COMP9414: Artificial Intelligence

Logic and Prolog

Wayne Wobcke

Room J17-433
wobcke@cse.unsw.edu.au
Based on slides by Maurice Pagnucco

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Logic and Prolog

- Prolog stands for programming in logic
- How does the implementation of Prolog relate to logic?
- Prolog is based on resolution theorem proving in first-order logic
- In this lecture we will look at the relationship between automated reasoning in first-order logic and Prolog
- References:
  - Ivan Bratko, Prolog Programming for Artificial Intelligence, Addison-Wesley, 2001. (Chapter 2)

Overview

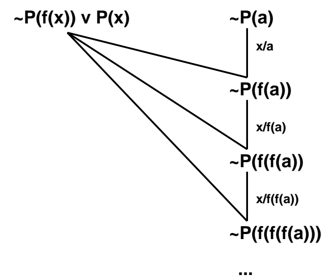
- Problems
- Undecidability of first-order logic
- Horn Clauses
- SLD Resolution
- Prolog
- Negation as Failure
- Conclusion

Soundness and Completeness Again

- First-order resolution refutation is sound, i.e. it preserves truth (if a set of premises are all true, any conclusion drawn from those premises **must** also be true)
- First-order resolution refutation is complete, i.e. it is capable of proving all consequences of any knowledge base (not shown here!)
- First-order resolution refutation is **not** decidable, i.e. there is **no** algorithm implementing resolution which when asked whether $S \vdash P$, can always answer ‘yes’ or ‘no’ (correctly)

Undecidability of First-Order Logic

- $KB = \{P(f(x)) \rightarrow P(x)\}$
- $Q = P(a)$?
- Obviously $KB \not\models Q$
- However, let us attempt to show this using resolution



Undecidability of First-Order Logic

- Can we determine in general when this problem will arise?
- **Answer:** no!
- There is no general procedure

  if (KB unsatisfiable)
  return Yes;
  else return No;

- Resolution refutation is complete so if KB is unsatisfiable, the search tree will contain the empty clause somewhere
- ...but if the search tree does not contain the empty clause the search may go on forever
- Even in the propositional case (which is decidable), complexity of resolution is $O(2^n)$ for problems of size n

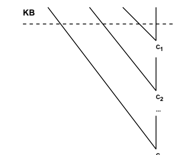
Horn Clauses

Idea: use less expressive language

- Review
  - literal — atomic formula or negation of atomic formula
  - clause — disjunction of literals
- Definite Clause – exactly one positive literal
  - e.g. $C \vee \neg A_1 \vee \ldots \vee \neg A_n$, i.e. $C \leftarrow A_1 \wedge \ldots \wedge A_n$ (Prolog rule)
- Negative Clause – no positive literals
  - e.g. $\neg Q$ (negation of a query)
- Horn Clause – clause with at most one positive literal

SLD Resolution — $\vdash_{SLD}$

- Selected literals Linear form Definite clauses resolution
- SLD refutation of a clause C from a set of clauses KB is a sequence of clauses such that
  1. First clause of sequence is C
  2. Each intermediate clause C_i is derived by resolving the previous clause C_{i-1} and a clause from KB
  3. The last clause in the sequence is $\Box$



- **Theorem.** For a definite KB and negative clause query Q: $KB \cup Q \vdash \Box$ if and only if $KB \cup Q \vdash_{SLD} \Box$

Prolog

- Horn clauses in first-order logic (facts and rules)
- SLD resolution
- Depth-first search strategy with backtracking
- User control
  - Ordering of clauses in Prolog database (facts and rules)
  - Ordering of subgoals in body of a rule
  - Cut (!) operator
  - Negation as failure
- That is, Prolog is a restricted form of first-order logic (Horn clauses) and puts more control of the theorem proving process into the hands of the programmer allowing them to use problem-specific knowledge to reduce search

Abstract Prolog Interpreter

Input: A query Q and a logic program KB
Output: 'yes' if Q follows from KB, 'no' otherwise

    Initialise current goal set to $\{Q\}$;
    while the current goal set is not empty do
        Choose G from the current goal set; (first in goal set)
        Choose a copy G' :- $B_1, \ldots, B_n$ of a rule from KB for which
            most general unifier of G, G' is θ; (try all in KB)
        (if no such rule, undo unifications and try alternative rules)
        Apply θ to the current goal set;
        Replace $G\theta$ by $B_1\theta, \ldots, B_n\theta$ in current goal set;
    if current goal set is empty,
        output yes;
    else output no;

- Depth-first, left-right with backtracking

Negation as Failure

- Prolog does not implement classical negation
- Prolog `not` is known as negation as failure
-
```
not(G) :- G, !, fail. % If G succeeds return no
not(G).               % else return yes
```
- $KB \vdash$ `not`(G) — cannot prove G
- $KB \vdash \neg G$ — can prove $\neg G$
- They are not the same
- Negation as failure is finite failure

Soundness and Completeness Again

- Prolog including cut and negation as failure is **not** sound, i.e. it does not preserve truth
- Pure Prolog (without cut and negation as failure) is **not** complete, i.e. it is incapable of proving all consequences of any knowledge base (this is because of the search order)
- Even pure Prolog is **not** decidable, i.e. the Prolog implementation of resolution when asked whether $KB \vdash Q$, can not always answer 'yes' or 'no' (correctly)

Conclusion

- First-order logic is an expressive formal language and allows for powerful reasoning
- Theorem proving is undecidable in general
- Other options:
  - Search heuristics (ordering of predicates, subgoals, breadth-first search)
  - Sacrifice expressivity (e.g. Horn clauses although still undecidable in first-order case)
  - User control (cut operator)
- Prolog is based on SLD resolution in first-order Horn clause logic and allows programmer to use knowledge about domain to control search
- Blend of theory and pragmatics

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