Your questions will mostly be of the following type: What happens to an algorithm presented in class if some small change is made. Examples are below.

1. **Skip Lists** Possible extensions:
   - Extending Skip Lists randomised data structure so that order statistic (i.e., finding the \(i^{th}\) element in size) also runs in time \(O(\log n)\).
   - Extending Skip Lists randomised data structure so that finding the median of all elements runs in time \(O(1)\).

2. **Order Statistic**
   - Randomised version: Tweak the algorithm so that it performs well even if the array has lots of repetitions.
   - Deterministic algorithm: estimate the run time of the algorithm if you split elements in groups of 7 elements rather than 5, as we did.
   - Why splitting elements in group of 3 does not work.

3. **Karger’s MinCut algorithm**
   - Tweak Karger’s MinCut algorithm so that the probability of success is \(1 - \frac{1}{n^2}\) while maintaining the run time of \(O(n^2(\log n)^3)\)
   - Tweak Karger’s MinCut algorithm so that the probability of success is \(1 - \frac{1}{n\log n}\) with a run time of \(O(n^2(\log n)^4)\)

4. **Method of random projections**
   - Universal hashing What happens if you project onto two randomly chosen vectors instead of just one?
   - Johnson-Lindenstrauss Lemma You have to investigate various properties of objects obtained by drawing random points from a unit ball in a high dimensional space. For example, assume you draw two points uniformly randomly from the unit ball. What is the expected distance between them?
   - Clustering Clustering in spaces of extremely high dimension:
     - Preprocessing to reduce dimensionality via the Johnson-Lindenstrauss Lemma
     - Preprocessing to reduce dimensionality via the Singular Value Decomposition (SVD), by projecting onto subspace spanned by singular vectors corresponding to a few largest singular values.

5. **Markov Chains and Random Walks on Graphs**
   - PageRank as a stationary distribution of a Markov chain.
     - How would the expected duration of surfing between two consecutive random jumps change if the teleportation factor \(\alpha\) were increased from 0.85 to 0.95.
     - Would such an increase make the PageRank more stable or less stable with respect to the changes in the web structure as new pages are added and some old ones deleted.
     - Possibly other applications of PageRank algorithm, for example to ranking scientific paper using the citations of papers.
Hidden Markov Models and The Viterbi Algorithm  You might be given a simple hidden Markov Model and a sequence of a few observations; your task would be to run the Viterbi Algorithm by hand and determine the most likely sequence of states which causes the given observations.

(6) The Discrete Fourier Transform, the Discrete Cosine Transform and the Convolution

- Explain why JPEG and MP3 use the Discrete Cosine Transform to compress an image or a sound instead of the Discrete Fourier Transform
- Basic applications of convolution, such as the following problems:
  (a) Describe how you would compute in time \( O(n \log n) \) all elements of the sequence \( F(0), F(1), F(2), \ldots, F(2n) \) where
    
    \[
    F(m) = \sum_{i+j=m} i^3 j^2 \quad \text{for} \ 0 \leq i,j \leq n
    \]
  (ii) 
    
    \[
    F(m) = \sum_{i+j=m} \log(j + 1)^i \quad \text{for} \ 0 \leq i,j \leq n
    \]

(7) Recommender Systems Assume you have the Netflix data of customer evaluation of movies. You pre-process user evaluations by first subtracting the mean of all ratings of all movies and then remove the systematic biases of all users and all movies.

  (a) You would like to group users into 10 groups such that customers in each group have similar tastes. How would you accomplish this?
  (b) You then group all the movies into 10 groups such that the movies in each group have similar (preprocessed) ratings across all users.
  (c) Finally, you assign each movie to the group of users obtained in (7a) which gives to that movie the highest average score thus obtaining another grouping of all movies into 10 groups. Do you think that such a grouping of all movies should be similar to the one obtained in (7b)?

(8) Transmit Power Control Assume that a matrix \( A \) has only positive entries and that its spectral radius is less than one. Show that then matrix \( (I - A) \) is invertible and its inverse \( (I - A)^{-1} \) also has only positive entries.