# Justified Representation in Approval-Based Committee Voting

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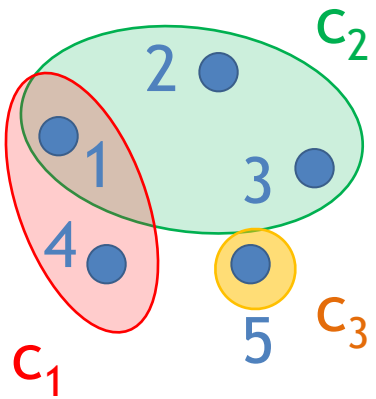
# Voting with Approval Ballots

- A set of alternatives $C$
- $n$ voters $\{1, \ldots, n\}$
- Each voter approves a subset of candidates $A_i \subseteq C$
- <u>Goal</u>: select $k$ winners




1: $c_1$, $c_2$
2: $c_2$
3: $c_2$
4: $c_1$
5: $c_3$

# Outline

- Approval-based multiwinner rules
- Justified Representation (JR)
- Which rules satisfy JR?
- Extended Justified Representation (EJR)

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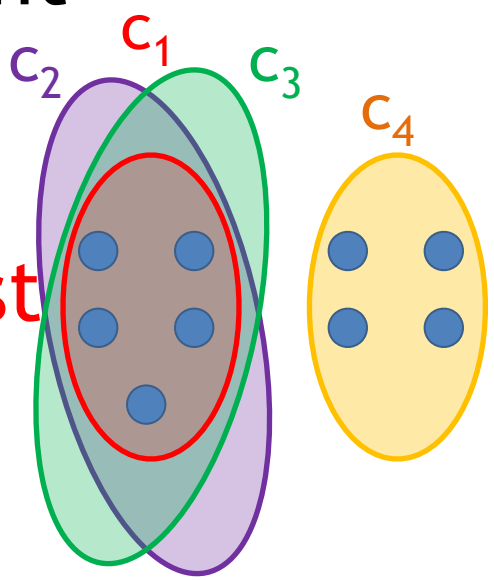
# Approval Voting (AV)

- Each candidate gets one point from each voter who approves her
- k candidates with the highest score are selected
  - ties broken deterministically



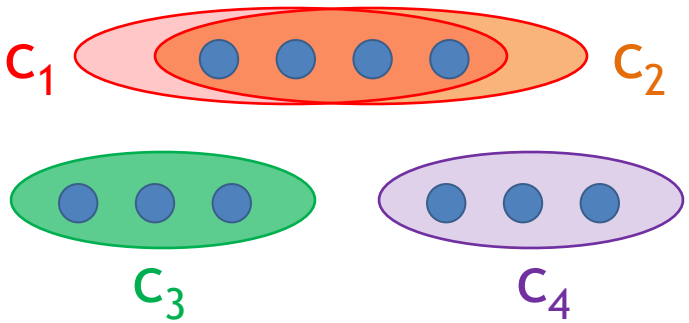

for k=3
AV outputs
$\{c_1, c_2, c_3\}$

# Satisfaction Approval Voting (SAV)

- Brams & Kilgour ’14

- Voter $i$ scores committee $W$ as $|A_i \cap W| / |A_i|$

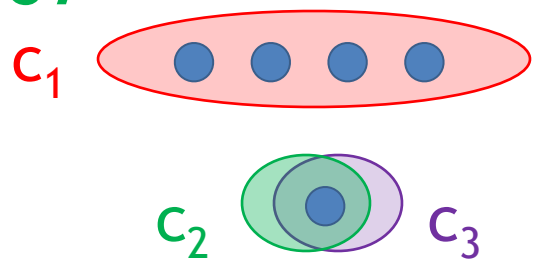
- <u>Goal</u>: select a size-$k$ committee with the maximum score




