



COMP4161: Advanced Topics in Software Verification



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Content

→ Foundations & Principles

- Intro, Lambda calculus, natural deduction [1,2]
- Higher Order Logic, Isar (part 1) [2,3^a]
- Term rewriting [3,4]

→ Proof & Specification Techniques

- Inductively defined sets, rule induction, datatype induction, primitive recursion [4,5]
- General recursive functions, termination proofs [7^b]
- Proof automation, Hoare logic, proofs about programs, invariants [8]
- C verification [9,10]
- Practice, questions, examp prep [10^c]

^aa1 due; ^ba2 due; ^ca3 due

Deep Embeddings

We used a **datatype** *com* to represent the **syntax** of IMP.

→ We then defined semantics over this datatype.

This is called a **deep embedding**:

→ separate representation of language terms and their semantics.

Advantages:

→ Prove general theorems about the **language**, not just of programs.

→ e.g. expressiveness, correct compilation, inference completeness ...

→ usually by structural induction over the syntax type.

Disadvantages:

→ Semantically equivalent programs are not obviously equal.

→ e.g. "IF True THEN SKIP ELSE SKIP = SKIP" is not a true theorem.

→ Many concepts already present in the logic must be reinvented.

Shallow Embeddings

Shallow Embedding: represent only the semantics, directly in the logic.

- A definition for each language construct, giving its **semantics**.
- Programs are represented as instances of these definitions.

Example: program semantics as functions $state \Rightarrow state$

$$\text{SKIP} \equiv \lambda s. s$$

$$\text{IF } b \text{ THEN } c \text{ ELSE } d \equiv \lambda s. \text{if } b \text{ s then } c \text{ s else } d \text{ s}$$

- “IF True THEN SKIP ELSE SKIP = SKIP” is now a true statement.
- can use the simplifier to do semantics-preserving program rewriting.

Today: a shallow embedding for (interesting parts of) C semantics

Records in Isabelle

Records are n -tuples with named components

Example:

$$\mathbf{record} \ A = \quad a :: \mathit{nat}$$
$$\quad \quad \quad b :: \mathit{int}$$

- Selectors: $a :: A \Rightarrow \mathit{nat}$, $b :: A \Rightarrow \mathit{int}$, $a \ r = \mathit{Suc} \ 0$
- Constructors: $(| a = \mathit{Suc} \ 0, b = -1 |)$
- Update: $r(| a := \mathit{Suc} \ 0 |)$, $b_update \ (\lambda b. b + 1) \ r$

Records are extensible:

$$\mathbf{record} \ B = A +$$
$$\quad \quad \quad c :: \mathit{nat} \ \mathit{list}$$
$$(| a = \mathit{Suc} \ 0, b = -1, c = [0, 0] |)$$

Demo

Nondeterministic State Monad with Failure

Shallow embedding suitable for (a useful fragment of) C.

Can express lots of C ideas:

- Access to `volatile` variables, external APIs: **Nondeterminism**
- Undefined behaviour: **Failure**
- Early exit (`return`, `break`, `continue`): **Exceptional control flow**

Relatively straightforward Hoare logic

Used extensively in the seL4 microkernel verification work.

AutoCorres: verified translation from deeply embedded C to monadic representation

- Specifically designed for humans to do proofs over.

State Monad: Motivation

Model the **semantics** of a (deterministic) computation as a function

$$'s \Rightarrow ('a \times 's)$$

The computation operates over a **state** of type `'s`:

→ Includes all global variables, external devices, etc.

The computation also yields a **return value** of type `'a`:

→ models e.g. exit status and return values

return – the computation that leaves the state unchanged and returns its argument:

$$\text{return } x \equiv \lambda s. (x, s)$$

State Monad: Basic Operations

get – returns the entire state without modifying it:

$$\text{get} \equiv \lambda s. (s, s)$$

put – replaces the state and returns the unit value ():

$$\text{put } s \equiv \lambda_. ((), s)$$

bind – sequences two computations; 2nd takes the first's result:

$$c \gg= d \equiv \lambda s. \text{let } (r, s') = c \text{ s in } d r s'$$

gets – returns a projection of the state; leaves state unchanged:

$$\text{gets } f \equiv \text{get} \gg= (\lambda s. \text{return } (f s))$$

modify – applies its argument to modify the state; returns ():

$$\text{modify } f \equiv \text{get} \gg= (\lambda s. \text{put } (f s))$$

Monads, Laws

Formally: a monad \mathbf{M} is a type constructor with two operations.

