

## COMP9414: Artificial Intelligence - Solutions Week 6

### First-Order Logic

1. (i) All birds fly.  
(If an object  $x$  is a bird, then it flies.)
- (ii) Everyone has a mother.
- (iii) There is someone who is everyone's mother.
2. (i)  $\forall x (cat(x) \rightarrow mammal(x))$
- (ii)  $\neg \exists x (cat(x) \wedge reptile(x))$   
or, equivalently,  $\forall x (cat(x) \rightarrow \neg reptile(x))$
- (iii)  $\forall x (computer\_scientist(x) \rightarrow \exists y (operating\_system(y) \wedge likes(x, y)))$
3. (i) CNF( $\forall x (bird(x) \rightarrow flies(x))$ )  
 $\equiv \forall x (\neg bird(x) \vee flies(x))$  (Remove  $\rightarrow$ )  
 $\equiv \neg bird(x) \vee flies(x)$  (Drop  $\forall$ )
- (ii) CNF( $\exists x \forall y \forall z (person(x) \wedge ((likes(x, y) \wedge y \neq z) \rightarrow \neg likes(x, z)))$ )  
 $\equiv \exists x \forall y \forall z (person(x) \wedge (\neg(likes(x, y) \wedge y \neq z) \vee \neg likes(x, z)))$  (Remove  $\rightarrow$ )  
 $\equiv \exists x \forall y \forall z (person(x) \wedge (\neg likes(x, y) \vee y = z \vee \neg likes(x, z)))$  (De Morgan)  
 $\equiv \forall y \forall z (person(c) \wedge (\neg likes(c, y) \vee y = z \vee \neg likes(c, z)))$  (Skolem constant  $c$ )  
 $\equiv person(c) \wedge (\neg likes(c, y) \vee y = z \vee \neg likes(c, z))$  (Drop  $\forall$ )
4. (i) CNF( $\forall x (P(x) \rightarrow Q(x))$ )  
 $\equiv \forall x (\neg P(x) \vee Q(x))$  (Remove  $\rightarrow$ )  
 $\equiv \neg P(x) \vee Q(x)$  (Drop  $\forall$ )

$$\begin{aligned}
 & \text{CNF}(\neg \forall y (\neg Q(y) \rightarrow \neg P(y))) \\
 & \equiv \neg \forall y (\neg \neg Q(y) \vee \neg P(y)) \text{ (Remove } \rightarrow \text{)} \\
 & \equiv \exists y \neg (\neg \neg Q(y) \vee \neg P(y)) \text{ (De Morgan)} \\
 & \equiv \exists y \neg (Q(y) \vee \neg P(y)) \text{ (Double Negation)} \\
 & \equiv \exists y (\neg Q(y) \wedge \neg \neg P(y)) \text{ (De Morgan)} \\
 & \equiv \exists y (\neg Q(y) \wedge P(y)) \text{ (Double Negation)} \\
 & \equiv \neg Q(c) \wedge P(c) \text{ (Skolemisation)}
 \end{aligned}$$

Proof:

1.  $\neg P(x) \vee Q(x)$  (Premise)
2.  $\neg Q(c)$  (Negated Conclusion)
3.  $P(c)$  (Negated Conclusion)
4.  $\neg P(c)$  (1, 2 Resolution  $\{x/c\}$ )
5.  $\square$  (3, 4 Resolution)

- (ii) (Works exactly as in (i).)

$$\begin{aligned}
 & \text{CNF}(\forall x (P(x) \rightarrow Q(x))) \\
 & \equiv \forall x (\neg P(x) \vee Q(x)) \text{ (Remove } \rightarrow \text{)} \\
 & \equiv \neg P(x) \vee Q(x) \text{ (Drop } \forall \text{)}
 \end{aligned}$$

$$\begin{aligned}
 & \text{CNF}(\neg \forall x (\neg Q(x) \rightarrow \neg P(x))) \\
 & \equiv \neg \forall x (\neg \neg Q(x) \vee \neg P(x)) \text{ (Remove } \rightarrow \text{)} \\
 & \equiv \neg \forall x (Q(x) \vee \neg P(x)) \text{ (Double Negation)} \\
 & \equiv \exists x \neg (Q(x) \vee \neg P(x)) \text{ (De Morgan)} \\
 & \equiv \exists x (\neg Q(x) \wedge \neg \neg P(x)) \text{ (De Morgan)}
 \end{aligned}$$

$$\equiv \exists x (\neg Q(x) \wedge P(x)) \text{ (Double Negation)}$$

$$\equiv \neg Q(c) \wedge P(c) \text{ (Skolemisation)}$$

Proof:

1.  $\neg P(x) \vee Q(x)$  (Premise 3)
2.  $\neg Q(c)$  (Negated Conclusion)
3.  $P(c)$  (Negated Conclusion)
4.  $\neg P(c)$  (1, 2 Resolution  $\{x/c\}$ )
5.  $\square$  (3, 4 Resloution)

$$\text{(iii) CNF}(\forall x (P(x) \rightarrow Q(x)))$$

$$\equiv \forall x (\neg P(x) \vee Q(x)) \text{ (Remove } \rightarrow)$$

$$\equiv \neg P(x) \vee Q(x) \text{ (Drop } \forall)$$

$$\text{CNF}(P(a))$$

$$\equiv P(a)$$

$$\text{CNF}(\neg Q(a))$$

$$\equiv \neg Q(a)$$

Proof:

