Overview

In this assignment you will implement a type inference pass for MinHS. The language used in this assignment differs from the language of the first assignment in two respects: it has a polymorphic type system, and it has aggregate data structures.

The assignment requires you to:

- Implement type synthesis for polymorphic MinHS with sum and product data types.
- (0.1% bonus) adjust the type inference pass to include various simple syntax extensions from Assignment 1.
- (0.2% bonus) extend the type inference pass to elaborate mutually recursive \texttt{letrec} bindings.
- (0.2% bonus) adjust the type inference pass to allow optional type annotations provided by the user.

Each of these parts is explained in detail below.

The parser, and an interpreter backend, are provided for you. You do not have to change anything in any module other than \texttt{TyInfer.hs} (even for the bonus parts).

Your type inference pass should annotate the abstract syntax with type information where it is missing. The resulting abstract syntax should be fully annotated and correctly typed. Your assignment will only be tested on correct programs, and will be judged correct if it produces correctly typed annotated abstract syntax, up to \(\alpha\)-renaming of type variables.

Please use the Ed forum for questions about this assignment.

Submission

Submit your (modified) \texttt{TyInfer.hs} using the CSE give system, by typing the command

\texttt{give cs3161 TyInfer TyInfer.hs}

or by using the CSE give web interface.
1 Task 1

Task 1 is worth 100% of the marks of this assignment. You are to implement type inference for MinHS with aggregate data structures. The following cases must be handled:

- the MinHS language of the first task of assignment 1 (without \(n\)-ary functions or \texttt{letrecs}, or lists)
- product types: the 0-tuple and 2-tuples.
- sum types
- polymorphic functions

These cases are explained in detail below. The abstract syntax defining these syntactic entities is in \texttt{Syntax.hs}. You should not need to modify the abstract syntax definition in any way.

Your implementation is to follow the definition of algebraic data types found in the lecture on data structures, and the rules defined in the lectures on type inference. Additional material can be found in the lecture notes on polymorphism, and the reference materials. The full set of rules is outlined in this specification.

2 Bonus Tasks

These tasks are all optional, and are worth a total of an additional 10%.

2.1 Bonus Task 1: Simple Syntax Extensions

This bonus task is worth an additional 3%. In this task, you should implement type inference for \(n\)-ary functions, and multiple bindings in the one \texttt{let} expression, with the same semantics as the extension tasks for Assignment 1.

You will need to develop the requisite extensions to the type inference algorithm yourself, but the extensions are very similar to the existing rules.

2.2 Bonus Task 2: Recursive Bindings

This bonus task is worth an additional 3%. In this task you must implement type inference for \texttt{letrec} constructs.

Each binding will require a fresh unification variable, similar to multiple-binding \texttt{let} expressions, however for \texttt{letrec} we do not wish types to be generalised.

2.3 Bonus Task 3: User-provided type signatures

This bonus task is worth an additional 4%. In this task you are to extend the type inference pass to accept programs containing \textit{some} type information. You need to combine this with the results of your type inference pass to produce the final type for each declaration. That is, you need to be able to infer correct types for programs like:

```
main = let f :: (Int -> Int)
    = recfun g x = x;
    in f 2;
```

You must ensure that the type signatures provided are not overly general. For example, the following program should be a type error, as the user-provided signature is too general:

```
main :: (forall a. a) = 3;
```
3 Algebraic Data Types

This section covers the extensions to the language of the first assignment. In all other respects (except lists) the language remains the same, so you can consult the reference material from the first assignment for further details on the language.

