Closed product:

All the behaviours that p||q could engage in, if they are the only processes in the world.

If you have some send/receives that can’t be matched by others in our closed world, they are ignored.
c <= 0

q1

q2

q3

x = x + 1

p1

p2

p3

y = -53
Look for matching pairs of inputs and outputs.

\[ C \leq 0 \]

\[ q_1 \xrightarrow{c \Rightarrow x} q_2 \xrightarrow{x \leftarrow x + 1} q_3 \]

\[ \langle p_1, q_1 \rangle \]
\[ \langle p_1, q_2 \rangle \]
\[ \langle p_1, q_3 \rangle \]

\[ \langle p_2, q_1 \rangle \]
\[ \langle p_2, q_2 \rangle \]
\[ \langle p_2, q_3 \rangle \]

\[ \langle p_3, q_1 \rangle \]
\[ \langle p_3, q_2 \rangle \]
\[ \langle p_3, q_3 \rangle \]

\[ y \leftarrow 53 \]
Look for **matching pairs** of inputs and outputs.

A send and receive together becomes an assignment.

- $C \leq 0$
- $q_1 \rightarrow q_2 \rightarrow q_3$
- $C = \Rightarrow x$
- $x \leftarrow x + 1$
- $p_1 \rightarrow <p_1, q_1>, <p_1, q_2>, <p_1, q_3>$
- $p_2 \rightarrow <p_2, q_1>, <p_2, q_2>, <p_2, q_3>$
- $p_3 \rightarrow <p_3, q_1>, <p_3, q_2>, <p_3, q_3>$
- $y \leftarrow 53$
- $x \leftarrow 0$
- A send and receive together becomes an assignment.
C ≤ 1
D ≤ 1

x < -1
ty < -1

Semantically matching
Not semantically matching
Look for matching pairs. There aren’t any :(  

Unary closed product

```
p1
  c<=0
  p2
    y<-53
    p3
```

```
p1
  p1
    p2
      y<-53
      p3
```
Forward messages only if they satisfy some predicate.
\[ C \leq f(i) \]

\[ i++ \]

\[ C \leq f(i) \]

\[ i++ \]

\[ f_1 \]

\[ f_2 \]

\[ f_3 \]

\[ D \leq x \]

\[ !P(x) \]

\[ C \Rightarrow x \]

\[ P(x) \]

\[ z + y \]

\[ D \Rightarrow y \]

\[ c_1 \]

\[ c_2 \]
Invariant: when we’re in \(<p1,f1,c1>\) then:
\[
z = \sum_{n : 0 \leq n < i \land P(f(i))} f(n)
\]