1.  $\neg P(x) \vee Q(x)$  (Premise)
2.  $P(a)$  (Premise)
3.  $\neg Q(a)$  (Negated Conclusion)
4.  $Q(a)$  (1, 2 Resolution  $\{x/a\}$ )
6.  $\square$  (3, 4 Resolution)

$$\text{(iv) CNF}(\forall x (P(x) \rightarrow Q(x)))$$

$$\equiv \forall x (\neg P(x) \vee Q(x)) \text{ (Remove } \rightarrow)$$

$$\equiv \neg P(x) \vee Q(x) \text{ (Drop } \forall)$$

$$\text{CNF}(\exists x P(x))$$

$$\equiv P(a) \text{ (Skolemisation)}$$

$$\text{CNF}(\neg \exists x Q(x))$$

$$\equiv \forall x \neg Q(x) \text{ (De Morgan)}$$

$$\equiv \neg Q(x) \text{ (Drop } \forall)$$

Proof:

1.  $\neg P(x) \vee Q(x)$  (Premise)
2.  $P(a)$  (Premise)
3.  $\neg Q(y)$  (Copy of Negated Conclusion)
4.  $Q(a)$  (1, 2 Resolution  $\{x/a\}$ )
5.  $\square$  (3, 4 Resolution  $\{y/a\}$ )

$$\text{(v) CNF}(\forall x (P(x) \rightarrow Q(x)))$$

$$\equiv \forall x (\neg P(x) \vee Q(x)) \text{ (Remove } \rightarrow)$$

$$\equiv \neg P(x) \vee Q(x) \text{ (Drop } \forall)$$

$$\text{CNF}(\forall x (Q(x) \rightarrow R(x)))$$

$$\equiv \forall x (\neg Q(x) \vee R(x)) \text{ (Remove } \rightarrow)$$

$$\equiv \neg Q(x) \vee R(x) \text{ (Drop } \forall)$$

$$\text{CNF}(\neg \forall x (P(x) \rightarrow R(x)))$$

$$\equiv \neg \forall x (\neg P(x) \vee R(x)) \text{ (Remove } \rightarrow)$$

$$\equiv \exists x (\neg(\neg P(x) \vee R(x))) \text{ (De Morgan)}$$

$$\equiv \exists x (\neg \neg P(x) \wedge \neg R(x)) \text{ (De Morgan)}$$

$\equiv \exists x (P(x) \wedge \neg R(x))$  (Double Negation)  
 $\equiv P(c) \wedge \neg R(c)$  (Skolemisation)

Proof:

1.  $\neg P(x) \vee Q(x)$  (Premise)
2.  $\neg Q(y) \vee R(y)$  (Copy of Premise)
3.  $P(c)$  (Negated Conclusion)
4.  $\neg R(c)$  (Negated Conclusion)
5.  $\neg P(y) \vee R(y)$  (1, 2 Resolution  $\{x/y\}$ )
6.  $R(c)$  (3, 5 Resolution  $\{y/c\}$ )
9.  $\square$  (4, 6 Resolution)

5. (i) (A)  $\exists x (cs(x) \wedge \forall y (os(y) \rightarrow likes(x, y)))$   
 (B)  $os(Linux)$   
 (C)  $\exists z likes(z, Linux)$
- (ii) (A)  $cs(c) \wedge (\neg os(y) \vee likes(c, y))$  (Skolemisation and Drop  $\forall$ )  
 (B)  $os(Linux)$   
 (C)  $\neg likes(z, Linux)$  (De Morgan and Drop  $\forall$ )
1.  $cs(c)$  (Premise A)
  2.  $\neg os(y) \vee likes(c, y)$  (Premise A)
  3.  $os(Linux)$  (Premise B)
- (iii) 4.  $\neg likes(z, Linux)$  (Negated Consequence)  
 5.  $likes(c, Linux)$  (2, 3 Resolution  $\{y/Linux\}$ )  
 6.  $\square$  (4, 5 Resolution  $\{z/c\}$ )
- (iv) Yes.  $A, B, \neg C$  in (ii) are Horn clauses so there must be an SLD resolution of the empty clause if there is a resolution of the empty clause, and there is as we have seen in (iii). The following is an SLD resolution of the empty clause starting with the negated consequence (line 4).
1.  $cs(c)$  (Premise A)
  2.  $\neg os(y) \vee likes(c, y)$  (Premise A)
  3.  $os(Linux)$  (Premise B)
  4.  $\neg likes(z, Linux)$  (Negated Consequence)
  5.  $\neg os(Linux)$  (4, 2 Resolution  $\{z/c, y/Linux\}$ )
  6.  $\square$  (5, 3 Resolution)
- (v)  $A, B \models C$