Exercise 1.

(a) Prove by induction that

\[1 \cdot 1! + 2 \cdot 2! + \ldots + n \cdot n! = (n + 1)! - 1 \quad \text{for } n \geq 1\]

(b) Given the recursive definition,

(B) \( s_1 = 1 \)

(R) \( s_{n+1} = \frac{1}{1 + s_n} \)

prove by induction that

\[s_n = \frac{\text{FIB}(n)}{\text{FIB}(n + 1)} \quad \text{for } n \geq 1\]

Exercise 2. Prove that in any rooted tree, the number of leaves is one more than the number of nodes with a right sibling.

*Exercise 3. Prove by induction that every connected graph \( G = (V, E) \) must satisfy \( e(G) \geq v(G) - 1 \).

*Exercise 4. Let \( T(n) \) be defined by the recurrence

\[T(n) = T(n - 1) + g(n) \quad \text{for } n > 1\]

Prove by induction on \( i \) that if \( 1 \leq i < n \), then

\[T(n) = T(n - i) + \sum_{j=0}^{i-1} g(n - j)\]