & \spadesuit & \heartsuit & \heartsuit \spadesuit & \heartsuit \spadesuit & \heartsuit \spadesuit & \heartsuit \spadesuit & \cdots
\end{array}
\]

\[\rho \models \heartsuit ? \]
\[\rho \models \spadesuit ? \]
More Exercises

Let $\rho$ be this behaviour:

![Diagram showing heart and spade symbols repeated with ellipsis]

$\rho \models \heartsuit$?
$\rho \models \spadesuit$?
$\rho \models \bigcirc \spadesuit$?
More Exercises

Let $\rho$ be this behaviour:

![Heart Card Symbols]

$\rho \models \heartsuit ?$
$\rho \models \spadesuit ?$
$\rho \models \bigcirc \spadesuit ?$
$\rho \models \diamondsuit \heartsuit ?$
More Exercises

Let $\rho$ be this behaviour:

\[
\begin{align*}
\rho |\ = & \heartsuit ? \\
\rho |\ = & \spadesuit ? \\
\rho |\ = & \bigcirc \spadesuit ? \\
\rho |\ = & \lozenge \heartsuit ? \\
\rho|_3 |\ = & \lozenge (\heartsuit \land \neg \spadesuit) ?
\end{align*}
\]
More Exercises

Let $\rho$ be this behaviour:

$$
\begin{align*}
\rho |\models & \spadesuit ? \\
\rho |\models & \clubsuit ? \\
\rho |\models & \bigcirc \spadesuit ? \\
\rho |\models & \Diamond \heartsuit ? \\
\rho |_{3} \models & \Diamond (\heartsuit \land \neg \spadesuit) ? \\
\rho |\models & \Diamond \Box (\heartsuit \land \spadesuit) ?
\end{align*}
$$
More Exercises

Let $\rho$ be this behaviour:

\[ \begin{align*}
   \rho |&= \heartsuit? \\
   \rho |&= \spadesuit? \\
   \rho |&= \bigcirc \spadesuit? \\
   \rho |&= \Diamond \heartsuit? \\
   \rho|_3 |&= \Diamond (\heartsuit \land \neg \spadesuit)? \\
   \rho |&= \Diamond \Box (\heartsuit \land \spadesuit)? \\
   \rho |&= \Box (\heartsuit U \spadesuit)? 
\end{align*} \]
More Exercises

Let $\rho$ be this behaviour:

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\rho|_3 &| = \Diamond (\heartsuit \land \neg \spadesuit)? \\
\rho &| = \Diamond \Box (\heartsuit \land \spadesuit)? \\
\rho &| = \Box (\heartsuit U \spadesuit)?
\end{align*}
$$

More Derived Operators

- Define “Infinitely Often” in LTL.
- Define “Almost Globally” in LTL (always true from some point onwards).
We can see that it is always possible for a run to move to the terminated state. How do we express this in LTL?
We can see that it is always possible for a run to move to the terminated state. How do we express this in LTL? We can’t! — it is a branching time property.

Branching Time
Dealing with branching time properties requires a different logic called CTL (Computation Tree Logic). Learn about it in COMP3153/9153 or COMP6752.
A counting argument for mechanical aids

How many scenarios are there for a program with $n$ finite processes consisting of $m$ atomic actions each?
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$$\frac{(nm)!}{m!^n}$$
A counting argument for mechanical aids

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So, for 6 processes consisting of 6 sequential atomic actions each, that’s merely 2,670,177,736,637,149,247,308,800 scenarios.
A counting argument for mechanical aids

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\[
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Do come back when you’re done testing!
Sobering Conclusion

For any realistic concurrent program, even if there is some synchronisation that helps to reduce the number of interleavings, it is *infeasible to test* all possible scenarios. Consequently, we need to apply smarter techniques than brute-force testing to establish properties of concurrent programs. *Formal methods* let us reason about programs, or, if that is too hard, about *abstractions* of programs.
Industrially applicable formal methods

To verify that some program $P$ has a certain property $\varphi$ (i.e. $P \models \varphi$) we can use:

- **model checking** — exhaustively searching through (an efficient representation of) $P$'s state space to find a *counter example* to $\varphi$
- **theorem proving** — construct a (formal) proof of $\varphi$

To be relevant in practice, these techniques must be supported by tools.
Model Checking

**Pros:** easy to use push-button technology; instructive counter examples (error traces) help debugging

**Cons:** *state (space) explosion problem*
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**Question**

Where can I learn more about model checking?
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**Answer**
COMP3153/9153 *Algorithmic Verification* (should run in T2)
(Interactive) Theorem Proving

Pros: no (theoretical) limits on state spaces
Cons: requires expert users (e.g. skilled computer scientists, mathematicians, or logicians) to hand-crank through proofs
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**Question**
Where can I learn more about interactive theorem proving?

