Working Paper

Computing and Testing Pareto Optimal Committees

Haris Aziz · Jérôme Monnot

Abstract Selecting a set of alternatives based on the preferences of agents is an important problem in committee selection and beyond. Among the various criteria put forth for desirability of a committee, Pareto optimality is a minimal and important requirement. As asking agents to specify their preferences over exponentially many subsets of alternatives is practically infeasible, we assume that each agent specifies a weak order on single alternatives, from which a preference relation over subsets is derived using some preference extension. We consider five prominent extensions (responsive, downward lexicographic, upward lexicographic, best, and worst). For each of them, we consider the corresponding Pareto optimality notion, and we study the complexity of computing and verifying Pareto optimal outcomes. We also consider strategic issues: for four of the set extensions, we present a linear-time, Pareto optimal and strategyproof algorithm that even works for weak preferences.

Keywords committee selection · multiwinner voting · Pareto optimality · algorithms and complexity · set extensions.

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1 Introduction

Pareto optimality is a central concept in economics and has been termed the *"single most important tool of normative economic analysis"* (Moulin, 2003). An outcome

H. Aziz
Data61, CSIRO and UNSW, Sydney 2052 , Australia
Tel.: +61-2-8306 0490
Fax: +61-2-8306 0405
E-mail: haris.aziz@unsw.edu.au

J. Monnot
LAMSADE, Université Paris-Dauphine
Paris, France
E-mail: jerome.monnot@lamsade.dauphine.fr

is Pareto optimal if there does not exist another outcome that all agents like at least as much and at least one agent strictly prefers. Although Pareto optimality has been considered extensively in single-winner voting and other social choice settings such as fair division or hedonic games, it has received only little attention in *multiwinner voting*, in which the outcomes are *sets* of alternatives. Multiwinner voting applies to selecting a set of plans or a committee, hiring team members, movie recommendations, and more. For convenience, we use the terminology "committee" even if our results have an impact far beyond committee elections (Faliszewski et al., 2017; Aziz et al., 2017).

In *single-winner voting* setting, agents express preferences over alternatives and a single alternative is selected. Pareto optimality in this context is straightforward to define, achieve, and verify. In *multiwinner voting*, a well-known difficulty is that it is unrealistic to assume that agents will report preferences over all possible committees, since there is an exponential number of them. For this reason, most approaches assume that they only report a small part of their preferences, and that some *extension principle* is used to induce a preference over all possible subsets from this 'small input' over single alternatives (Barberà et al., 2004). Such preference extensions are also widely used in other social choice settings such as fair division or matching. The most two widely used choices of 'small inputs' in multiwinner voting are *rankings (linear orders) over alternatives* and *sets of approved alternatives*. In this paper we make a choice that generalizes both of them: agents report *weak orders over single alternatives*. Then we consider five prominent preference extension principles: the *responsive* extension, where a set of alternatives S is at least as preferred as a set of alternatives T if S is obtained from T by repeated replacements of an alternative by another alternative which is at least as preferred; the *optimistic*, or *'best'* (respectively *pessimistic*, or *'worst'*) extension, which orders subsets of alternatives according to their most (respectively, least) preferred element; the *downward lexicographic* extension, a lexicographic refinement of the optimistic extension, and the *upward lexicographic* extension, a lexicographic refinement of the pessimistic (worst) extension.

The responsive extension (Barberà et al., 2004; Roth and Sotomayor, 1990) can be seen as the ordinal counterpart of additivity. The downward lexicographic extension has been considered in various papers (Bossert, 1995; Lang et al., 2012; Klamler et al., 2012). The 'best' set extension has been considered in a number of approaches such as full proportional representation (Chamberlin and Courant, 1983; Monroe, 1995) and other committee voting settings Elkind et al. (2015). The 'worst' set extension, also used by Klamler et al. (2012) and Skowron et al. (2015b), captures settings where the impact of a bad alternative in the selection overwhelms the benefits of good alternatives: for instance, when the decision about a crucial issue will be made by *one* of the members of the committee but the agent ignores which one and is risk-averse; or the case of a parent's preferences over a set of movies to be watched by a child. The 'best' and 'worst' set extensions have been used in coalition formation (Aziz and Savani, 2016; Cechlárová, 2008).

