



COMP4161: Advanced Topics in Software Verification

C

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Last Time



- Deep and shallow embeddings
- Isabelle records
- Nondeterministic State Monad with Failure
- Monadic Weakest Precondition Rules

Content



- Intro & motivation, getting started [1]
- Foundations & Principles
 - Lambda Calculus, natural deduction [1,2]
 - Higher Order Logic, Isar (part 1) [3^a]
 - Term rewriting [4]
- Proof & Specification Techniques
 - Inductively defined sets, rule induction [5]
 - Datatypes, recursion, induction, Isar (part 2) [6, 7^b]
 - Hoare logic, proofs about programs, invariants [8]
 - C verification [9]
 - Practice, questions, exam prep [10^c]

^aa1 due; ^ba2 due; ^ca3 due

wp



apply (wp *extra_wp_rules*)

Tactic for automatic application of **weakest precondition rules**

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- Originally developed by Thomas Sewell, NICTA/Data61, for the seL4 proofs
- Knows about a huge collection of existing wp rules for monads
- Works best when precondition is a schematic variable
- related tool: **wpc** for Hoare reasoning over **case** statements

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Today we will learn about AutoCorres and C verification.

Demo

Introduction to AutoCorres and wp

A Brief Overview of C and Simpl

C



Main new problems in verifying C programs:

- expressions with side effects
- more control flow (do/while, for, break, continue, return)
- local variables and blocks
- functions & procedures
- concrete C data types
- C memory model and C pointers

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C is not a nice language for reasoning.

Things are going to get ugly.

AutoCorres will help.

C Parser: translates C into Simpl



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C Parser: parses C, produces Simpl definitions in Isabelle

- written by Michael Norrish, NICTA/Data61 and ANU
- Handles a non-trivial subset of C
- Originally written to verify seL4's C implementation
- AutoCorres is built on top of the C Parser

Commands in Simpl



```
datatype ('s, 'p, 'f) com =  
  Skip  
| Basic "'s  $\Rightarrow$  's"  
| Spec "('s * 's) set"  
| Seq "('s, 'p, 'f) com" "('s, 'p, 'f) com"  
| Cond "'s set" "('s, 'p, 'f) com" "('s, 'p, 'f) com"  
| While "'s set" "('s, 'p, 'f) com"  
| Call 'p  
| DynCom "'s  $\Rightarrow$  ('s, 'p, 'f) com"  
| Guard 'f "'s set" "('s, 'p, 'f) com"  
| Throw  
| Catch "('s, 'p, 'f) com" "('s, 'p, 'f) com"
```

's = state, 'p = procedure names, 'f = faults

Expressions with side effects



```
a = a * b;    x = f(h);    i = ++i - i++;    x = f(h) + g(x);
```


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Alternative:

Explicitly model nondeterministic order of execution in expressions.

Control flow



```
do { c } while (condition);
```

automatically translates into:

```
c; while (condition) { c }
```

Similarly:

```
for (init; condition; increment) { c }
```

becomes

```
init; while (condition) { c; increment; }
```

More control flow: break/continue



```
while (condition) {  
    foo;  
    if (Q) continue;  
    bar;  
    if (P) break;  
}
```

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Non-local control flow: **continue** goes to condition, **break** goes to end.

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→ throw exception '**continue**', catch at end of body.

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Non-local control flow: **continue** goes to condition, **break** goes to end.
Can be modelled with exceptions:

- throw exception '**continue**', catch at end of body.
- throw exception '**break**', catch after loop.

Break/continue



Break/continue example becomes:

```
try {  
    while (condition) {  
        try {  
            foo;  
            if (Q) { exception = 'continue'; throw; }  
            bar;  
            if (P) { exception = 'break'; throw; }  
        } catch { if (exception == 'continue') SKIP else throw; }  
    }  
} catch { if (exception == 'break') SKIP else throw; }
```

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```

This is not C any more. But it models C behaviour!

Need to be careful that only the translation has access to exception state.

Return



```
if (P) return x;  
foo;  
return y;
```

Similar non-local control flow.

Return



```
if (P) return x;  
foo;  
return y;
```

Similar non-local control flow. **Similar solution:** use throw/try/catch

```
try {  
    if (P) { return_val = x; exception = 'return'; throw; }  
    foo;  
    return_val = y; exception = 'return'; throw;  
} catch {  
    SKIP  
}
```

AutoCorres

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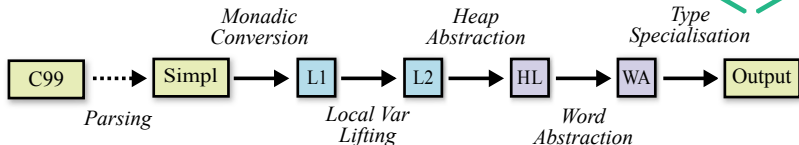
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For each Simpl definition C and its generated shallow embedding A :