3.1 Product Types

We only have 2-tuples in MinHS, and the Unit type, which could be viewed as a 0-tuple.

\[
\begin{align*}
\text{Types} & \quad \tau \rightarrow \tau_1 \times \tau_2 \\
& \quad | \quad 1 \\
\text{Expressions} & \quad exp \rightarrow (e_1, e_2) \\
& \quad | \quad \text{fst } e | \text{snd } e \\
& \quad | \quad ()
\end{align*}
\]

3.2 Sum Types

Sum types in MinHS follow the presentation in the lectures.

\[
\begin{align*}
\text{Types} & \quad \tau \rightarrow \tau + \tau \\
\text{Expressions} & \quad exp \rightarrow \text{Inl } e_1 \\
& \quad | \quad \text{Inr } e_2 \\
& \quad | \quad \text{case } e \text{ of} \\
& \quad & \quad \text{Inl } x \rightarrow e_1 ; \\
& \quad & \quad \text{Inr } y \rightarrow e_2 ;
\end{align*}
\]

3.3 Polymorphism

The extensions to allow polymorphism are relatively minor. Two new type forms have been introduced: the TypeVar \( t \) form, and the Forall \( t e \) form.

\[
\begin{align*}
\text{Types} & \quad \tau \rightarrow \text{forall } \tau_1 \ldots \tau_n \\
\text{For example, consider the following code fragment before and after type inference:}
\end{align*}
\]

```haskell
main =
  let f = recfun g x = x;
  in if f True
      then f (Inl 1)
      else f (Inr ())

main :: (Int + 1) =
  let f :: forall a. (a -> a) = recfun g :: (a -> a) x = x;
  in if f True
      then f (Inl 1)
      else f (Inr ())
```

3
4 Type Inference Rules

Sections coloured in blue can be viewed as inputs to the algorithm, while red text can be viewed as outputs.

Constants and Variables

\[\Gamma \vdash n : \text{Int} \quad \Gamma \vdash x : \forall \alpha_1 \ldots \forall \alpha_n. \tau \in \Gamma \]

\[\beta_i \text{ fresh} \]

(unquantify in TyInfer.hs provides an easy way to implement this)

Constructors and Primops

\[\text{constructorType}(C) = \forall \alpha_1 \ldots \forall \alpha_n. \tau \quad \text{primopType}(o) = \forall \alpha_1 \ldots \forall \alpha_n. \tau \]

\[\beta_i \text{ fresh} \]

(\text{constructorType} and \text{primopType} are defined in TyInfer.hs)

Application

\[\frac{\Gamma \vdash e_1 : \tau_1 \quad T \Gamma \vdash e_2 : \tau_2 \quad T' \Gamma \vdash \tau_1 \sim (\tau_2 \rightarrow \alpha)}{U \Gamma \vdash \text{Apply} e_1 e_2 : U \alpha} \quad \alpha \text{ fresh} \]

If-Then-Else

\[\frac{\Gamma \vdash e : \tau \quad \tau \sim \text{Bool} \quad T \Gamma \vdash e_1 : \tau_1 \quad T \Gamma \vdash e_2 : \tau_2 \quad T' \Gamma \vdash \tau_1 \sim \tau_2}{U \Gamma \vdash \text{If} \ e_1 e_2 : U \tau_2} \]

Case

\[\frac{\Gamma \vdash e : \tau \quad T \Gamma \vdash e_1 : \tau_1 \quad T \Gamma \vdash e_2 : \tau_2 \quad T \Gamma \vdash \alpha_1 + \alpha_2 \sim \alpha_2 \tau_2 \quad T \Gamma \vdash \alpha_1 \sim \alpha_1 \tau_1}{U \Gamma \vdash \text{Case} e x.e_1 y.e_2 : U \tau_2} \quad \alpha_1, \alpha_2 \text{ fresh} \]

Recursive Functions

\[\frac{T(\Gamma \cup \{x : \alpha_1\} \cup \{f : \alpha_2\}) \vdash e : \tau \quad T \Gamma \vdash \text{Recfun} f x.e : U(T \alpha_1 \rightarrow \tau)}{U \Gamma \vdash e : \text{Recfun} f x.e : U(T \alpha_1 \rightarrow \tau)} \quad \alpha_1, \alpha_2 \text{ fresh} \]