Although set extensions have been implicitly or explicitly considered in multiwinner voting, most of the computational work has dealt with *specific* voting rules (see the related work section). Instead, we concentrate on Pareto optimality, consider the computation and verification of Pareto optimal committees, as well as the exis-

Set Extension	Computation	Verification
Responsive (RS)	in P^{IC} (Th. 2)	coNP-C (Th. 3 and 6), W[2]-hard
Downward Lexicographic (DL)	in P^{IC} (Th. 2)	coNP-C (Cor. 1), W[2]-hard
Upward Lexicographic (UL)	in P^{IC} (Th. 2)	coNP-C (Cor. 2), W[2]-hard
Best ($\mathcal{B}$)	NP-hard (Th. 11) in P for strict prefs (Th. 12)	coNP-C, W[2]-hard (Th. 9)
Worst ($\mathcal{W}$)	in P^{IC} (Th. 15)	in P (Th. 14)

Table 1 Complexity of computing and verifying Pareto optimal committees for general partial preferences. P^{IC} (coined by Christos Papadimitriou in a seminar at Simons Institute in 2015) indicates a class of problems in which agents provide the input and the problems admit a strategyproof and polynomial-time algorithm.

tence of a polynomial-time and strategyproof algorithm that returns Pareto optimal outcomes.

Contributions We consider Pareto optimality with respect to the five aforementioned preference set extensions. We present various connections between the Pareto optimality notions. For each of the notions, we undertake a detailed study of complexity of computing and verifying Pareto optimal outcomes. Table 1 summarizes the complexity results for partial ordering and Table 2 proposes dichotomy complexity results, that is coNP-complete (coNP-C in short) versus polynomial (P in short) for dichotomous preferences depending on the size of top equivalence class or for linear preferences.

We show that there exist linear-time Pareto optimal and strategyproof algorithms for committee voting even for weak preferences for four of the five set extensions. The algorithms can be considered as careful adaptations of serial dictatorship for committee voting.

For responsive, downward lexicographic, upward lexicographic, best and worst extensions, we give a *complete* characterization of the complexity of testing Pareto optimality when preferences are dichotomous or linear. More precisely, we prove that for the three first extensions (i.e., RS, DL and UL) the size of top equivalence class parameter, i.e., *topwidth* (*tw* in short) is two: unless P = NP, Pareto optimality can be tested in polynomial time if and only if the size of the first equivalence classes is at most two. For the 'best' extension, unless P = NP, Pareto optimality can be tested in polynomial time if and only if the size of the first equivalence classes is at most one. In contrast to the other extensions, for the 'worst' extension, both problems of computing and verifying Pareto optimal outcomes admit polynomial-time algorithms for every partial ordering. These results are summarized in Table 2.

Note for the 'best' extension, we also show that even computing a Pareto optimal outcome is NP-hard. An important take-home message of the results is that testing Pareto optimality or obtaining Pareto improvements over status-quo committees can be computationally hard even when computing *some* Pareto optimal committee is easy.

Set Extension	Dichotomous Preferences	Linear Preferences
Responsive (RS)	coNP-C if $tw \geq 3$ (Th. 3) and P if $tw \leq 2$ (Th. 5)	coNP-C (Th. 6)
Downward Lexicographic (DL)	coNP-C if $tw \geq 3$ (Th. 3) and P if $tw \leq 2$ (Th. 5)	coNP-C (Th. 7)
Upward Lexicographic (UL)	coNP-C if $tw \geq 3$ (Th. 3) and P if $tw \leq 2$ (Th. 5)	coNP-C (Th. 8)
Best ($\mathcal{B}$)	coNP-C if $tw \geq 2$ (Th. 9) and P if $tw = 1$ (Th. 10)	coNP-C (Th. 13)
Worst ($\mathcal{W}$)	P (Th. 14)	P (Th. 14)

Table 2 Complexity of testing Pareto optimality: complete characterization for dichotomous preferences according to $topwidth = tw$ parameter and for linear preferences.

2 Related Work

A first related stream of work involves studying *specific committee elections rules* from a computational point of view (generally with little or no focus on Pareto optimality). Our focus on determining whether a committee is Pareto optimal or on finding a Pareto optimal committee, is in some sense orthogonal to the study of committee election rules. The simplest (and most widely used) rules for electing a committee, called *best-k* rules, compute a score for each alternative based on the ranks, and the alternatives with the best k scores are elected Elkind et al. (2014); Faliszewski et al. (2016). Scoring-based extension principles have also been used by Darmann (2013). Note that the output of a best-k rule is obviously Pareto-optimal for the preferences induced by this scoring function, but not necessarily with respect to other set extensions.