for $k=2$
AV outputs
$\{c_1, c_2\}$,
SAV outputs
$\{c_3, c_4\}$

# Minimax Approval Voting (MAV)

- Brams, Kilgour & Sanver '07



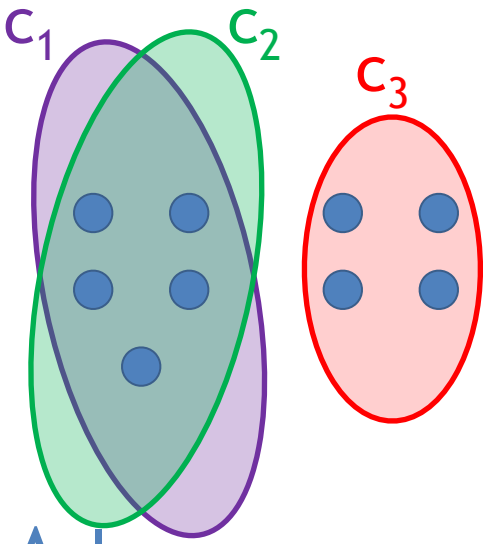

- Distance from ballot $A_i$ to a committee $W$:
  $d(A_i, W) = |A_i \setminus W| + |W \setminus A_i|$

- <u>Goal</u>: select a size-$k$ committee that minimizes $\max_i d(A_i, W)$

for $k=1$
AV outputs
$c_1$,
MAV outputs
$c_2$ or $c_3$

# Proportional Approval Voting (PAV)

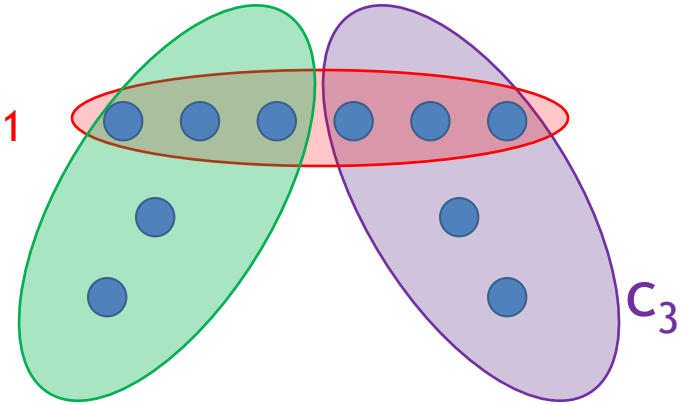
- Simmons '01
- Voter $i$ derives utility of 1 from her 1st approved candidate, 1/2 from 2nd, 1/3 from 3rd, etc.
- $u_i(W) = 1 + 1/2 + \ldots + 1/|W \cap A_i|$
- <u>Goal</u>: select a size-$k$ committee $W$ that maximizes $u(W) = \Sigma_i u_i(W)$




for $k=2$
AV outputs $\{c_1, c_2\}$,
PAV outputs $\{c_1, c_3\}$ or $\{c_2, c_3\}$

# Reweighted Approval Voting (RAV)

- Thiele, early 20th century
- Sequential version of PAV
- Initialize:
  $\omega(i) = 1$ for all $i$, $W = \emptyset$
- Repeat $k$ times:
  - add to $W$ a candidate with max approval weight
    $\omega(c) = \Sigma_{\text{ i approves c}} \omega(i)$
  - update the weight of each voter to $\omega(i) = 1/(1+|A_i \cap W|)$




for $k=2$
PAV outputs $\{c_2, c_3\}$ ,
RAV outputs $\{c_1, c_2\}$ or $\{c_1, c_3\}$

# Generalizing PAV and RAV: Arbitrary Weights

- PAV and RAV both use weight vector $(1, 1/2, 1/3, \ldots)$
- We can use an arbitrary weight vector $(w_1, w_2, \ldots)$ with $w_1 = 1$ ,$w_1 \geq w_2 \geq \ldots$ instead:
  $(w_1, w_2, \ldots)$-PAV and $(w_1, w_2, \ldots)$-RAV
- $(1, 0, \ldots)$-RAV: choose candidates one by one to cover as many uncovered voters as possible at each step (Greedy Approval Voting (GAV))