Let Bindings

\[\frac{\Gamma \vdash e_1 : \tau \quad T' \Gamma \vdash \text{Let} e_1 x.e_2 : \tau' \quad T \Gamma \vdash e : \tau \quad T \Gamma \vdash e_1 : \tau}{U \Gamma \vdash \text{Let} e_1 x.e_2 : \tau' \quad \Gamma \vdash e : \text{Generalise}(\Gamma, \tau)} \quad \text{where Generalise}(\Gamma, \tau) = \forall (TV(\tau) \setminus TV(\Gamma)) . \tau \]

4.1 Implementing the algorithm

The type inference rules imply an algorithm, where the expression and the environment can be seen as input, and the substitution and the type of the expression can be seen as output, as seen from the color coding above.

Such an algorithm would probably have a type signature like:
inferExp :: Gamma -> Exp -> TC (Type, Subst)

However our goal isn’t just to infer the type of the expression, but to also elaborate the expression to its explicitly-typed equivalent. In other words, we want to add typing annotations to our expressions, meaning we must return a new expression as well as the type and substitution.

inferExp :: Gamma -> Exp -> TC (Exp, Type, Subst)

The cases for let and recfun (and letrec, in the bonus) must add a (correct) type signature to the resultant expression, and all other cases must be sure to return a new expression consisting of the updated subexpressions.

Because we add annotations to the expressions on the way, those type annotations wouldn’t contain the information we get from typing successive expressions. Consider the following example:

\{+ : \text{Int} \rightarrow \text{Int} \rightarrow \text{Int}, x : a\} \vdash (\text{let } z = x \text{ in } x, x+1)

When typing \text{let } z = x \text{ in } x, we only know that \(x\) is of type \(a\), so we would add the type annotation \text{let } z :: a = x \text{ in } x.\ Only when we type the second expression, \(x+1\), we know that \(a\) has to be \text{Int} (which will be reflected in \(T'\)). That’s why, when we’re done typing the top-level binding, we have to traverse the whole binding again, applying the substitution to each type annotation anywhere in the binding.

The function \texttt{allTypes}, defined in Syntax.hs, allows you to update each type annotation in an expression, making it easy to perform this substitution.

5 Substitution

Substitutions are implemented as an abstract data type, defined in Subst.hs. \texttt{Subst} is an instance of the \texttt{Monoid} type class, which is defined in the standard library as follows:

```haskell
class Monoid a where
  mappend :: a -> a -> a -- also written as the infix operator <>
  mempty :: a
```

For the \texttt{Subst} instance, \texttt{mempty} corresponds to the empty substitution, and \texttt{mappend} is substitution composition. That is, applying the substitution \(a <> b\) is the same as applying \(a\) and \(b\) simultaneously. It should be reasonably clear that this instance obeys the monoid laws:

\[
\begin{align*}
  \text{mempty} <> x &= x & \text{ -- left identity} \\
  x <> \text{mempty} &= x & \text{ -- right identity} \\
  x <> (y <> z) &= (x <> y) <> z & \text{ -- associativity}
\end{align*}
\]

It is also commutative (\(x <> y = y <> x\)) assuming that the substitutions are disjoint (i.e that \(\text{dom}(x) \cap \text{dom}(y) = \emptyset\)). In the type inference algorithm, your substitutions are all applied in order and thus should be disjoint, therefore this property should hold.

You can use this \(<>\) operator to combine multiple substitutions into your return substitution.

You can construct a singleton substitution, which replaces one variable, with the \(=:\) operator, so the substitution \("a" =: \text{TypeVar "b"} \) \(<> \) \("b" =: \text{TypeVar "c"}\) is a substitution which replaces \(a\) with \(c\) and \(b\) with \(c\).