**Answer**
COMP4161 *Advanced Verification* (should run in T3)
SPIN

A model checker for concurrent systems with a lot of useful features and support for LTL model checking.

http://www.spinroot.com

Programs are modelled in the Promela language.
Promela in brief

- A kind of weird hybrid of C and Guarded Command Language.
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- Variables can be of several types: *bit*, *byte*, *int* and so on, as well as *channels*. 

Correctness claims can be expressed in many different ways.
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Warning

Variables of non-fixed size like int are of machine determined size, like C.
Example 1: Hello World

Johannes will demonstrate the basics of proctype and run using some simple examples.
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Take-away
You can use SPIN to randomly simulate Promela programs as well as model check them.
Example 2: Counters

Johannes will demonstrate a program that exhibits non-deterministic behaviour due to scheduling.
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Johannes will demonstrate a program that exhibits non-deterministic behaviour due to scheduling.

Explicit non-determinism

You can also add explicit non-determinism using if and do blocks:

```promela
if
:: (n % 2 != 0) -> n = 1;
:: (n >= 0) -> n = n - 2;
:: (n % 3 == 0) -> n = 3;
:: else -> skip;
fi
```
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:: else -> skip;
fi
```

What would happen without the else line?
The arrows in the previous slide are just sugar for semicolons:

```promela
if
:: (n % 2 != 0); n = 1;
:: (n >= 0); n = n - 2;
:: (n % 3 == 0); n = 3;
fi
```

A boolean expression by itself forms a guard. Execution can only progress past a guard if the boolean expression evaluates to true (non-zero).

If the entire system cannot make progress, that is called deadlock. SPIN can detect deadlock in Promela programs.
mtype = \{RED, YELLOW, GREEN\};
active proctype TrafficLight() {
    mtype state = GREEN;
    do
    :: (state == GREEN) -> state = YELLOW;
    :: (state == YELLOW) -> state = RED;
    :: (state == RED) -> state = GREEN;
    od
}

Non-determinism can be avoided by making guards mutually exclusive. Exit loops with break.
Volatile Variables

\[
\begin{array}{|c|c|}
\hline
\text{var } y, z \leftarrow 0, 0 \\
\hline
p_1: \text{ var } x; & q_1: \ y \leftarrow 1; \\
p_2: \ x \leftarrow y + z; & q_2: \ z \leftarrow 2; \\
\hline
\end{array}
\]

Question

What are the possible final values of \( x \)?
Volatile Variables

**Question**

What are the possible final values of $x$?
What about $x = 2$? Is that possible?

