Projection in the Epistemic Situation Calculus with Belief Conditionals

Christoph Schwering and Gerhard Lakemeyer

RWTH Aachen University, Germany

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Belief Revision in Dynamic Environments

Suppose we want to have dinner at a restaurant:

- We don’t know that the restaurant is Italian
- We believe:
  1. usually, the specialty is burger
  2. but in Italian rest.s, it’s pizza or pasta
- We can take action:
  1. order the specialty
  2. ask if restaurant is Italian
Belief Revision in Dynamic Environments

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**Belief projection:** After actions $n_1, \ldots, n_k$, do we believe $\alpha$?

E.g.: After we order the specialty and then find out the restaurant is Italian, do we believe that we will get a dish but don’t know which?
Belief Revision in the Situation Calculus

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Belief projection: After actions $n_1, \ldots, n_k$, do we believe $\alpha$?

E.g.: Truth in a model (in a variant of Shapiro et al. [AlJ-2011]):

$$f, w \models [\text{odr}] [\text{ask}] B (\exists x. D(x) \land \neg B D(x))$$
Belief Revision in the Situation Calculus

Suppose we want to have dinner at a restaurant:

- We don’t know that the restaurant is Italian
- We believe:
  1. usually, the specialty is burger \( \text{TRUE} \Rightarrow S(x) \equiv x = \text{burger} \)
  2. but in Italian rest.s, it’s pizza or pasta \( I \Rightarrow S(\text{pizza}) \lor S(\text{pasta}) \)
- We can take action:
  1. order the specialty \( \Box[a]D(x) \equiv a = \text{odr} \land S(x) \lor D(x) \)
  2. ask if restaurant is Italian \( \Box SF(a) \equiv a = \text{ask} \supset I \)

**Belief projection:** After actions \( n_1, \ldots, n_k \), do we believe \( \alpha \)?

E.g.: Entailments of action theory:

\[
\Omega \land I \land \mathbf{O}(\Omega, \Gamma) \models [\text{odr}][\text{ask}]\mathbf{B}(\exists x. D(x) \land \neg \mathbf{B}D(x))
\]
Belief Revision in the Situation Calculus

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**Solution:**
1. elimination of actions using regression
2. elimination of beliefs by reduction to first-order reasoning
Elimination of Actions: The Objective Case

Regression (due to Reiter):
- Push actions inwards
- Replace $[r]F(t)$ and $SF(r)$ with the RHS from the action theory
- Yields a formula without actions
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\[
[\text{odr}][\text{ask}]\exists x.D(x)
\]

\[
\rightarrow \exists x.[\text{odr}][\text{ask}]D(x)
\]
Elimination of Actions: The Objective Case

Regression (due to Reiter):

► Push actions inwards
► Replace \([r]F(t)\) and \(SF(r)\) with the RHS from the action theory
► Yields a formula without actions

\[
\begin{align*}
\text{o}dr\text{[ask]}\exists x.D(x) \\
\Rightarrow\exists x.\text{o}dr\text{[ask]}D(x) \\
\Rightarrow\exists x.\text{o}dr(\text{ask} = \text{o}dr \land S(x) \lor D(x)) \\
\Rightarrow\exists x.\text{o}drD(x)
\end{align*}
\]

\[
\square [a]D(x) \equiv a = \text{o}dr \land S(x) \lor D(x)
\]
Elimination of Actions: The Objective Case

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\[
\begin{align*}
[\text{odr}][\text{ask}] & \exists x. D(x) \\
\rightarrow & \exists x. [\text{odr}][\text{ask}] D(x) \\
\rightarrow & \exists x. [\text{odr}] (\text{ask} = \text{odr} \land S(x) \lor D(x)) \\
\rightarrow & \exists x. [\text{odr}] D(x) \\
\rightarrow & \exists x. (\text{odr} = \text{odr} \land S(x) \lor D(x)) \\
\rightarrow & \exists x. (S(x) \lor D(x))
\end{align*}
\]
Elimination of Actions: The Subjective Case
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How to push actions inwards of $\mathbf{B}$ modalities?