- AutoCorres proves an Isabelle theorem stating that C **refines** A
- Every behaviour of C has a corresponding behaviour of A
- Refinement guarantees that properties proved about A will also hold for C .
- (Provided that A never fails. c.f. Total Correctness)

AutoCorres Process



L1: initial monadic shallow embedding

L2: local variables introduced by λ -bindings

HL: heap state abstracted into a set of **typed heaps**

WA: machine words abstracted to idealised integers or nats

Output: human-readable output with **type strengthening**, polish

On-the-fly proof:

Simpl refines **L1** refines **L2** refines **HL** refines **WA** refines **Output**

Example: C99



We will use the following example program to illustrate each of the phases.

```
unsigned some_func(unsigned *a, unsigned *b, unsigned c) {  
    unsigned *p = NULL;  
  
    if (c > 10u){  
        p = a;  
    } else {  
        p = b;  
    }  
  
    return *p;  
}
```

Example: Simpl



```
some_func_body ≡
TRY
  `p := ptr_coerce (Ptr (scast 0));;
  IF 0xA < `c THEN
    `p := `a
  ELSE
    `p := `b
  FI;;
  Guard C_Guard {c_guard `p}
  (creturn global_exn_var_'_update ret__unsigned_'_update
   (λs. h_val (hrs_mem (t_hrs_' (globals s))) (p_' s))));;
  Guard DontReach {} SKIP
CATCH SKIP END
```

Example: L1

(monadic shallow embedding)



```
l1_some_func ≡ L1_seq (L1_init ret__unsigned_'_update)
  (L1_seq (L1_modify (p_'_update (λ_. ptr_coerce (Ptr (scast 0)))))
    (L1_seq (L1_condition (λs. 0xA < c_' s)
      (L1_modify (λs. s[p_] := a_' s)))
      (L1_modify (λs. s[p_] := b_' s))))
    (L1_seq (L1_guard (λs. c_guard (p_' s)))
      (L1_seq (L1_modify (λs. s[ret__unsigned_] :=
        h_val (hrs_mem (t_hrs_' (globals s))) (p_' s))))
        (L1_modify (global_exn_var_'_update (λ_. Return))))))
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        h_val (hrs_mem (t_hrs_ ' (globals s))) (p_ ' s)]))
        (L1_modify (global_exn_var_ 'update (λ_. Return))))))
```

State type is the same as Simpl, namely a record with fields:

- **globals**: heap and type information
- **a_**, **b_**, **c_**, **p_** (parameters and local variables)
- **ret__unsigned_**, **global_exn_var_** (return value, exception type)

Example: L2

(local variables lifted)



```
l2_some_func a b c ≡  
L2_seq (L2_condition (λs. 0xA < c)  
        (L2_gets (λs. a) ['p'])  
        (L2_gets (λs. b) ['p'])))  
(λp. L2_seq (L2_guard (λs. c_guard p))  
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```

State is a record with just the **globals** field

- function now takes its parameters as arguments
- local variable **p** now passed via λ -binding
- **L2_gets** annotated with local variable names
- This ensures preservation by later AutoCorres phases

Example: HL

(heap abstracted into typed heaps)



```
hl_some_func a b c ≡  
L2_seq (L2_condition (λs. 0xA < c)  
        (L2_gets (λs. a) ['p']))  
        (L2_gets (λs. b) ['p']))  
(λr. L2_seq (L2_guard (λs. is_valid_w32 s r)  
        (λ_. L2_gets (λs. heap_w32 s r) ['ret'])))
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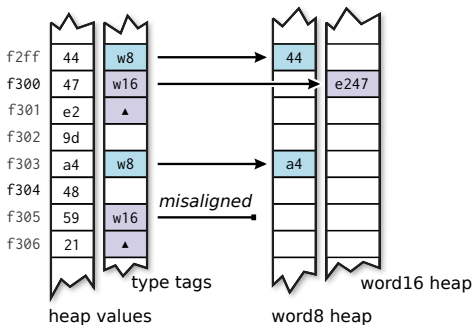
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State is a record with a set of **is_valid_** and **heap_** fields:

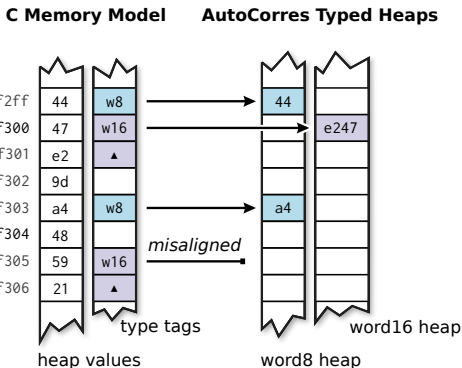
- These store **pointer validity** and **heap contents** respectively, per type
- above example has only 32-bit word pointers

Heap Abstraction

C Memory Model AutoCorres Typed Heaps



Heap Abstraction



C Memory Model: by Harvey Tuch

- **Heap** is a mapping from 32-bit addresses to bytes: 32 word \Rightarrow 8 word
- **Heap Type Description** stores type information for each heap location