```plaintext
<table>
<thead>
<tr>
<th>$\text{var } y, z \leftarrow 0, 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_1$: $\text{var } x;$</td>
</tr>
<tr>
<td>$p_2$: $x \leftarrow y + z;$</td>
</tr>
<tr>
<td>$q_1$: $y \leftarrow 1;$</td>
</tr>
<tr>
<td>$q_2$: $z \leftarrow 2;$</td>
</tr>
</tbody>
</table>
```
Volatile Variables

<table>
<thead>
<tr>
<th></th>
<th>var y, z ← 0, 0</th>
</tr>
</thead>
<tbody>
<tr>
<td>p₁:</td>
<td>var x;</td>
</tr>
<tr>
<td>p₂:</td>
<td>x ← y + z;</td>
</tr>
<tr>
<td>q₁:</td>
<td>y ← 1;</td>
</tr>
<tr>
<td>q₂:</td>
<td>z ← 2;</td>
</tr>
</tbody>
</table>

**Question**

What are the possible final values of x?
What about x = 2? Is that possible?

It *is* possible, as we cannot guarantee that the statement p₂ is executed *atomically* — that is, as one step.
Volatile Variables

<table>
<thead>
<tr>
<th>var y, z ← 0, 0</th>
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</thead>
<tbody>
<tr>
<td>p₁: var x;</td>
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</tbody>
</table>

Question

What are the possible final values of x?
What about x = 2? Is that possible?
It is possible, as we cannot guarantee that the statement p₂ is executed atomically — that is, as one step.

Typically, we require that each statement only accesses (reads from or writes to) at most one shared variable at a time. Otherwise, we cannot guarantee that each statement is one atomic step. This is called the limited critical reference restriction.
Ensuring Atomicity

We will often have multiple actions that we wish to group into one step, i.e. to execute \textit{atomically}.

\textbf{Example (Counters)}

In our counter example, if each process executes the loop body atomically the result number can be guaranteed.

In Promela we can simply state this requirement, but in real programming languages we must use \textit{synchronisation} techniques to achieve this.
Grouping statements in Promela with \texttt{atomic} prevents them from being interrupted.

If a statement in an atomic block is \texttt{blocked}, atomicity is temporarily suspended and another process may run.
**atomic and d_step**

Grouping statements with d_step is more efficient than atomic, as it groups them all into **one transition**.

Non-determinism (if,do) is not allowed in d_step. If a statement in the block blocks, a **runtime error** is raised.
Atomicity

In the Real World™, we don’t have the luxury of atomic and d_step blocks. To solve this for real systems, we need solutions to the critical section problem.
Atomicity

In the Real World™, we don’t have the luxury of atomic and \texttt{d\_step} blocks. To solve this for real systems, we need solutions to the \textit{critical section problem}.

A sketch of the problem can be outlined as follows:

<table>
<thead>
<tr>
<th>forever do</th>
<th>forever do</th>
</tr>
</thead>
<tbody>
<tr>
<td>\textit{non-critical section}</td>
<td>\textit{non-critical section}</td>
</tr>
<tr>
<td>\textit{pre-protocol}</td>
<td>\textit{pre-protocol}</td>
</tr>
<tr>
<td>\textit{critical section}</td>
<td>\textit{critical section}</td>
</tr>
<tr>
<td>\textit{post-protocol}</td>
<td>\textit{post-protocol}</td>
</tr>
</tbody>
</table>
**Atomicity**

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<tr>
<td>\textit{critical section}</td>
</tr>
<tr>
<td>\textit{post-protocol}</td>
</tr>
</tbody>
</table>

The non-critical section models the possibility that a process may do something else. It can take any amount of time (even infinite).
### Atomicity

In the Real World™, we don’t have the luxury of atomic and d-step blocks. To solve this for real systems, we need solutions to the *critical section problem*. A sketch of the problem can be outlined as follows:

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</tr>
</thead>
<tbody>
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<td>non-critical section</td>
</tr>
<tr>
