

COMP9414: Artificial Intelligence - Solutions Week 5

Propositional Logic

1. (i) $(\neg Ja \wedge \neg Jo) \rightarrow T$

Where:

Ja: Jane is in town

Jo: John is in town

T: we will play tennis

(ii) $R \vee \neg R$

Where:

R: it will rain today

(iii) $\neg S \rightarrow \neg P$

Where:

S: you study

P: you will pass this course

2. (i) $P \rightarrow Q$

$\neg P \vee Q$ (remove \rightarrow)

(ii) $(P \rightarrow \neg Q) \rightarrow R$

$\neg(\neg P \vee \neg Q) \vee R$ (remove \rightarrow)

$(\neg\neg P \wedge \neg\neg Q) \vee R$ (De Morgan)

$(P \wedge Q) \vee R$ (Double Negation)

$(P \vee R) \wedge (Q \vee R)$ (Distribute \vee over \wedge)

(iii) $\neg(P \wedge \neg Q) \rightarrow (\neg R \vee \neg Q)$

$\neg\neg(P \wedge \neg Q) \vee (\neg R \vee \neg Q)$ (remove \rightarrow)

$(P \wedge \neg Q) \vee (\neg R \vee \neg Q)$ (Double Negation)

$(P \vee \neg R \vee \neg Q) \wedge (\neg Q \vee \neg R \vee \neg Q)$ (Distribute \vee over \wedge)

This can be further simplified to: $(P \vee \neg R \vee \neg Q) \wedge (\neg Q \vee \neg R)$

And in fact this can be simplified to $\neg Q \vee \neg R$ since $\neg Q \vee \neg R \vdash P \vee \neg R \vee \neg Q$

<i>P</i>	<i>Q</i>	$P \rightarrow Q$	$\neg Q$	$\neg P$
<i>T</i>	<i>T</i>	<i>T</i>	<i>F</i>	<i>F</i>
<i>T</i>	<i>F</i>	<i>F</i>	<i>T</i>	<i>F</i>
<i>F</i>	<i>T</i>	<i>T</i>	<i>F</i>	<i>T</i>
<i>F</i>	<i>F</i>	<i>T</i>	<i>T</i>	<i>T</i>

3. (i)

In all rows where both $P \rightarrow Q$ and $\neg Q$ are true, $\neg P$ is also true. Therefore, inference is valid.

<i>P</i>	<i>Q</i>	$\neg P$	$\neg Q$	$P \rightarrow Q$	$\neg Q \rightarrow \neg P$
<i>T</i>	<i>T</i>	<i>F</i>	<i>F</i>	<i>T</i>	<i>T</i>
<i>T</i>	<i>F</i>	<i>F</i>	<i>T</i>	<i>F</i>	<i>F</i>
<i>F</i>	<i>T</i>	<i>T</i>	<i>F</i>	<i>T</i>	<i>T</i>
<i>F</i>	<i>F</i>	<i>T</i>	<i>T</i>	<i>T</i>	<i>T</i>

(ii)

In all rows where both $P \rightarrow Q$ is true, $\neg Q \rightarrow \neg P$ is also true. Therefore, inference is valid.

<i>P</i>	<i>Q</i>	<i>R</i>	$P \rightarrow Q$	$Q \rightarrow R$	$P \rightarrow R$
<i>T</i>	<i>T</i>	<i>T</i>	<i>T</i>	<i>T</i>	<i>T</i>
<i>T</i>	<i>T</i>	<i>F</i>	<i>T</i>	<i>F</i>	<i>F</i>
<i>T</i>	<i>F</i>	<i>T</i>	<i>F</i>	<i>T</i>	<i>T</i>
<i>T</i>	<i>F</i>	<i>F</i>	<i>F</i>	<i>T</i>	<i>F</i>
<i>F</i>	<i>T</i>	<i>T</i>	<i>T</i>	<i>T</i>	<i>T</i>
<i>F</i>	<i>T</i>	<i>F</i>	<i>T</i>	<i>F</i>	<i>T</i>
<i>F</i>	<i>F</i>	<i>T</i>	<i>T</i>	<i>T</i>	<i>T</i>
<i>F</i>	<i>F</i>	<i>F</i>	<i>T</i>	<i>T</i>	<i>T</i>

(iii)

In all rows where both $P \rightarrow Q$ and $Q \rightarrow R$ are true, $P \rightarrow R$ is also true. Therefore, inference is valid.