Klamler *et al.* Klamler et al. (2012) compute optimal committees under a weight constraint for a *single* agent (therefore optimality is equivalent to Pareto optimality), using several preference extensions including ‘worst’, ‘best’, and downward lexicographic.

The ‘best’ ($\mathcal{B}$) extension principle has been used in a number of papers on committee elections by *full proportional representation*, starting with (Chamberlin and Courant, 1983) and studied from a computational point of view in a long series of papers (*e.g.*, (Procaccia et al., 2008; Lu and Boutilier, 2011; Betzler et al., 2013; Skowron et al., 2015a; Elkind and Ismaili, 2015). These rules obviously output Pareto optimal committees *for* $\mathcal{B}$, but not necessarily for other extensions.

Some of the set extensions considered in this paper have corresponding analogues when extending preferences over alternatives to preferences over ‘lotteries over alternatives.’ In particular, the RS set extension corresponds to *SD (stochastic dominance)* lottery extension. Also the DL and UL set extensions considered in this paper correspond to DL and UL lottery extensions considered in works in probabilistic social choice (Brandl, 2013; Aziz et al., 2013b; Cho, 2016).

Some works are based on the *Hamming extension*. Each agent specifies his ideal committee and he prefers committees with less Hamming distance from the ideal committee. The Hamming distance notion can be used to define specific rules such as *minimax approval voting* (Brams et al., 2007), which selects the committee minimizing the maximum Hamming distance for the agents. Although the output of minimax approval voting is not always Pareto-optimal for the Hamming extension, there are good Pareto-optimal approximations of it Caragiannis et al. (2010). Note that for dichotomous preferences, the Hamming extension coincides with the responsive and the downward lexicographic extensions, therefore our computational results for responsive set extension for dichotomous preferences also hold for the Hamming and downward lexicographic extensions.

A second line of work concerns understanding the classes of rules that result in Pareto optimal outcomes. Most works along this line bear on a different type of committee elections, called *designated-seat voting*, where candidates must declare the seat they contest (Benoît and Kornhauser, 2010).[1] Results about the existence or non-existence of Pareto optimal rules have been presented Özkal-Sanver and Sanver (2006); Benoît and Kornhauser (2010); Cuhadaroğlu and Lainé (2012).

3 Setup

We consider a set of agents $N = \{1, \ldots, n\}$, a set of alternatives $A = \{a_1, \ldots, a_m\}$ and a preference profile $\succsim = (\succsim_1, \ldots, \succsim_n)$ such that each $\succsim_i$ is a complete and transitive relation over A. We write $a \succsim_i b$ to denote that agent i values a at least as much as b and use $\succ_i$ for the strict part of $\succsim_i$, i.e., $a \succ_i b$ iff $a \succsim_i b$ but not $b \succsim_i a$. Finally, $\sim_i$ denotes i's indifference relation, i.e., $a \sim_i b$ iff both $a \succsim_i b$ and $b \succsim_i a$.

The relation $\succsim_i$ results in equivalence classes $E_i^1, E_i^2, \ldots, E_i^{k_i}$ for some k_i such that $a \succ_i a'$ if $a \in E_i^l$ and $a' \in E_i^{l'}$ for some $l < l'$. We will use these equivalence classes to represent the preference relation of an agent as a preference list $i : E_i^1, E_i^2, \ldots, E_i^{k_i}$. For example, we will denote the preferences $a \sim_i b \succ_i c$ by the list $i : \{a, b\}, \{c\}$. An agent i's preferences are *strict* if the size of each equivalence class is 1. When all agent preferences are strict we also refer to them as *linear preferences*. An agent i's preferences are *dichotomous* if he partitions the alternatives into just two equivalence classes, i.e., $k_i = 2$. Let $Topwidth(\succsim)$ be the maximum size of the most preferred equivalence class, i.e., $Topwidth(\succsim) = \max_{i \leq n} |E_i^1|$. For any $S \subseteq A$, we will denote by $\max_{\succsim_i}(S)$ and $\min_{\succsim_i}(S)$ the alternatives in S that are maximally and minimally preferred by i respectively. Thus, if q and r are respectively the smallest and the largest indices such that $E_i^q \cap S \neq \emptyset$ and $E_i^r \cap S \neq \emptyset$, then $\max_{\succsim_i}(S) = E_i^q \cap S$ and $\min_{\succsim_i}(S) = E_i^r \cap S$. For $k \leq m$, let $S_k(A) = \{W \subseteq A : |W| = k\}$.

