Solutions

Exercise 1.

(a) No, for example: $P(\heartsuit A_t = 2 \land \heartsuit A_{t-1} = 1) = 0$ (you can’t draw $\heartsuit A$ twice) while $P(\heartsuit A_{t-1} = 2) > 0$ and $P(\heartsuit A_t = 1) > 0$, hence $P(\heartsuit A_{t-1} = 2) \cdot P(\heartsuit A_t = 1) > 0$.

(b) Expected number of attempts to draw a non-ace:

$$1 \cdot \frac{48}{52} + 2 \cdot \frac{4}{52} \cdot \frac{48}{51} + 3 \cdot \frac{4}{52} \cdot \frac{3}{51} \cdot \frac{48}{50} + 4 \cdot \frac{4}{52} \cdot \frac{2}{51} \cdot \frac{3}{50} \cdot \frac{48}{49} + 5 \cdot \frac{4}{52} \cdot \frac{3}{51} \cdot \frac{2}{50} \cdot \frac{1}{49} \approx 1.0816$$

(c) The possible outcomes are:

- 1 drawing attempt: any card of value $\geq 5$
- 2 drawing attempts: a 2 followed by a card of value $\geq 3$, or a 3 or 4 followed by any card
- 3 drawing attempts: two 2s followed by any card

Hence,

$$1 \cdot \frac{40}{52} + 2 \cdot \left( \frac{4}{52} \cdot \frac{48}{51} + \frac{8}{52} \cdot \frac{51}{51} \right) + 3 \cdot \frac{4}{52} \cdot \frac{3}{51} \cdot \frac{50}{50} \approx 1.2353$$

(d) If the cards are put back, then each draw is independent since the deck from which a card is randomly selected is the same for each drawing attempt.

Each event of drawing a non-ace has the probability $p = \frac{48}{52} = \frac{12}{13}$. Hence, the expected number of drawing attempts is $\frac{1}{p} = \frac{13}{12} = 1.0833$.

Expected number of draws to obtain a sum $\geq 5$:

$$1 \cdot \frac{40}{52} + 4 \cdot \left( \frac{4}{52} \cdot \frac{48}{51} + \frac{9}{52} \cdot \frac{50}{51} \right) + 9 \cdot \frac{4}{52} \cdot \frac{3}{51} \cdot \frac{49}{49} + 25 \cdot \frac{4}{52} \cdot \frac{3}{51} \cdot \frac{2}{50} \cdot \frac{1}{49} - 1.0816^2 \approx 0.085$$

(e) Variance for the number of draws to get a non-ace:

$$1 \cdot \frac{48}{52} + 4 \cdot \frac{4}{52} \cdot \frac{48}{51} + 9 \cdot \frac{4}{52} \cdot \frac{3}{51} \cdot \frac{48}{50} + 16 \cdot \frac{4}{52} \cdot \frac{3}{51} \cdot \frac{2}{50} \cdot \frac{48}{49} + 25 \cdot \frac{4}{52} \cdot \frac{3}{51} \cdot \frac{2}{50} \cdot \frac{1}{49} - 1.0816^2 \approx 0.085$$

Variance for the number of draws to obtain a sum $\geq 5$:

$$1 \cdot \frac{40}{52} + 4 \cdot \left( \frac{4}{52} \cdot \frac{48}{51} + \frac{8}{52} \cdot \frac{51}{51} \right) + 9 \cdot \frac{4}{52} \cdot \frac{3}{51} \cdot \frac{50}{50} - 1.2353^2 \approx 0.189$$