

## Local illumination equation

$$I = I_a k_a + \sum_{k=1 \dots n} f_{\text{att}}(d_k) I_k (k_d \vec{N} \cdot \vec{L}_k + k_s (\vec{V} \cdot \vec{R}_k)^n)$$

where

$k_a$	=	ambient reflection coefficient
$k_d$	=	diffuse reflection coefficient
$k_s$	=	specular reflection coefficient
$\vec{N}$	=	surface normal
$I_a$	=	Ambient light intensity
$I_k$	=	Intensity of light source $k$
$\vec{L}_k$	=	Direction to light source $k$
$\vec{V}$	=	Direction of viewer
$\vec{R}_k$	=	$2\vec{N}(\vec{N} \cdot \vec{L}_k) - \vec{L}_k$
$n$	=	Phong exponent
$d_k$	=	distance to light source $k$
$f_{\text{att}}()$	=	light attenuation function

All the vectors ( $N, L_k, V$ ) should be normalized.

## Bezier curve

$$B(t) = \sum_{i=0 \dots 3} b_i(t) p_i$$

where

$$\begin{aligned} b_0(t) &= (1-t)^3 \\ b_1(t) &= 3t(1-t)^2 \\ b_2(t) &= 3t^2(1-t) \\ b_3(t) &= t^3 \\ p_i &= \text{Control point } i \end{aligned}$$

## B spline curve

$$B_j(t) = \sum_{i=0 \dots 3} b_i(t) p_{i+j}$$

where

$$\begin{aligned} b_0(t) &= (-t^3 + 3t^2 - 3t + 1)/6 \\ b_1(t) &= (3t^3 - 6t^2 + 4)/6 \\ b_2(t) &= (-3t^3 + 3t^2 + 3t + 1)/6 \\ b_3(t) &= t^3/6 \\ p_i &= \text{Control point } i \end{aligned}$$

## 2D Transformations

$$\begin{array}{ccc}
 \text{Translation } (t_x, t_y) & \text{Scaling } (s_x, s_y) & \text{Rotation by } \theta \\
 \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}
 \end{array}$$

$$\begin{array}{ccc}
 \text{Window-Viewport} & & \text{where} \\
 \begin{bmatrix} a & 0 & c \\ 0 & e & f \\ 0 & 0 & 1 \end{bmatrix} & a = (V_r - V_l)/(W_r - W_l) & \\
 & c = V_l - aW_l & \\
 & e = (V_t - V_b)/(W_t - W_b) & \\
 & f = V_b - eW_b & 
 \end{array}$$

## Vector operations

$$\begin{aligned}
 \hat{a} &= (a_x, a_y, a_z) \\
 \hat{a} \cdot \hat{b} &= a_x b_x + a_y b_y + a_z b_z \\
 \|\hat{a}\| &= \sqrt{a_x^2 + a_y^2 + a_z^2} \\
 \hat{a} \times \hat{b} &= (a_y b_z - a_z b_y, a_z b_x - a_x b_z, a_x b_y - a_y b_x)
 \end{aligned}$$

## 3D Transformations

$$\begin{array}{ccc}
 T(t_x, t_y, t_z) & S(s_x, s_y, s_z) & R_X(\theta) \\
 \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta & 0 \\ 0 & \sin \theta & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
 R_Y(\theta) & R_Z(\theta) & \text{perspective} \\
 \begin{bmatrix} \cos \theta & 0 & \sin \theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin \theta & 0 & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} \cos \theta & -\sin \theta & 0 & 0 \\ \sin \theta & \cos \theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 1 \end{bmatrix}
 \end{array}$$

## Local to World Transformation

$$M_{UVN \rightarrow XYZ} = \begin{bmatrix} u_x & v_x & n_x & O_x \\ u_y & v_y & n_y & O_y \\ u_z & v_z & n_z & O_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

## Viewing Transformation

$$\begin{aligned}
 \vec{n} &= V\vec{P}N / \|V\vec{P}N\| \\
 \vec{u} &= V\vec{u}p \times V\vec{P}N / \|V\vec{u}p \times V\vec{P}N\| \\
 \vec{v} &= \vec{n} \times \vec{u}
 \end{aligned}$$

$$M_{XYZ \rightarrow UVN} = \begin{bmatrix} u_x & u_y & u_z & 0 \\ v_x & v_y & v_z & 0 \\ n_x & n_y & n_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} T(-o_x, -o_y, -o_z)$$

## Linear Interpolation

Parametric equation of line from  $\vec{a}$  to  $\vec{b}$  is

$$L(t) = \vec{a} + (\vec{b} - \vec{a})t, \quad 0 \leq t \leq 1$$

This is also the formula for linear interpolation.