$\text{return} :: \alpha \Rightarrow \mathbf{M} \alpha$ $\text{bind} :: \mathbf{M} \alpha \Rightarrow (\alpha \Rightarrow \mathbf{M} \beta) \Rightarrow \mathbf{M} \beta$

Infix Notation: $a \gg= b$ is infix notation for $\text{bind } a \ b$

Do-Notation: $a \gg= (\lambda x. b \ x)$ is often written as **do** $\{ x \leftarrow a; b \ x \}$

Monad Laws:

return-left: $(\text{return } x \gg= f) = f \ x$

return-right: $(m \gg= \text{return}) = m$

bind-assoc: $((a \gg= b) \gg= c) = (a \gg= (\lambda x. b \ x \gg= c))$

State Monad: Example

A fragment of C:

```
void f(int *p) {  
    int x = *p;  
    if (x < 10)  
{  
        *p = x+1;  
    }  
}
```

record state =

hp :: int ptr \Rightarrow int

f :: “int ptr \Rightarrow (state \Rightarrow (unit,state))”

f p \equiv

do {

x \leftarrow gets (λ s. hp s p);

if x < 10 **then**

modify (hp_update (λ h. (h(p := x + 1))))

else

return ()

}

State Monad with Failure

Computations can **fail**: $'s \Rightarrow (('a \times 's) \times \underline{\text{bool}})$

bind – fails when either computation fails

$\text{bind } a \ b \equiv \text{let } ((r,s'),f) = a \ s; ((r'',s''),f') = b \ r \ s' \ \text{in } ((r'',s''), f \vee f')$

fail – the computation that always fails:

$\text{fail} \equiv \lambda s. (\text{undefined}, \text{True})$

assert – fails when given condition is False:

$\text{assert } P \equiv \text{if } P \ \text{then } \text{return } () \ \text{else } \text{fail}$

guard – fails when given condition applied to the state is False:

$\text{guard } P \equiv \text{get } \gg= (\lambda s. \text{assert } (P \ s))$

Guards

Used to assert the absence of **undefined behaviour** in C

→ pointer validity, absence of divide by zero, signed overflow, etc.

```
f p ≡  
  do {  
    y ← guard (λs. valid s p);  
    x ← gets (λs. hp s p);  
    if x < 10 then  
      modify (hp_update (λh. (h(p := x + 1))))  
    else  
      return ()  
  }
```

Nondeterministic State Monad with Failure

Computations can be **nondeterministic**: $'s \Rightarrow (('a \times 's) \text{ set} \times \text{bool})$

Nondeterminism: computations return a **set** of possible results.

→ Allows **underspecification**: e.g. malloc, external devices, etc.

bind – runs 2nd computation for all results returned by the first:

$$\text{bind } a \ b \equiv \ \lambda s. (\{(r'', s''). \exists (r', s') \in \text{fst } (a \ s). (r'', s'') \in \text{fst } (b \ r' \ s')\}, \\ \text{snd } (a \ s) \vee (\exists (r', s') \in \text{fst } (a \ s). \text{snd } (b \ r' \ s')))$$

All non-failing computations so far are **deterministic**:

→ e.g. $\text{return } x \equiv \lambda s. (\{(x, s)\}, \text{False})$

→ Others are similar.

select – nondeterministic selection from a set:

$$\text{select } A \equiv \lambda s. ((A \times \{s\}), \text{False})$$

Demo

While Loops

Monadic while loop, defined **inductively**.

$$\begin{aligned} \text{whileLoop} &:: ('a \Rightarrow 's \Rightarrow \text{bool}) \Rightarrow \\ &('a \Rightarrow ('s \Rightarrow ('a \times 's) \text{ set} \times \text{bool})) \Rightarrow \\ &('a \Rightarrow ('s \Rightarrow ('a \times 's) \text{ set} \times \text{bool})) \end{aligned}$$

$\text{whileLoop } C B$

- **condition** C : takes **loop parameter** and **state** as arguments, returns **bool**
- **monadic body** B : takes **loop parameter** as argument, return-value is the **updated** loop parameter
- **fails** if the loop body ever fails or if the loop never terminates

