COMP 4161

NICTA Advanced Course

Advanced Topics in Software Verification

Gerwin Klein, June Andronick, Toby Murray
Binary Search (java.util.Arrays)

```java
public static int binarySearch(int[] a, int key) {
    int low = 0;
    int high = a.length - 1;
    while (low <= high) {
        int mid = (low + high) / 2;
        int midVal = a[mid];
        if (midVal < key)
            low = mid + 1
        else if (midVal > key)
            high = mid - 1;
        else
            return mid; // key found
    }
    return -(low + 1); // key not found.
}
```

6: int mid = (low + high) / 2;

http://googleresearch.blogspot.com/2006/06/extra-extra-read-all-about-it-nearly.html
Organisatorials

**When**
- Tue: 9:00 – 10:30
- Thu: 9:00 – 10:30

**Where**
- Tue: Webster 256
- Thu: ASBus 115

http://www.cse.unsw.edu.au/~cs4161/
Members of the seL4 verification team

- Functional correctness of a C microkernel
  Isabelle/HOL model ↔ Haskell model ↔ C code
- 10 000 KLOC / 300 000 lines of proof script (!)
- 25 person years / $6 million

What you will learn

➜ how to use a theorem prover
➜ background, how it works
➜ how to prove and specify
➜ how to reason about programs

Health Warning

Theorem Proving is addictive
Content — Using Theorem Provers

→ Intro & motivation, getting started [today]

→ Foundations & Principles
  • Lambda Calculus, natural deduction [2,3,4a]
  • Higher Order Logic [5,6b,7]
  • Term rewriting [8,9,10c]

→ Proof & Specification Techniques
  • Isar [11,12d]
  • Inductively defined sets, rule induction [13e,15]
  • Datatypes, recursion, induction [16,17f,18,19]
  • Calculational reasoning, mathematics style proofs [20]
  • Hoare logic, proofs about programs [21g,22,23]

a1 out; b1 due; c2 out; a2 due; session break; f a3 out; g a3 due
What you should do to have a chance of succeeding

→ attend lectures
→ try Isabelle early
→ redo all the demos alone
→ try the exercises/homework we give, when we do give some
→ DO NOT CHEAT
  • Assignments and exams are take-home. This does NOT mean you can work in groups. Each submission is personal.
  • For more info, see Plagiarism Policy

\(^a\) http://www.cse.unsw.edu.au/people/studentoffice/policies/yellowform.html#assign
some material (in using-theorem-provers part) shamelessly stolen from

Tobias Nipkow, Larry Paulson, Markus Wenzel

David Basin, Burkhard Wolff

Don’t blame them, errors are mine
What is a proof?

to prove

→ from Latin probare (test, approve, prove)
→ to learn or find out by experience (archaic)
→ to establish the existence, truth, or validity of
   (by evidence or logic)
   *prove a theorem, the charges were never proved in court*

pops up everywhere

→ politics (weapons of mass destruction)
→ courts (beyond reasonable doubt)
→ religion (god exists)
→ science (cold fusion works)
What is a mathematical proof?

In mathematics, a proof is a demonstration that, given certain axioms, some statement of interest is necessarily true. (Wikipedia)

**Example:** $\sqrt{2}$ is not rational.

Proof: assume there is $r \in \mathbb{Q}$ such that $r^2 = 2$.

Hence there are mutually prime $p$ and $q$ with $r = \frac{p}{q}$.

Thus $2q^2 = p^2$, i.e. $p^2$ is divisible by 2.

2 is prime, hence it also divides $p$, i.e. $p = 2s$.

Substituting this into $2q^2 = p^2$ and dividing by 2 gives $q^2 = 2s^2$. Hence, $q$ is also divisible by 2. Contradiction. Qed.
Nice, but..

- still not rigorous enough for some
  - what are the rules?
  - what are the axioms?
  - how big can the steps be?
  - what is obvious or trivial?
- informal language, easy to get wrong
- easy to miss something, easy to cheat

**Theorem.** A cat has nine tails.

**Proof.** No cat has eight tails. Since one cat has one more tail than no cat, it must have nine tails.
What is a formal proof?

A derivation in a formal calculus

Example: $A \land B \rightarrow B \land A$ derivable in the following system

Rules:

- \[ \frac{X \in S}{S \vdash X} \] (assumption)
- \[ \frac{S \vdash X \rightarrow Y}{S \vdash X \rightarrow Y} \] (impl)
- \[ \frac{S \vdash X \quad S \vdash Y}{S \vdash X \land Y} \] (conj)
- \[ \frac{S \vdash X \land Y \quad S \vdash Z}{S \cup \{X, Y\} \vdash Z} \] (conjE)

Proof:

1. \{A, B\} \vdash B (by assumption)
2. \{A, B\} \vdash A (by assumption)
3. \{A, B\} \vdash B \land A (by conjl with 1 and 2)
4. \{A \land B\} \vdash B \land A (by conjE with 3)
5. \{\} \vdash A \land B \rightarrow B \land A (by impl with 4)
What is a theorem prover?