Example: WA

(words abstracted to ints and nats)



```
wa_some_func a b c ≡  
L2_seq (L2_condition (λs. 10 < c)  
        (L2_gets (λs. a) [''p''])  
        (L2_gets (λs. b) [''p''])))  
(λr. L2_seq (L2_guard (λs. is_valid_w32 s r))  
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Word abstraction: $C \text{ int} \rightarrow \text{Isabelle int}$, $C \text{ unsigned} \rightarrow \text{Isabelle nat}$

- Guards inserted to ensure absence of unsigned underflow and overflow
- Signed under/overflow already has guards (it has undefined behaviour)

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In the example, the **unsigned** argument **c** is now of type **nat**

- The function also returns a nat result
- The heap is not abstracted, hence the call to **unat**

Example: Output

(type strengthening and polish)



```
some_func' a b c ≡  
DO p ← oreturn (if 10 < c then a else b);  
    oguard (λs. is_valid_w32 s p);  
    ogets (λs. unat (heap_w32 s p))  
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Type Strengthening:

- Tries to convert output to a more restricted monad
- The above is in the **option** monad because it doesn't modify the state, but might fail
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Polish:

- Simplify output as much as possible
- The **condition** has been rewritten to a **return** because the condition **10 < c** doesn't depend on the state

Type Strengthening

Example:

```
unsigned zero(void){ return 0u; }
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Monad Type	Kind	Type	Example
pure	Pure function	'a	0
gets	Read-only, non-failing	's \Rightarrow 'a	$\lambda s. 0$
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Can be controlled by the **ts_force** option of AutoCorres

(Reader) Option Monad



Another standard monad, familiar from e.g. Haskell

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$\text{return } x \equiv \lambda s. \text{Some } x$

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$\text{obind } a \ b \equiv \lambda s. \text{case } a \ s \text{ of } \text{None} \Rightarrow \text{None} \mid \text{Some } r \Rightarrow b \ r \ s$

→ **Infix notation:** $|>>$

→ **Do notation:** DO ... OD

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Hoare Logic:

$$\text{valid } P \ f \ Q \equiv \forall s \ r. P \ s \wedge f \ s = \text{Some } r \longrightarrow Q \ r \ s$$

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$$\text{ovoid } (P \ x) \ (\text{oreturn } x) \ P$$

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$$\text{oreturn } x \equiv \lambda s. \text{Some } x$$

Bind:

$$\text{obind } a \ b \equiv \lambda s. \text{case } a \ s \text{ of } \text{None} \Rightarrow \text{None} \mid \text{Some } r \Rightarrow b \ r \ s$$

→ **Infix notation:** $|>>$

→ **Do notation:** DO ... OD

Hoare Logic:

$$\text{ovalid } P \ f \ Q \equiv \forall s \ r. P \ s \wedge f \ s = \text{Some } r \longrightarrow Q \ r \ s$$

$$\text{ovalid } (P \ x) \ (\text{oreturn } x) \ P \quad \frac{\bigwedge r. \text{ovalid } (R \ r) \ (g \ r) \ Q \quad \text{ovalid } P \ f \ R}{\text{ovalid } P \ (f \ |>> \ g) \ Q}$$

Exception Monad



Exceptions used to model early return, break and continue.

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Exception Monad: $'s \Rightarrow ((\underline{'e + 'a}) \times 's) \text{ set } \times \text{bool}$

- Instance of the nondeterministic state monad: return-value type is **sum type** $'e + 'a$
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$\text{returnOk } x \equiv \text{return (Inr } x)$ $\text{throwError } e \equiv \text{return (Inl } e)$

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Basic Monadic Operations

$$\begin{aligned} \text{returnOk } x &\equiv \text{return (Inr } x) & \text{throwError } e &\equiv \text{return (Inl } e) \\ \text{lift } b &\equiv (\lambda x. \text{ case } x \text{ of Inl } e \Rightarrow \text{throwError } e \mid \text{Inr } r \Rightarrow b \ r) \end{aligned}$$

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Hoare Rules for Exceptions



New kind of Hoare triples to model normal and exceptional cases:

$$\{P\} f \{Q\}, \{E\}$$

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Weakest Precondition Rules:

$$\frac{}{\{P x\} \text{returnOk } x \{P\}, \{E\}}$$

$$\frac{}{\{E e\} \text{throwError } e \{P\}, \{E\}}$$

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Weakest Precondition Rules:

$$\begin{array}{c} \frac{}{\{P x\} \text{returnOk } x \{P\}, \{E\}} \qquad \frac{}{\{E e\} \text{throwError } e \{P\}, \{E\}} \\[1em] \frac{\bigwedge x. \{R x\} b x \{Q\}, \{E\} \quad \{P\} a \{R\}, \{E\}}{\{P\} a \gg=E b \{Q\}, \{E\}} \end{array}$$

(other rules analogous)

Today we have seen



- The automated proof method **wp**
- The C Parser and translating C into Simpl
- AutoCorres and translating Simpl into monadic form
- The option and exception monads