The \texttt{Subst} module also includes a variety of functions for running substitutions on types, quantified types, and environments.
6 Unification

The unification algorithm is the structural matching one originally due to Robinson and discussed in lectures:

- **input:** two type terms \( t_1 \) and \( t_2 \), where forall quantified variables have been replaced by fresh, unique variables

- **output:** the most general unifier of \( t_1 \) and \( t_2 \) (if it exists). The unifier is a data structure or function that specifies the substitutions to take place to unify \( t_1 \) and \( t_2 \).

6.1 Unification Cases

For \( t_1 \) and \( t_2 \):

1. *both are type variables \( v_1 \) and \( v_2 \):*
   - if \( v_1 = v_2 \), return the empty substitution
   - otherwise, return \([v_1 := v_2]\)

2. *both are primitive types*
   - if they are the same, return the empty substitution
   - otherwise, there is no unifier

3. *both are product types, with \( t_1 = (t_{11} \ast t_{12}) \), \( t_2 = (t_{21} \ast t_{22}) \)*
   - compute the unifier \( S \) of \( t_{11} \) and \( t_{21} \)
   - compute the unifier \( S' \) of \( S t_{12} \) and \( S t_{22} \)
   - return \( S \leftrightarrow S' \)

4. *function types and sum types (as for product types)*

5. *only one is a type variable \( v \), the other an arbitrary type term \( t \)*
   - if \( v \) occurs in \( t \), there is no unifier
   - otherwise, return \([v := t]\)

6. *otherwise, there is no unifier*

Functions in the `Data.List` library are useful for implementing the occurs check.

Once you generate a unifier (also a substitution), you then need to apply that unifier to your types, to produce the unified type.

7 Errors and Fresh Names

Thus far, the following type signature would be sufficient for implementing our type inference function:

\[
\text{inferExp :: Gamma -> Exp -> (Exp, Type, Subst)}
\]

Unification is a partial function, however, so we want a principled way to handle the error cases, rather than just bail out with `error` calls.

To achieve this, we’ll adjust the basic, pure signature for type inference to include the possibility of a `TypeError`:

\[
\text{inferExp :: Gamma -> Exp -> (Exp, Type, Subst, TypeError)}
\]
inferExp :: Gamma -> Exp -> Either TypeError (Exp, Type, Subst)

Even this, though, is not sufficient, as we cannot generate fresh, unique type variables for use as
unification variables:

fresh :: Type -- it is impossible for fresh to return a different
fresh = ? -- value each time!

To solve this problem, we could pass an (infinite) list of unique names around our program, and
fresh could simply take a name from the top of the list, and return a new list with the name
removed:

fresh :: [Id] -> ([Id],Type)
fresh (x:xs) = (xs,TypeVar x)

This is quite awkward though, as now we have to manually thread our list of identifiers throughout
the entire inference algorithm:

inferExp :: Gamma -> Exp -> [Id] -> Either TypeError ([Id], (Exp, Type, Subst))

To resolve this, we bundle both the [Id] state transformer and the Either TypeError x error
handling into one abstract type, called TC (defined in TCMonad.hs)

newtype TC a = TC ([Id] -> Either TypeError ([Id], a))

One can think of TC a abstractly as referring to a stateful action that will, if executed, return a
value of type a, or throw an exception.

As the name of the module implies, TC is a Monad, meaning that it exposes two functions
(return and >>=) as part of its interface.

return :: a -> TC a
return = ...
(>>>) :: TC a -> TC b -> TC b
a >>= b = a >>= const b
(>>=) :: TC a -> (a -> TC b) -> TC b
a >>= b = ...

The function return is, despite its name, just an ordinary function which lifts pure values into a TC
action that returns that value. The function (>>>) (read then), is a kind of composition operator,
which produces a TC action which runs two TC actions in sequence, one after the other, returning
the value of the last one to be executed. Lastly, the function (>>=), more general than (>>), allows
the second executed action to be determined by the return value of the first.