<td>pre-protocol</td>
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</tr>
<tr>
<td>post-protocol</td>
<td>post-protocol</td>
</tr>
</tbody>
</table>

The non-critical section models the possibility that a process may do something else. It can take any amount of time (even infinite). Our task is to find a pre- and post-protocol such that certain atomicity properties are satisfied.
Desiderata

We want to ensure two main properties and two secondary ones:

- **Mutual Exclusion** No two processes are in their critical section at the same time.
Desiderata

We want to ensure two main properties and two secondary ones:

- **Mutual Exclusion** No two processes are in their critical section at the same time.

- **Eventual Entry** (or *starvation-freedom*) Once it enters its pre-protocol, a process will eventually be able to execute its critical section.

**Question**

Which is safety and which is liveness?

Eventual Entry is liveness, the rest are safety.
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Which is safety and which is liveness?
Eventual Entry is liveness, the rest are safety.
First Attempt

We can implement `await` using primitive machine instructions or OS syscalls, or even using a busy-waiting loop.

<table>
<thead>
<tr>
<th>var</th>
<th>turn ← 1</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>forever do</strong></td>
<td><strong>forever do</strong></td>
</tr>
<tr>
<td>p₁</td>
<td>non-critical section</td>
</tr>
<tr>
<td>p₂</td>
<td><code>await</code> <code>turn = 1</code>;</td>
</tr>
<tr>
<td>p₃</td>
<td>critical section</td>
</tr>
<tr>
<td>p₄</td>
<td><code>turn ← 2</code></td>
</tr>
<tr>
<td>q₁</td>
<td>non-critical section</td>
</tr>
<tr>
<td>q₂</td>
<td><code>await</code> <code>turn = 2</code>;</td>
</tr>
<tr>
<td>q₃</td>
<td>critical section</td>
</tr>
<tr>
<td>q₄</td>
<td><code>turn ← 1</code></td>
</tr>
</tbody>
</table>

**Question**

Mutual Exclusion?
First Attempt

We can implement \texttt{await} using primitive machine instructions or OS syscalls, or even using a busy-waiting loop.

<table>
<thead>
<tr>
<th>var</th>
<th>turn $\leftarrow 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>\textbf{forever do}</td>
<td></td>
</tr>
<tr>
<td>$p_1$</td>
<td>\textit{non-critical section}</td>
</tr>
<tr>
<td>$p_2$</td>
<td>\texttt{await turn} $= 1$;</td>
</tr>
<tr>
<td>$p_3$</td>
<td>\textit{critical section}</td>
</tr>
<tr>
<td>$p_4$</td>
<td>\texttt{turn} $\leftarrow 2$</td>
</tr>
<tr>
<td>\textbf{forever do}</td>
<td></td>
</tr>
<tr>
<td>$q_1$</td>
<td>\textit{non-critical section}</td>
</tr>
<tr>
<td>$q_2$</td>
<td>\texttt{await turn} $= 2$;</td>
</tr>
<tr>
<td>$q_3$</td>
<td>\textit{critical section}</td>
</tr>
<tr>
<td>$q_4$</td>
<td>\texttt{turn} $\leftarrow 1$</td>
</tr>
</tbody>
</table>

\textbf{Question}

Mutual Exclusion? Yup!
First Attempt

We can implement **await** using primitive machine instructions or OS syscalls, or even using a busy-waiting loop.

<table>
<thead>
<tr>
<th>forever do</th>
<th>forever do</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>p</strong>₁</td>
<td><strong>q</strong>₁</td>
</tr>
<tr>
<td><em>non-critical section</em></td>
<td><em>non-critical section</em></td>
</tr>
<tr>
<td><strong>p</strong>₂</td>
<td><strong>q</strong>₂</td>
</tr>
<tr>
<td><strong>await</strong></td>
<td><strong>await</strong></td>
</tr>
<tr>
<td><em>turn = 1</em>;</td>
<td><em>turn = 2</em>;</td>
</tr>
<tr>
<td><strong>p</strong>₃</td>
<td><strong>q</strong>₃</td>
</tr>
<tr>
<td><em>critical section</em></td>
<td><em>critical section</em></td>
</tr>
<tr>
<td><strong>p</strong>₄</td>
<td><strong>q</strong>₄</td>
</tr>
<tr>
<td><em>turn ← 2</em></td>
<td><em>turn ← 1</em></td>
</tr>
</tbody>
</table>

**Question**

Mutual Exclusion? **Yup!**

Other criteria?
First Attempt

We can implement `await` using primitive machine instructions or OS syscalls, or even using a busy-waiting loop.