**Theorem:**

$$\models \Box [a] \mathbf{B}(\phi \Rightarrow \psi) \equiv \neg \mathbf{S}F(a) \land \mathbf{B}(\neg \mathbf{S}F(a) \land [a] \phi \Rightarrow [a] \psi) \lor \mathbf{S}F(a) \land \mathbf{B}(\mathbf{S}F(a) \land [a] \phi \Rightarrow [a] \psi)$$

When no actions in front of $\mathbf{B}$ left, continue regression inside $\mathbf{B}$. 
Elimination of Actions: The Subjective Case

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**Theorem:**

$$\models \Box [a] \mathcal{B} (\phi \Rightarrow \psi) \equiv \neg SF(a) \land \mathcal{B}(\neg SF(a) \land [a]\phi \Rightarrow [a]\psi) \lor$$

$$SF(a) \land \mathcal{B}( SF(a) \land [a]\phi \Rightarrow [a]\psi)$$

When no actions in front of $\mathcal{B}$ left, continue regression inside $\mathcal{B}$.

$$[\text{odr}][\text{ask}] \mathcal{B} (\exists x. (D(x) \land \neg BD(x)))$$

$$\rightarrow [\text{odr}][\text{ask}] \mathcal{B} (\text{TRUE} \Rightarrow \exists x. (D(x) \land \neg BD(x)))$$
Elimination of Actions: The Subjective Case

How to push actions inwards of $\mathbf{B}$ modalities?

**Theorem:**

$$
\models \Box [a] \mathbf{B}(\phi \Rightarrow \psi) \equiv \neg SF(a) \land \mathbf{B}(\neg SF(a) \land [a] \phi \Rightarrow [a] \psi) \lor
SF(a) \land \mathbf{B}(SF(a) \land [a] \phi \Rightarrow [a] \psi)
$$

When no actions in front of $\mathbf{B}$ left, continue regression inside $\mathbf{B}$.

$$
[\text{odr}][\text{ask}] \mathbf{B}(\exists x. (D(x) \land \neg \mathbf{B}D(x)))
$$

$$
\rightarrow [\text{odr}][\text{ask}] \mathbf{B}(\text{true} \Rightarrow \exists x. (D(x) \land \neg \mathbf{B}D(x)))
$$

$$
\rightarrow [\text{odr}]([\neg SF(\text{ask}) \land \mathbf{B}(\neg SF(\text{ask}) \Rightarrow [\text{ask}] \exists x. (D(x) \land \neg \mathbf{B}D(x)))] \lor
SF(\text{ask}) \land \mathbf{B}(SF(\text{ask}) \Rightarrow [\text{ask}] \exists x. (D(x) \land \neg \mathbf{B}D(x)))
$$
Elimination of Actions: The Subjective Case

How to push actions inwards of $\mathbf{B}$ modalities?

Theorem:

$$
\models \Box [a] \mathbf{B}(\phi \Rightarrow \psi) \equiv \neg SF(a) \land \mathbf{B}(-SF(a) \land [a]\phi \Rightarrow [a]\psi) \lor
SF(a) \land \mathbf{B}( SF(a) \land [a]\phi \Rightarrow [a]\psi)
$$

When no actions in front of $\mathbf{B}$ left, continue regression inside $\mathbf{B}$.

$$
[\mathit{odr}][\mathit{ask}] \mathbf{B}(\exists x. (D(x) \land \neg \mathbf{B}D(x)))
\rightarrow [\mathit{odr}][\mathit{ask}] \mathbf{B}(\mathit{true} \Rightarrow \exists x. (D(x) \land \neg \mathbf{B}D(x)))
\rightarrow [\mathit{odr}](\neg SF(\mathit{ask}) \land \mathbf{B}(\neg SF(\mathit{ask}) \Rightarrow [\mathit{ask}]\exists x. (D(x) \land \neg \mathbf{B}D(x))) \lor
SF(\mathit{ask}) \land \mathbf{B}( SF(\mathit{ask}) \Rightarrow [\mathit{ask}]\exists x. (D(x) \land \neg \mathbf{B}D(x))))
\rightarrow \ldots \lor [\mathit{odr}] SF(\mathit{ask}) \land SF(\mathit{odr}) \land \mathbf{B}( [\mathit{odr}] SF(\mathit{ask}) \Rightarrow \exists x. ([\mathit{odr}][\mathit{ask}] D(x) \land \neg [\mathit{odr}][\mathit{ask}]\mathbf{B}D(x)))
$$
Elimination of Actions: The Subjective Case

How to push actions inwards of B modalities?