Example: $\text{whileLoop } (\lambda p s. \text{hp } s \ p = 0) (\lambda p. \text{return } (\text{ptrAdd } p \ 1))$
 p

Defining While Loops Inductively

Two-part definition: results and termination

Results: $\text{while_results} :: ('a \Rightarrow 's \Rightarrow \text{bool}) \Rightarrow$
 $('a \Rightarrow ('s \Rightarrow ('a \times 's) \text{ set} \times \text{bool})) \Rightarrow$
 $((('a \times 's) \text{ option}) \times (('a \times 's) \text{ option})) \text{ set}$

$$\frac{\neg C r s}{(\text{Some } (r,s), \text{Some } (r,s)) \in \text{while_results } C B} \text{ (terminate)}$$

$$\frac{C r s \text{ snd } (B r s)}{(\text{Some } (r,s), \text{None}) \in \text{while_results } C B} \text{ (fail)}$$

$$\frac{C r s \quad (r',s') \in \text{fst } (B r s) \quad (\text{Some } (r', s'), z) \in \text{while_results } C B}{(\text{Some } (r,s), z) \in \text{while_results } C B} \text{ (loop)}$$

Defining While Loops Inductively

Termination:

$$\text{while_terminates} :: ('a \Rightarrow 's \Rightarrow \text{bool}) \Rightarrow$$
$$('a \Rightarrow ('s \Rightarrow ('a \times 's) \text{ set} \times \text{bool}))) \Rightarrow$$
$$'a \Rightarrow 's \Rightarrow \text{bool}$$
$$\frac{\neg C r s}{\text{while_terminates } C B r s} \text{ (terminate)}$$
$$\frac{C r s \quad \forall (r', s') \in \text{fst } (B r s). \text{ while_terminates } C B r' s'}{\text{while_terminates } C B r s} \text{ (loop)}$$

$\text{whileLoop } C B \equiv$

$$(\lambda r s. (\{(r', s'). (\text{Some } (r, s), \text{Some } (r', s')) \in \text{while_results } C B\},$$
$$(\text{Some } (r, s), \text{None}) \in \text{while_results} \vee$$
$$\neg \text{while_terminates } C B r s))$$

Hoare Logic over Nondeterministic State Monads

Partial correctness:

$$\{P\} m \{Q\} \equiv \forall s. P s \longrightarrow \forall (r,s') \in \text{fst } (m s). Q r s'$$

→ Post-condition Q is a predicate of return-value and result state.

Weakest Precondition Rules

$$\{\lambda s. P x s\} \text{ return } x \{ \lambda r s. P r s \} \quad \{\lambda s. P s s\} \text{ get } \{P\} \quad \{\lambda s. P () x\} \text{ put } x \{P\}$$

$$\{\lambda s. P (f s) s\} \text{ gets } f \{P\} \quad \{\lambda s. P () (f s)\} \text{ modify } f \{P\}$$

$$\{\lambda s. P \longrightarrow Q () s\} \text{ assert } P \{Q\} \quad \{\lambda _. \text{ True}\} \text{ fail } \{Q\}$$

More Hoare Logic Rules

$$\frac{P \implies \{Q\} f \{S\} \quad \neg P \implies \{R\} g \{S\}}{\{\lambda s. (P \longrightarrow Q s) \wedge (\neg P \longrightarrow R s)\} \text{ if } P \text{ then } f \text{ else } g \{S\}}$$

$$\frac{\bigwedge x. \{B x\} g x \{C\} \quad \{A\} f \{B\}}{\{A\} \text{ do}\{ x \leftarrow f; g x \} \{C\}}$$

$$\frac{\{R\} m \{Q\} \quad \bigwedge s. P s \implies R s}{\{P\} m \{Q\}}$$

$$\frac{\bigwedge r. \{\lambda s. I r s \wedge C r s\} B \{I\} \quad \bigwedge r s. \llbracket I r s; \neg C r s \rrbracket \implies Q r s}{\{I r\} \text{ whileLoop } C B r \{Q\}}$$

Demo

We have seen today

- Deep and shallow embeddings
- Isabelle records
- Nondeterministic State Monad with Failure
- Monadic Weakest Precondition Rules