Implementation of a formal logic on a computer.

➔ fully automated (propositional logic)
➔ automated, but not necessarily terminating (first order logic)
➔ with automation, but mainly interactive (higher order logic)

➔ based on rules and axioms
➔ can deliver proofs

There are other (algorithmic) verification tools:

➔ model checking, static analysis, ...
➔ usually do not deliver proofs
Why theorem proving?

→ Analysing systems/programs thoroughly
→ Finding design and specification errors early
→ High assurance (mathematical, machine checked proof)
→ it’s not always easy
→ it’s fun
Main theorem proving system for this course

Isabelle

→ used here for applications, learning how to prove
What is Isabelle?

A generic interactive proof assistant

→ **generic:**
  not specialised to one particular logic
  (two large developments: HOL and ZF, will mainly use HOL)

→ **interactive:**
  more than just yes/no, you can interactively guide the system

→ **proof assistant:**
  helps to explore, find, and maintain proofs
Why Isabelle?

- free
- widely used systems
- active development
- high expressiveness and automation
- reasonably easy to use
- (and because we know it best ;-))
If I prove it on the computer, it is correct, right?
If I prove it on the computer, it is correct, right?

No, because:

1. hardware could be faulty
2. operating system could be faulty
3. implementation runtime system could be faulty
4. compiler could be faulty
5. implementation could be faulty
6. logic could be inconsistent
7. theorem could mean something else
If I prove it on the computer, it is correct, right?

No, but:

probability for

- OS and H/W issues reduced by using different systems
- runtime/compiler bugs reduced by using different compilers
- faulty implementation reduced by right architecture
- inconsistent logic reduced by implementing and analysing it
- wrong theorem reduced by expressive/intuitive logics

No guarantees, but assurance immensely higher than manual proof
If I prove it on the computer, it is correct, right?

Soundness architectures

<table>
<thead>
<tr>
<th>Approach</th>
<th>System(s)</th>
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<tbody>
<tr>
<td>careful implementation</td>
<td>PVS</td>
</tr>
<tr>
<td>LCF approach, small proof kernel</td>
<td>HOL4, Isabelle</td>
</tr>
<tr>
<td>explicit proofs + proof checker</td>
<td>Coq, Twelf, Isabelle, HOL4</td>
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Meta Logic

**Meta language:**
The language used to talk about another language.

**Examples:**
English in a Spanish class, English in an English class

**Meta logic:**
The logic used to formalize another logic

**Example:**
Mathematics used to formalize derivations in formal logic
Meta Logic – Example

Formulae: \[ F ::= V \mid F \rightarrow F \mid F \land F \mid False \]

Syntax:
\[ V ::= [A - Z] \]

Derivable: \[ S \vdash X \quad X \text{ a formula, } S \text{ a set of formulae} \]

logic / meta logic

\[ \begin{array}{l}
X \in S \\
\hline
S \vdash X
\end{array} \quad \begin{array}{l}
S \cup \{X\} \vdash Y \\
\hline
S \vdash X \rightarrow Y
\end{array} \]

\[ \begin{array}{l}
S \vdash X \quad S \vdash Y \\
\hline
S \vdash X \land Y
\end{array} \quad \begin{array}{l}
S \cup \{X, Y\} \vdash Z \\
\hline
S \cup \{X \land Y\} \vdash Z
\end{array} \]
Isabelle’s Meta Logic
Syntax: $\forall x. F$  
($F$ another meta level formula)

in ASCII:  `!!x. F`

→ universal quantifier on the meta level
→ used to denote parameters
→ example and more later
Syntax: \[ A \implies B \] \quad (A, B \text{ other meta level formulae})

in ASCII: \[ A \implies B \]

Binds to the right:

\[
A \implies B \implies C = A \implies (B \implies C)
\]

Abbreviation:

\[
[A; B] \implies C = A \implies B \implies C
\]

→ read: \( A \) and \( B \) implies \( C \)