4. (i) $\text{CNF}(P \rightarrow Q)$

$$\equiv \neg P \vee Q$$

$$\text{CNF}(\neg Q)$$

$$\equiv \neg Q$$

$$\text{CNF}(\neg\neg P)$$

$$\equiv P \text{ (Double Negation)}$$

Proof:

1. $\neg P \vee Q$ (Hypothesis)
2. $\neg Q$ (Hypothesis)
3. P (Negation of Conclusion)
4. Q 1, 3 Resolution
5. \square 2, 4 Resolution

(ii) $\text{CNF}(P \rightarrow Q)$

$$\equiv \neg P \vee Q$$

$$\text{CNF}(\neg(\neg Q \rightarrow \neg P))$$

$$\equiv \neg(\neg\neg Q \vee \neg P) \text{ (Remove } \rightarrow)$$

$$\equiv \neg(Q \vee \neg P) \text{ (Double Negation)}$$

$$\equiv \neg Q \wedge \neg\neg P \text{ (De Morgan)}$$

$$\equiv \neg Q \wedge P \text{ (Double Negation)}$$

Proof:

1. $\neg P \vee Q$ (Hypothesis)
2. $\neg Q$ (Negation of Conclusion)
3. P (Negation of Conclusion)
4. $\neg P$ 1, 2 Resolution
5. \square 3, 4 Resolution

(iii) $P \rightarrow Q, Q \rightarrow R \vdash P \rightarrow R$

$$\text{CNF}(P \rightarrow Q)$$

$$\equiv \neg P \vee Q$$

$$\text{CNF}(Q \rightarrow R)$$

$$\equiv \neg Q \vee R$$

$$\text{CNF}(\neg(P \rightarrow R))$$

$$\equiv \neg(\neg P \vee R) \text{ (Remove } \rightarrow)$$

$$\equiv \neg\neg P \wedge \neg R \text{ (De Morgan)}$$

$$\equiv P \wedge \neg R \text{ (Double Negation)}$$

Proof:

1. $\neg P \vee Q$ (Hypothesis)
2. $\neg Q \vee R$ (Hypothesis)
3. P (Negation of Conclusion)
4. $\neg R$ (Negation of Conclusion)
5. Q 1, 3 Resolution
6. R 2, 5 Resolution
7. \square 4, 6 Resolution

5. (i)

P	Q	$\neg P$	$P \vee Q$	$(P \vee Q) \wedge \neg P$	$((P \vee Q) \wedge \neg P) \rightarrow Q$
T	T	F	T	F	T
T	F	F	T	F	T
F	T	T	T	T	T
F	F	T	F	F	T

Last column is always true no matter what truth assignment to the atoms P and Q . Therefore $((P \vee Q) \wedge \neg P) \rightarrow Q$ is a tautology.

(ii) $S = ((P \rightarrow Q) \wedge \neg(P \rightarrow R)) \rightarrow (P \rightarrow Q)$

P	Q	R	$P \rightarrow Q$	$P \rightarrow R$	$\neg(P \rightarrow R)$	$(P \rightarrow Q) \wedge \neg(P \rightarrow R)$	S
T	T	T	T	T	F	F	T
T	T	F	T	F	T	T	T
T	F	T	F	T	F	F	T
T	F	F	F	F	T	F	T
F	T	T	T	T	F	F	T
F	T	F	T	T	F	F	T
F	F	T	T	T	F	F	T
F	F	F	T	T	F	F	T

Last column is always true no matter what truth assignment to the atoms P , Q and R . Therefore $((P \rightarrow Q) \wedge \neg(P \rightarrow R)) \rightarrow (P \rightarrow Q)$ is a tautology.

(iii)

P	$\neg P$	$\neg P \wedge P$	$\neg(\neg P \wedge P)$	$\neg(\neg P \wedge P) \wedge P$
T	F	F	T	T
F	T	F	T	F

Last column is not always true. Therefore $\neg(\neg P \wedge P) \wedge P$ is not a tautology.