4 Set Extensions and Pareto Optimality

Set Extensions Set extensions are used for reasoning about the preferences of an agent over *sets* of alternatives given their preferences over single alternatives. For fixed-size committee voting, the *responsive extension* (RS) is very natural and has been applied in various matching settings as well (Barberà et al., 2004; Roth and Sotomayor, 1990). For all $V, W \in S_k(A)$, we say that $W \succsim_i^{RS} V$ if and only if there is an

[1] If there are exactly two candidates per seat, then designated voting is equivalent to *multiple referenda*, where a decision has to be taken on each of a series of yes-no issues.

injection f from V to W such that for each $a \in V$, agent i weakly prefers $f(a)$ to a, i.e. $f(a) \succsim_i a$.

We define the *best* set extension and the *worst* set extension which are denoted $\mathcal{B}$ and $\mathcal{W}$ respectively. For all $W, V \in S_k(A)$, $W \succsim_i^{\mathcal{B}} V$ if and only if $w \succsim_i v$ for $w \in \max_{\succsim_i}(W)$ and $v \in \max_{\succsim_i}(V)$. On the other side, $W \succsim_i^{\mathcal{W}} V$ if and only if $w \succsim_i v$ for $w \in \min \succsim_i(W)$ and $v \in \min_{\succsim_i}(V)$.

In the *downward lexicographic (DL)* extension, an agent prefers a committee that selects more alternatives from his most preferred equivalence class, in case of equality, the one with more alternatives for the second most preferred equivalence class, and so on. Formally, $W \succ_i^{DL} V$ iff for the smallest (if any) l with $|W \cap E_i^l| \neq |V \cap E_i^l|$ we have $|W \cap E_i^l| > |V \cap E_i^l|$.

In the *upward lexicographic (UL)* extension, an agent prefers a committee that selects less alternatives from his least preferred equivalence class, in case of equality, the one with less alternatives for the second least preferred equivalence class, and so on. Formally, $W \succ_i^{UL} V$ iff for the largest (if any) l with $|W \cap E_i^l| \neq |V \cap E_i^l|$ we have $|W \cap E_i^l| < |V \cap E_i^l|$.

Remark 1 Consider an agent i with preferences $\succsim_i$ over A. Let $S, T \subset W$ such that $|S| = |T| = k$. Then,

- $S \succsim_i^{RS} T \implies S \succsim_i^{DL} T \implies S \succsim_i^{\mathcal{B}} T$
- $S \succsim_i^{RS} T \implies S \succsim_i^{UL} T \implies S \succsim_i^{\mathcal{W}} T$
- $S \succ_i^{RS} T \implies S \succ_i^{DL} T$
- $S \succ_i^{RS} T \implies S \succ_i^{UL} T$

The relations follow from the definitions.

Efficiency based on Set Extensions With each set extension $\mathcal{E}$, we can define Pareto optimality with respect to $\mathcal{E}$. For two committees $W, W' \in S_k(A)$, we write $W' \succsim^{\mathcal{E}} W$ if $\forall i \in N$, $W' \succsim_i^{\mathcal{E}} W$ and $W' \succ^{\mathcal{E}} W$ if $\exists j \in N$, $W' \succ_j^{\mathcal{E}} W$ and $\forall i \in N \setminus \{j\}$, $W' \succsim_i^{\mathcal{E}} W$. A committee $W \in S_k(A)$ is *Pareto optimal* with respect to $\mathcal{E}$, or simply $\mathcal{E}$*-efficient*, if there exists no committee $W' \in S_k(A)$ such that $W' \succ^{\mathcal{E}} W$. Note that for each of our set extensions, $\mathcal{E}$-efficiency coincides with standard Pareto optimality when $k = 1$. An outcome is a Pareto improvement over another if each agent weakly improves and at least one agent strictly improves.

Example 1 Consider the preference profile:

$$1 : a, b, c, d$$
$$2 : d, c, b, a$$

Suppose $k = 2$. Then,

- The unique $\mathcal{B}$-efficient committee is $\{a, d\}$
- The unique $\mathcal{W}$-efficient committee is $\{b, c\}$.
- The DL-efficient committees are $\{a, d\}$, $\{a, b\}$, and $\{d, c\}$.
- The UL-efficient committees are $\{b, c\}$, $\{a, b\}$, and $\{d, c\}$.
- The RS-efficient committees are $\{a, d\}$, $\{b, c\}$, $\{a, b\}$, and $\{d, c\}$,

Remark 2 Consider a committee S.