→ used to write down rules, theorems, and proof states
Example: a theorem

**Mathematics:** if \( x < 0 \) and \( y < 0 \), then \( x + y < 0 \)

**Formal Logic:** \( \vdash x < 0 \land y < 0 \rightarrow x + y < 0 \)

**Variation:** \( x < 0; y < 0 \vdash x + y < 0 \)

**Isabelle:**

- **Lemma:** "\( x < 0 \land y < 0 \rightarrow x + y < 0 \)"
- **Variation:** "\( [x < 0; y < 0] \Rightarrow x + y < 0 \)"
- **Variation:** "\( \text{lemma} \) assumes "\( x < 0 \)" and "\( y < 0 \)" shows "\( x + y < 0 \)""
Example: a rule

logic: \[
\frac{X \quad Y}{X \land Y}
\]

variation: \[
\frac{S \vdash X \quad S \vdash Y}{S \vdash X \land Y}
\]

Isabelle: \[
\llbracket X; Y \rrbracket \implies X \land Y
\]
Example: a rule with nested implication

\begin{align*}
\text{logic:} \\
& \begin{array}{c}
X \quad Y \\
\vdots \\
X \lor Y \\
\hline \\
Z \\
\end{array} \\
& \begin{array}{c}
X \\
\vdots \\
Y \\
\hline \\
Z \\
\end{array} \\
& \begin{array}{c}
X \lor Y \\
\hline \\
Z \\
\end{array}
\end{align*}

\text{variation:}
\begin{align*}
S \cup \{X\} \vdash Z \\
S \cup \{Y\} \vdash Z \\
S \cup \{X \lor Y\} \vdash Z
\end{align*}

\text{Isabelle:}
\begin{align*}
\& [X \lor Y; X \Rightarrow Z; Y \Rightarrow Z] \Rightarrow Z
\end{align*}
Syntax: $\lambda x. F$  
(F another meta level formula)

in ASCII: %x . F

- lambda abstraction
- used for functions in object logics
- used to encode bound variables in object logics
- more about this in the next lecture
ENOUGH THEORY!
GETTING STARTED WITH ISABELLE
System Architecture

Proof General – user interface

HOL, ZF – object-logics

Isabelle – generic, interactive theorem prover

Standard ML – logic implemented as ADT

User can access all layers!
System Requirements

➔ Linux, Windows, FreeBSD, MacOS X or Solaris
➔ Standard ML
    (PolyML fastest, SML/NJ supports more platforms)
➔ Emacs (for ProofGeneral) or Java (for jEdit)

Premade packages for Linux, Mac, and Windows + info on:
http://mirror.cse.unsw.edu.au/pub/isabelle/download.html
Available from http://isabelle.in.tum.de

→ Learning Isabelle
  • Tutorial on Isabelle/HOL (LNCS 2283)
  • Tutorial on Isar
  • Tutorial on Locales

→ Reference Manuals
  • Isabelle/Isar Reference Manual
  • Isabelle Reference Manual
  • Isabelle System Manual

→ Reference Manuals for Object-Logics
ProofGeneral

- User interface for Isabelle
- Runs under XEmacs or Emacs
- Isabelle process in background

Interaction via

- Basic editing in XEmacs (with highlighting etc)
- Buttons (tool bar)
- Key bindings
- ProofGeneral Menu (lots of options, try them)
X-Symbol Cheat Sheet

Input of funny symbols in ProofGeneral

- via menu (“X-Symbol”)
- via ASCII encoding (similar to \LaTeX): \texttt{\textbackslash and}, \texttt{\textbackslash or}, …
- via abbreviation: /\, \textbackslash /, \textbackslash \rightarrow, …
- via rotate: \texttt{1 C-}. = \lambda (cycles through variations of letter)

<table>
<thead>
<tr>
<th>(\forall)</th>
<th>(\exists)</th>
<th>(\lambda)</th>
<th>(-)</th>
<th>(\land)</th>
<th>(\lor)</th>
<th>(\rightarrow)</th>
<th>(\Rightarrow)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>\texttt{\textbackslash forall}</td>
<td>\texttt{\textbackslash exists}</td>
<td>\texttt{\textbackslash lambda}</td>
<td>\texttt{\textbackslash not}</td>
<td>/\</td>
<td>\textbackslash /</td>
<td>\textbackslash \rightarrow</td>
</tr>
<tr>
<td>2</td>
<td>\texttt{ALL}</td>
<td>\texttt{EX}</td>
<td>%</td>
<td>~</td>
<td>&amp;</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

① converted to X-Symbol  
② stays ASCII
DEMO
Exercises

→ Download and install Isabelle from
  http://mirror.cse.unsw.edu.au/pub/isabelle/
→ Switch on X-Symbol in ProofGeneral
→ Step through the demo files from the lecture web page
→ Write your own theory file, look at some theorems in the library, try 'find theorem’

→ How many theorems can help you if you need to prove something like “Suc(Suc x))”?
→ What is the name of the theorem for associativity of addition of natural numbers in the library?