(iv) $(P \vee Q) \rightarrow \neg(\neg P \wedge \neg Q)$

P	Q	$\neg P$	$\neg Q$	$P \vee Q$	$\neg P \wedge \neg Q$	$\neg(\neg P \wedge \neg Q)$	$(P \vee Q) \rightarrow \neg(\neg P \wedge \neg Q)$
T	T	F	F	T	F	T	T
T	F	F	T	T	F	T	T
F	T	T	F	T	F	T	T
F	F	T	T	F	T	F	T

6. (i) $\text{CNF}(\neg(((P \vee Q) \wedge \neg P) \rightarrow Q))$
 $\equiv \neg(\neg((P \vee Q) \wedge \neg P) \vee Q)$ (Remove \rightarrow)
 $\equiv \neg\neg((P \vee Q) \wedge \neg P) \wedge \neg Q$ (De Morgan)
 $\equiv (P \vee Q) \wedge \neg P \wedge \neg Q$ (Double Negation)

Proof:

1. $P \vee Q$ (Negated Conclusion)
2. $\neg P$ (Negated Conclusion)
3. $\neg Q$ (Negated Conclusion)
4. Q 1, 2 Resolution
5. \square 3, 4 Resolution

Therefore $\neg(((P \vee Q) \wedge \neg P) \rightarrow Q)$ is a tautology.

- (ii) $\text{CNF}(\neg(((P \rightarrow Q) \wedge \neg(P \rightarrow R)) \rightarrow (P \rightarrow Q)))$
 $\equiv \neg(\neg(\neg(P \vee Q) \wedge \neg(\neg P \vee R)) \vee (\neg P \vee Q))$ (Remove \rightarrow)
 $\equiv \neg\neg(\neg(P \vee Q) \wedge \neg(\neg P \vee R)) \wedge \neg(\neg P \vee Q)$ (De Morgan)
 $\equiv (\neg P \vee Q) \wedge (\neg\neg P \wedge \neg R) \wedge (\neg\neg P \wedge \neg Q)$ (Double Negation and De Morgan)
 $\equiv (\neg P \vee Q) \wedge (P \wedge \neg R) \wedge (P \wedge \neg Q)$ (Double Negation)

Proof:

1. $\neg P \vee Q$ (Negated Conclusion)
2. P (Negated Conclusion)
3. $\neg R$ (Negated Conclusion)
4. $\neg Q$ (Negated Conclusion)
5. Q 1, 2 Resolution
6. \square 4, 5 Resolution

Therefore $((P \rightarrow Q) \wedge \neg(P \rightarrow R)) \rightarrow (P \rightarrow Q)$ is a tautology.

- (iii) $\text{CNF}(\neg(\neg(\neg P \wedge P) \wedge P))$
 $\equiv \neg\neg(\neg P \wedge P) \vee \neg P$ (De Morgan)
 $\equiv (\neg P \wedge P) \vee \neg P$ (Double Negation)
 $\equiv (\neg P \vee \neg P) \wedge (P \vee \neg P)$ (Distribute \wedge over \vee)
 $\equiv \neg P$ (Can simplify to this by removing repetition and tautologies)

Proof:

1. $\neg P$ (Negated Conclusion)

Cannot obtain empty clause using resolution so $\neg(\neg P \wedge P) \wedge P$ is not a tautology.

- (iv) $\text{CNF}(\neg((P \vee Q) \rightarrow \neg(\neg P \wedge \neg Q)))$
 $\equiv \neg(\neg(P \vee Q) \vee \neg(\neg P \wedge \neg Q))$ (Remove \rightarrow)
 $\equiv \neg\neg(P \vee Q) \wedge \neg\neg(\neg P \wedge \neg Q)$ (De Morgan)
 $\equiv (P \vee Q) \wedge \neg P \wedge \neg Q$ (Double Negation)

Proof:

1. $P \vee Q$ (Negated Conclusion)
2. $\neg Q$ (Negated Conclusion)
3. $\neg P$ (Negated Conclusion)
4. Q 1, 2 Resolution
5. \square 3, 4, Resolution

Therefore $(P \vee Q) \rightarrow \neg(\neg P \wedge \neg Q)$ is a tautology.