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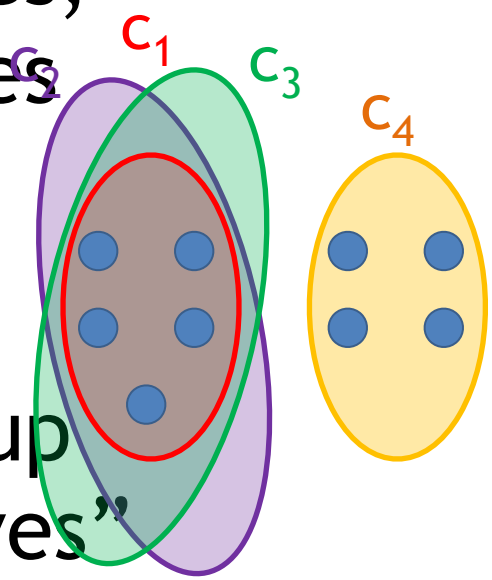
# Representation

- 5 voters get 3 representatives,
  4 voters get 0 representatives

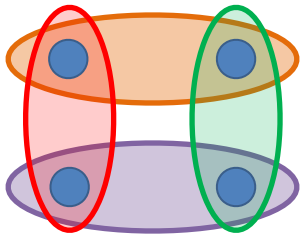



- Intuition: each cohesive group of voters of size n/k "deserves" at least one representative

for k=3
AV outputs
$\{c_1, c_2, c_3\}$

# Justified Representation

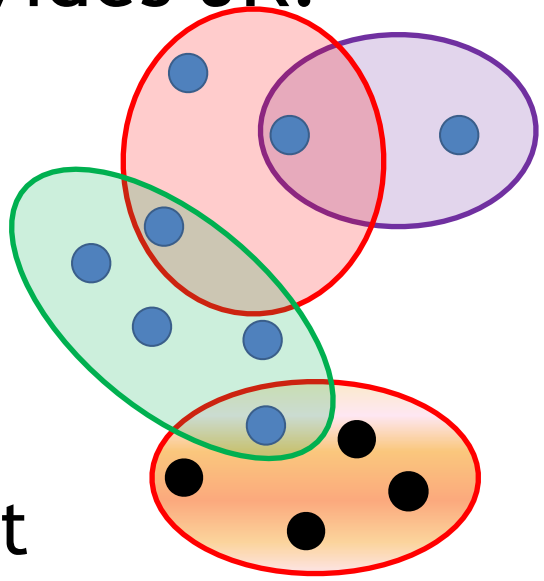
- <u>Definition</u>: a committee $W$ provides justified representation (JR) for a list of ballots $(A_1,\ldots, A_n)$ and committee size $k$ if for every set of voters $X$ with $|X| \geq n/k$ and $\cap_{i \in X} A_i \neq \emptyset$ it holds that $W$ contains at least one candidate from $\cup_{i \in X} A_i$.



- $k=1$: JR is satisfied unless there exists a candidate approved by all, but we pick a candidate not approved by anyone

# Can We Always Satisfy JR?

- <u>Claim</u>: GAV (aka (1, 0, …)-RAV) always outputs a committee that provides JR.
- Proof:
  - Suppose after $k$ steps we have $n/k$ uncovered voters who all approve $a$
  - $a$'s weight is $\geq n/k$
  - then at each step we chose a set that covered $\geq n/k$ uncovered voters
  - thus we should have covered all $n$ voters



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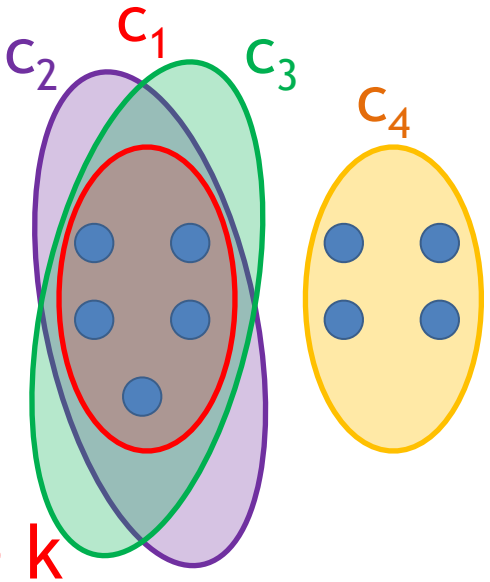
# Rules that fail JR