```
var turn ← 1

forever do
  p₁  non-critical section
  p₂  await turn = 1;
  p₃  critical section
  p₄  turn ← 2
forever do
  q₁  non-critical section
  q₂  await turn = 2;
  q₃  critical section
  q₄  turn ← 1
```

Question

Mutual Exclusion? Yup!
Other criteria? Nope!
First Attempt

We can implement `await` using primitive machine instructions or OS syscalls, or even using a busy-waiting loop.

```
var turn ← 1
forever do
  p1 non-critical section
  p2 await turn = 1;
  p3 critical section
  p4 turn ← 2
forever do
  q1 non-critical section
  q2 await turn = 2;
  q3 critical section
  q4 turn ← 1
```

Question

Mutual Exclusion? Yup!
Other criteria? Nope! What if `q1` never finishes?
## Second Attempt

<table>
<thead>
<tr>
<th>var ( \text{wantp}, \text{wantq} \leftarrow \text{False, False} )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>forever do</strong></td>
</tr>
<tr>
<td>( p_1 )</td>
</tr>
<tr>
<td>( p_2 )</td>
</tr>
<tr>
<td>( p_3 )</td>
</tr>
<tr>
<td>( p_4 )</td>
</tr>
<tr>
<td>( p_7 )</td>
</tr>
<tr>
<td><strong>forever do</strong></td>
</tr>
<tr>
<td>( q_1 )</td>
</tr>
<tr>
<td>( q_2 )</td>
</tr>
<tr>
<td>( q_3 )</td>
</tr>
<tr>
<td>( q_4 )</td>
</tr>
<tr>
<td>( q_7 )</td>
</tr>
</tbody>
</table>
Second Attempt

<table>
<thead>
<tr>
<th>var</th>
<th>wantp, wantq ← False, False</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>forever do</strong></td>
<td><strong>forever do</strong></td>
</tr>
<tr>
<td>p₁</td>
<td>non-critical section</td>
</tr>
<tr>
<td>p₂</td>
<td>await wantq = False;</td>
</tr>
<tr>
<td>p₃</td>
<td>wantp ← True;</td>
</tr>
<tr>
<td>p₄</td>
<td>critical section</td>
</tr>
<tr>
<td>p₇</td>
<td>wantp ← False</td>
</tr>
</tbody>
</table>

Mutual exclusion is violated if they execute in lock-step (i.e. p₁q₁p₂q₂p₃q₃ etc.)
### Third Attempt

<table>
<thead>
<tr>
<th>Forever do</th>
<th>forever do</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>p_1</strong></td>
<td><strong>q_1</strong></td>
</tr>
<tr>
<td><em>non-critical section</em></td>
<td><em>non-critical section</em></td>
</tr>
<tr>
<td><strong>p_2</strong></td>
<td><strong>q_2</strong></td>
</tr>
<tr>
<td><em>wantp ← True;</em></td>
<td><em>wantq ← True;</em></td>
</tr>
<tr>
<td><strong>p_3</strong></td>
<td><strong>q_3</strong></td>
</tr>
<tr>
<td><em>await wantq = False;</em></td>
<td><em>await wantp = False;</em></td>
</tr>
<tr>
<td><strong>p_4</strong></td>
<td><strong>q_4</strong></td>
</tr>
<tr>
<td><em>critical section</em></td>
<td><em>critical section</em></td>
</tr>
<tr>
<td><strong>p_7</strong></td>
<td><strong>q_7</strong></td>
</tr>
<tr>
<td><em>wantp ← False</em></td>
<td><em>wantq ← False</em></td>
</tr>
</tbody>
</table>