Theorem:

\[ \models \Box [a]B(\phi \Rightarrow \psi) \equiv \neg SF(a) \land B(\neg SF(a) \land [a]\phi \Rightarrow [a]\psi) \lor SF(a) \land B( SF(a) \land [a]\phi \Rightarrow [a]\psi) \]

When no actions in front of B left, continue regression inside B.

\[
\begin{align*}
[odr][ask]B(\exists x.(D(x) \land \neg BD(x))) \\
\rightarrow [odr][ask]B(\text{true} \Rightarrow \exists x.(D(x) \land \neg BD(x))) \\
\rightarrow [odr](\neg SF(\text{ask}) \land B(\neg SF(\text{ask}) \Rightarrow [\text{ask}]\exists x.(D(x) \land \neg BD(x)))) \lor SF(\text{ask}) \land B( SF(\text{ask}) \Rightarrow [\text{ask}]\exists x.(D(x) \land \neg BD(x)))) \\
\rightarrow ... \lor [odr]SF(\text{ask}) \land SF(odr) \land B( [odr]SF(\text{ask}) \Rightarrow \exists x.([odr][ask]D(x) \land \neg [odr][ask]BD(x))) \\
\rightarrow I \land B(I \Rightarrow \exists x.((S(x) \lor D(x)) \land \neg B(I \Rightarrow S(x) \lor D(x))))
\end{align*}
\]
Elimination of Beliefs: The Idea

- $B(\phi \Rightarrow \psi)$ iff most plausible $\phi$-worlds satisfy $\psi$

- Every sphere can be represented by an objective sentence $\gamma_i$

- $B(\phi \Rightarrow \psi)$ iff first $\phi$-consistent $\gamma_i$ entails $\phi \supset \psi$

$$
\begin{align*}
\gamma_0 &= \neg I \land (S(x) \equiv x = \text{burger}) \\
\gamma_1 &= I \supset (S(\text{pasta}) \lor S(\text{pizza})) \\
\gamma_2 &= \text{TRUE}
\end{align*}
$$

- Free variables: enumerate believed instances (due to Levesque)
Elimination of Beliefs: The Idea

- \( B(\phi \Rightarrow \psi) \) iff most plausible \( \phi \)-worlds satisfy \( \psi \)
- Every sphere can be represented by an objective sentence \( \gamma_i \)
- \( B(\phi \Rightarrow \psi) \) iff first \( \phi \)-consistent \( \gamma_i \) entails \( \phi \supset \psi \)
  
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  \]

- Free variables: enumerate believed instances (due to Levesque)

  \[
  I \land B(I \Rightarrow \exists x.((S(x) \lor D(x)) \land \neg B(I \Rightarrow S(x) \lor D(x))))
  \]

  \[
  \rightarrow I \land B(I \Rightarrow \exists x.((S(x) \lor D(x)) \land \neg \text{FALSE}))
  \]
Elimination of Beliefs: The Idea

- $B(\phi \Rightarrow \psi)$ iff most plausible $\phi$-worlds satisfy $\psi$

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\]

- Free variables: enumerate believed instances (due to Levesque)

\[
I \land B(I \Rightarrow \exists x.((S(x) \lor D(x)) \land \neg B(I \Rightarrow S(x) \lor D(x))))
\Rightarrow I \land B(I \Rightarrow \exists x.((S(x) \lor D(x)) \land \neg \text{FALSE}))
\Rightarrow I \land \text{TRUE}
\]
Conclusion

Solved **belief projection** in the Situation Calculus:

1. Elimination of actions: formula about initial beliefs
2. Elimination of beliefs: series of first-order entailments

**Working implementation** based on Lakemeyer and Levesque [KR-2014]

Future work:

- Progression of beliefs
- Regression for Spohn-style logics  e.g., Delgrande and Levesque [KR-2012]