- AV fails JR for $k \geq 3$
- SAV fails JR for $k \geq 2$
- MAV fails JR for $k \geq 2$
  - except if each ballot is of size k and ties are broken in favour of JR




for $k=3$
AV outputs
$\{c_1, c_2, c_3\}$

# PAV, RAV and JR

- <u>Theorem</u>: PAV satisfies JR
- <u>Theorem</u>: $(w_1, w_2, \ldots)$-PAV satisfies JR iff $w_j \leq 1/j$ for all $j$
- <u>Theorem</u>: RAV satisfies JR for $k = 2$ and fails JR for $k \geq 10$
  - $k = 3, \ldots, 9$ is open!
- <u>Theorem</u>: $(w_1, w_2, \ldots)$-RAV fails JR if $w_2 > 0$
  - $(1, 0, \ldots)$-RAV is GAV and satisfies JR

# Summary: JR

| | Satisfies JR | |
|---|---|---|
| AV | No | |
| SAV | No | |
| MAV | No | |
| PAV | Yes | |
| RAV | No | |
| GAV | Yes | |

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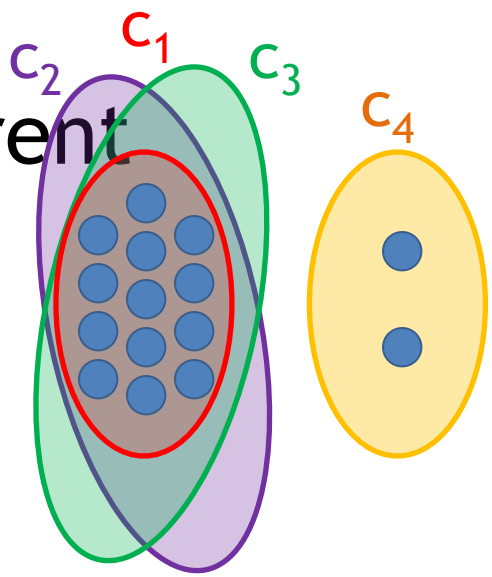
# Is JR Enough?

- Should we choose $c_4$ ???
- Perhaps a **very large** coherent group of voters "deserves" several representatives?
- <u>Idea</u>: if $n/k$ voters who agree on a candidate "deserve" one representative, then maybe $l \cdot n/k$ voters who agree on $l$ candidates "deserve" $l$ representatives?



# Extended Justified Representation

- <u>Definition</u>: a committee $W$ provides extended justified representation (EJR) for a list of ballots $(A_1,\ldots, A_n)$ and committee size $k$ if for every $l > 0$, every set of voters $X$ with $|X| \geq l \cdot n/k$ and $|\bigcap_{i \in X} A_i| \geq l$ it holds that $|W \cap A_i| \geq l$ for at least one $i \in X$.

- $l = 1$: justified representation

# Satisfying EJR

- Observation: GAV fails EJR
- Theorem: PAV satisfies EJR, but is NP-hard to compute
  - do any other rules satisfy EJR?
- Theorem: $(w_1, w_2, \ldots)$-PAV fails EJR if

  $$(w_1, w_2, \ldots) \neq (1, 1/2, 1/3, \ldots)$$
- Theorem: checking if a committee provides EJR is coNP-complete
- Open: complexity of finding an EJR committee

# Summary: EJR

| | Satisfies JR | Satisfies EJR |
|---|---|---|
| AV | No | No |
| SAV | No | No |
| MAV | No | No |
| PAV | Yes | Yes |
| RAV | No | No |
| GAV | Yes | No |

Thank you!