```plaintext
var wantp, wantq ← False, False
```

Now we have a deadlock (or stuck state) if they proceed in lock step.
Third Attempt

\[
\begin{array}{|c|c|}
\hline
\text{var} & \text{wantp, wantq} \leftarrow \text{False, False} \\
\hline
\text{forever do} & \text{forever do} \\
\hline
p_1 & \text{non-critical section} \\
p_2 & \text{wantp} \leftarrow \text{True;} \\
p_3 & \text{await wantq} = \text{False;} \\
p_4 & \text{critical section} \\
p_7 & \text{wantp} \leftarrow \text{False} \\
q_1 & \text{non-critical section} \\
q_2 & \text{wantq} \leftarrow \text{True;} \\
q_3 & \text{await wantp} = \text{False;} \\
q_4 & \text{critical section} \\
q_7 & \text{wantq} \leftarrow \text{False} \\
\hline
\end{array}
\]

Now we have a deadlock (or stuck state) if they proceed in lock step.
### Fourth Attempt

<table>
<thead>
<tr>
<th>forever do</th>
<th>forever do</th>
</tr>
</thead>
<tbody>
<tr>
<td>p₁</td>
<td>non-critical section</td>
</tr>
<tr>
<td>p₂</td>
<td>wantp ← True;</td>
</tr>
<tr>
<td>p₃</td>
<td>while wantq do</td>
</tr>
<tr>
<td>p₄</td>
<td>wantp ← False;</td>
</tr>
<tr>
<td>p₅</td>
<td>wantp ← True</td>
</tr>
<tr>
<td>p₆</td>
<td>critical section</td>
</tr>
<tr>
<td>p₇</td>
<td>wantp ← False</td>
</tr>
<tr>
<td>q₁</td>
<td>non-critical section</td>
</tr>
<tr>
<td>q₂</td>
<td>wantq ← True;</td>
</tr>
<tr>
<td>q₃</td>
<td>while wantp do</td>
</tr>
<tr>
<td>q₄</td>
<td>wantq ← False;</td>
</tr>
<tr>
<td>q₅</td>
<td>wantq ← True</td>
</tr>
<tr>
<td>q₆</td>
<td>critical section</td>
</tr>
<tr>
<td>q₇</td>
<td>wantq ← False</td>
</tr>
</tbody>
</table>

We have replaced the deadlock with live lock (looping) if they continuously proceed in lock-step.
Fourth Attempt

<table>
<thead>
<tr>
<th>var wantp, wantq ← False, False</th>
</tr>
</thead>
<tbody>
<tr>
<td>forever do</td>
</tr>
<tr>
<td>p₁  non-critical section</td>
</tr>
<tr>
<td>p₂  wantp ← True;</td>
</tr>
<tr>
<td>p₃  while wantq do</td>
</tr>
<tr>
<td>p₄  wantp ← False;</td>
</tr>
<tr>
<td>p₅  wantp ← True</td>
</tr>
<tr>
<td>p₆  critical section</td>
</tr>
<tr>
<td>p₇  wantp ← False</td>
</tr>
<tr>
<td>forever do</td>
</tr>
<tr>
<td>q₁  non-critical section</td>
</tr>
<tr>
<td>q₂  wantq ← True;</td>
</tr>
<tr>
<td>q₃  while wantp do</td>
</tr>
<tr>
<td>q₄  wantq ← False;</td>
</tr>
<tr>
<td>q₅  wantq ← True</td>
</tr>
<tr>
<td>q₆  critical section</td>
</tr>
<tr>
<td>q₇  wantq ← False</td>
</tr>
</tbody>
</table>

We have replaced the deadlock with live lock (looping) if they continuously proceed in lock-step.
# Fifth Attempt

<table>
<thead>
<tr>
<th>forever do</th>
<th>forever do</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>p1</strong> non-critical section</td>
<td><strong>q1</strong> non-critical section</td>
</tr>
<tr>
<td><strong>p2</strong> wantp = True;</td>
<td><strong>q2</strong> wantq = True;</td>
</tr>
<tr>
<td><strong>p3</strong> while wantq do</td>
<td><strong>q3</strong> while wantp do</td>
</tr>
<tr>
<td><strong>p4</strong> if turn = 2 then</td>
<td><strong>q4</strong> if turn = 1 then</td>
</tr>
<tr>
<td><strong>p5</strong> wantp ← False;</td>
<td><strong>q5</strong> wantq ← False;</td>
</tr>
<tr>
<td><strong>p6</strong> await turn = 1;</td>
<td><strong>q6</strong> await turn = 2;</td>
</tr>
<tr>
<td><strong>p7</strong> wantp ← True</td>
<td><strong>q7</strong> wantq ← True</td>
</tr>
<tr>
<td><strong>p8</strong> critical section</td>
<td><strong>q8</strong> critical section</td>
</tr>
<tr>
<td><strong>p9</strong> turn ← 2</td>
<td><strong>q9</strong> turn ← 1</td>
</tr>
<tr>
<td><strong>p10</strong> wantp ← False</td>
<td><strong>q10</strong> wantq ← False</td>
</tr>
</tbody>
</table>

```plaintext
var wantp, wantq ← False, False
var turn ← 1
```
Reviewing this attempt

The fifth attempt (Dekker’s algorithm) works well except if the scheduler pathologically tries to run the loop at $q_3 \cdots q_7$ when $turn = 2$ over and over rather than run the process $p$ (or vice versa).

What would we need to assume to prevent this?
Reviewing this attempt

The fifth attempt (Dekker’s algorithm) works well except if the scheduler pathologically tries to run the loop at $q_3 \cdots q_7$ when $\text{turn} = 2$ over and over rather than run the process $p$ (or vice versa).

What would we need to assume to prevent this?

Fairness

The *fairness assumption* means that if a process can always make a move, it will eventually be scheduled to make that move.

With this assumption, Dekker’s algorithm is correct.
Expressing Fairness in LTL

Let enabled(\(\pi\)) and taken(\(\pi\)) be predicates true in a state iff an action \(\pi\) is enabled, resp., taken.

**Examples**

*Weak fairness* for action \(\pi\) is then expressible as:

\[ \square(\square \text{enabled}(\pi) \Rightarrow \Diamond \text{taken}(\pi)) \]

*Strong fairness* for action \(\pi\) is then expressible as:

\[ \square(\square \Diamond \text{enabled}(\pi) \Rightarrow \Diamond \text{taken}(\pi)) \]

Promela can assume weak fairness when checking models.
What now?

- Do the homework exercises out now and submit them by Thursday next week.
- Assignment 0 (warm-up) is out. You have enough knowledge to start it, but not yet enough to finish it.
- Get spin (and ispin) working on your development environment (or use VLAB)