The TCMonad.hs module also includes a few built-in actions:

typeError :: TypeError -> TC a -- throw an error
fresh :: TC Type -- return a fresh type variable

Haskell includes special syntactic sugar for monads, which allow for programming in a somewhat
imperative style. Called do notation, it is simple sugar for (>>) and (>>=).

do e  -- e
do e; v -- e >> do v
do p <- e; v -- e >>= \p -> do v
do let x = y; v -- let x = y in do v

This lets us write unification and type inference quite naturally. A simple example of the use of
the TC monad is already provided to you, with the unquantify function, which takes a type with
some number of quantifiers, and replaces all quantifiers with fresh variables (very useful in the type
inference cases for variables, constructors, and primops):
unquantify :: QType -> TC Type
unquantify (Ty t) = return t
unquantify (Forall x t) = do x' <- fresh
                           unquantify (substQType (x := x') t)

To run a TC action in your top level infer function, the runTC function can be used:

runTC :: TC a -> Either TypeError a

Please note: This function runs the TC action with the same source of fresh names each time!
Using it more than once in your program is not likely to give correct results.

8 Program structure

A program in MinHS may evaluate to any non-function type, including an aggregate type. This is
a valid MinHS program:

\[
\text{main} = (1, (\text{Inl True}, \text{False}));
\]

which can be elaborated to the following type:

\[
\begin{align*}
\text{main} & : \text{forall } t. \ (\text{Int} * ((\text{Bool} + t) * \text{Bool})) \\
& = (1, (\text{InL True}, \text{False}));
\end{align*}
\]

8.1 Type information

The most significant change to the language of assignment 1 is that the parser now accepts programs
missing some or all of their type information. Type declarations are no longer compulsory! Unless
you are attempting the bonus parts of the assignment, you can assume that no type information
will be provided in the program. You must reconstruct it all.

You can view the type information after your pass using `--dump type-infer`.

9 Implementing Type Inference

You are required to implement the function infer. Some stub code has been provided for you,
along with some type declarations, and the type signatures of useful functions you may wish to
implement. You may change any part of TyInfer.hs you wish, as long as it still provides the
function infer, of the correct type. The stub code is provided only as a hint, you are free to ignore
it.

10 Useful interfaces

You need to use environments to a limited extent. This follows the same interface for environments
you used in assignment 1, it is defined in Env.hs, and there are many examples throughout the
program.

11 Testing

Your assignments will be autotested rigorously. You are encouraged to autotest yourself. minhs comes
with a tester script, and you should add your own tests to this. Your assignment will be tested by
comparing the output of minhs --dump type-infer against the expected abstract syntax. Your
solution must be $\alpha$-equivalent to the expected solution. It it up to you to write your own tests for your submission.

Much like the previous assignment, you are given a suite of tests for Task 1. Unlike the previous assignment, the tests do not specify the expected results—note the lack of `.out` files. Beware: unless you add `.out` specifying the expected result yourself, the tests will report success no matter what you return.

In this assignment we make no use of the later phases of the compiler.

12 Building MinHS

Building MinHS is exactly the same as in Assignment 1.

To run the type inference pass and inspect its results, for cabal users type:

```
$ cabal run minhs-2 -- --dump type-infer foo.mhs
```

Users of stack should type:

```
$ stack exec minhs-2 -- --dump type-infer foo.mhs
```

You may wish to experiment with some of the debugging options to see, for example, how your program is parsed, and what abstract syntax is generated. Many `--dump` flags are provided, which let you see the abstract syntax at various stages in the compiler.

13 Late Penalty

You may submit up to five days (120 hours) late. Each day of lateness corresponds to a 5% reduction of your total mark. For example, if your assignment is worth 88% and you submit it two days late, you get 78%. If you submit it more than five days late, you get 0%.

Course staff cannot grant assignment extensions—if you need an extension, you have to apply for special consideration through the standard procedure. More information here: https://www.student.unsw.edu.au/special-consideration

14 Plagiarism

Many students do not appear to understand what is regarded as plagiarism. This is no defense. Before submitting any work you should read and understand the UNSW plagiarism policy https://student.unsw.edu.au/plagiarism.

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References
