Bayesian Statistics and Tracking

Due: Start of Wednesday lecture in week 4 (11 August)

1 Overview

The goal of this assignment is to test your understanding of Bayesian Statistics and Perception.

You should feel free to ask the Lecturer questions and discuss the assignment with others, but you should not copy someone else’s work. (e.g. You can get someone to show you how to answer a question, but you should write your answer up yourself from what you remember at least 30 minutes after they’ve left.)

Total marks for this assignment: 15. It will be scaled to 5% of your grade.

2 Bayesian Statistics

2.1 Question 2.1 (1 mark):

Give one representation for all the information known to an agent.

2.2 Question 2.2 (1 mark):

You and a friend have been designing a robot for the RoboCup robotic soccer competition. You’ve decided that you’re going to use Bayesian probabilities to estimate the robots location. The space of hypotheses that the robot is going to keep about its location is the 2D space of locations on the field (you’re ignoring angle, and other elements of the problem to get started).

You and your friend are arguing about the robot’s beliefs when it is first switched on. Which of the following is correct:

A Bayesian statistics says that the robot’s initial belief should have all its probability mass is on the point just next to the centre circle lined up for kickoff.

B Bayesian statistics says that the robot’s initial belief should have equal probability mass at each position on the field - the uniform distribution.

C Bayesian statistics says that the robot’s initial belief should be a mixture of the previous two options; a peak at the kickoff position, but non-zero at every location on the field.
D Bayesian statistics doesn’t say anything about the robot’s initial belief, that is up
to the designer.

2.3 Question 2.3 (1 mark):
You and a friend are trying to decide what to do on Friday night. She wants to go
dancing, and you want to go to the lab and play with robots (Hey - they’re cool). You
decide to flip a coin to see what to do. She flips and wins, and you go dancing.
Later, you decide to use your new found skills in Bayesian statistics and check to
see if the coin was biased. If the coin is biased you’d also like to find out how biased it
is (i.e. what is the probability it would come up ‘heads’ when flipped?).
What space of possible hypotheses/models could you use for this investigation (and
how is this space parameterised)?

2.4 Question 2.4 (1 mark):
You decide to use a uniform prior over the model space from the previous question. You
then flip the coin in the first trial. It shows heads.
What is \( p(\text{Heads}|m) \) for each of your models? (If your model space is continuous
then this will be a function.)

2.5 Question 2.5 (1 mark):
What is the posterior distribution over your model space given the single coin flip you’ve
seen?

2.6 Question 2.6 (1 mark):
You flip the coin again. Again it shows heads.
What is the posterior distribution over your model space when you incorporate this
new information?

2.7 Question 2.7 (1 mark):
You flip the coin a third time. This time it shows tails.
What is \( p(\text{Tails}|m) \) for each of your models?

2.8 Question 2.8 (1 mark):
You decide to use a one-dimensional Kalman filter to track the height of some wa-
ter in a rainwater tank. You remember from class that the pointwise product of two
Gaussians can be calculated as follows: \( G_1(\mu_1, \sigma_1^2).G_2(\mu_2, \sigma_2^2) = C.G_3(\mu_3, \sigma_3^2) \) where
\[
\mu_3 = \frac{\sigma_2^2 \mu_1 + \sigma_1^2 \mu_2}{\sigma_1^2 + \sigma_2^2} \quad \text{and} \quad \sigma_3^2 = \frac{\sigma_1^2 \sigma_2^2}{\sigma_1^2 + \sigma_2^2}
\]
From previous measurements your belief about the height of the water in the tank has $\mu = 2.9\text{m}$ with a variance $\sigma^2 = 0.01$. You receive an observation of water height of $3.1\text{m}$, the variance of observations is known to be about $\sigma^2 = 0.01$.

What is the mean and variance of the belief state distribution after incorporating this observation?

### 2.9 Question 2.9 (3 marks):

A friend is working on tracking cars on the road. He has defined a probability distribution over the number and location of cars using the following density function.

```java
class Car {
    double position;
    double speed;
}

class Density {

double factorial(int k) {
    if (k <= 0)
        return 1.0;
    else
        return k * factorial(k-1);
}

double poisson(double mean, double k) {
    return Math.pow(mean, k) * Math.exp(-mean) / factorial(k);
}

double normal(double mean, double stdDev, double x) {
    double d = (x - mean) / stdDev;
    double variance = stdDev * stdDev;
    return 1/Math.sqrt(2*Math.PI*variance) * Math.exp(-d*d/2.0);
}

double density(List<Car> carList) {
    int n = carList.size();
    double pn = poisson(5, n)
    double pCars = 1;
    for (int i=0; i<n; i++) {
        Car c = carList.get(i);
        double pCar = normal(10*i, 3, c.position) * normal(15, 2, c.speed);
        pCars *= pCar;
    }
    return pCars;
}
```
What is the probability that there are exactly 2 or exactly 3 cars, and every car is between location 10 and 20 and has a velocity between 10 and 20 $m.s^{-1}$?

If your answer requires integrating a continuous distribution, then you may give the answer informally in terms of the integral. e.g. $P = \int_{-3}^{6} N(5.2, 6.7)$. (Make sure you’re clear if the second argument to the normal is a standard deviation or a variance.) There are online calculators that can approximate this integral for you if you wish - search for “cumulative normal distribution function”.

2.10 Question 2.10 (1 mark):

You have a spinning device for a game. You know that internally it has three states, $A$, $B$, and $C$. Each time-step the state rotates one unit, $A \rightarrow B$, $B \rightarrow C$, $C \rightarrow A$. However, the window allowing you to read the spinner is blurry, and so you cannot read the state exactly. For each state you have a 60% chance of reading the correct state, and a 20% chance of seeing each of the other states.

You are watching this spinner spin. You want to try to predict what you will observe in the future.

What space of possible hypotheses/models could you use for this investigation (and how is this space parameterised)?

2.11 Question 2.11 (1 mark):

What initial probability distribution would you use over this hypothesis space?

2.12 Question 2.12 (1 mark):

Your first observation is $A$. What is the posterior distribution over the space of models?

2.13 Question 2.13 (1 mark):

What is the probability that you observe a $B$ on the next time-step?