- If S is DL-efficient, then S is RS-efficient
- If S is UL-efficient, then S is RS-efficient

The argument is as follows. By contraposed, suppose S is not RS-efficient, then there exists some other outcome T such that $T \succsim_i^{RS} S$ for all $i \in N$ and $T \succ_i^{RS} S$ for some $i \in N$. In that case $T \succsim_i^{DL} S$ for all $i \in N$ and $T \succ_i^{DL} S$ for some $i \in N$. Also $T \succsim_i^{UL} S$ for all $i \in N$ and $T \succ_i^{UL} S$ for some $i \in N$. Hence S is neither DL-efficient not UL-efficient.

Remark 3 There always exists a $\mathcal{B}$-efficient committee that is also DL-efficient: DL Pareto improvements over a $\mathcal{B}$-efficient does not harm any agent with respect to the $\mathcal{B}$ relation.

Remark 4 There always exists a $\mathcal{W}$-efficient committee that is also UL-efficient: UL Pareto improvements over a $\mathcal{W}$-efficient does not harm any agent with respect to the $\mathcal{W}$ relation.

In Figure 1, we illustrate the relations between the different efficiency notions. Later on in the paper we will present an algorithm that returns a committee that is UL-efficient and DL-efficient, and hence RS-efficient.

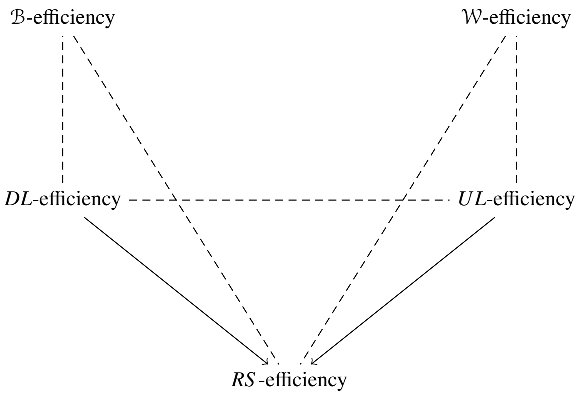

Fig. 1 Relations between the five notions of efficiency. An arrow from $\mathcal{E}_1$-efficiency to $\mathcal{E}_2$-efficiency means that $\mathcal{E}_1$-efficiency implies $\mathcal{E}_2$-efficiency; a dashed line means there always exists a committee that is both $\mathcal{E}_1$- and $\mathcal{E}_2$-efficient; absence of arrow or line means that the sets of $\mathcal{E}_1$- and $\mathcal{E}_2$-efficient committees can be disjoint.

We also make the following general observation.

Lemma 1 *If there is a polynomial-time algorithm to compute a Pareto improvement over a committee, then there exists a polynomial-time algorithm to compute an $\mathcal{E}$-efficient committee under set extensions $\mathcal{E} \in \{RS, DL, UL, \mathcal{W}, \mathcal{B}\}$.*

Proof. Here, we start from any committee and we recursively apply Pareto improvement until we reach a Pareto optimal committee. For the 'best' and 'worst' extensions, there can be at most mn Pareto improvements because for one agent there can be at most m improvements. Since an RS-improvement implies an DL-improvement, let us bound the number of Pareto-improvements with respect to DL. In each Pareto-improvement, for the agent who strictly improves, the most preferred equivalence class that has different number of alternative in the outcome increases by at least one. Therefore the most preferred equivalence class can be the improving class in at most m of the Pareto improvements. Similarly, the number of Pareto improvements in the subsequent less preferred equivalence class improves in a Pareto improvement can be at most m of the Pareto improvements. Therefore the total number of DL Pareto-improvements is bounded by m^2n. A similar argument holds for UL as well. □

We end this section by observing that, under any of the set extensions we consider, a set of Pareto optimal alternatives may be Pareto dominated. Consider the following example.

Example 2

$$1 : a, c, b, d \qquad\qquad 2 : a, d, b, c$$
$$3 : b, c, a, d \qquad\qquad 4 : b, d, a, c$$

The set $\{c, d\}$ consists of Pareto optimal alternatives but is Pareto dominated by $\{a, b\}$ under any of our set extensions.

5 A general algorithm to compute a DL/UL/RS-efficient committee

There is a trivial way to achieve Pareto optimality under the responsive set extension by taking any decreasing scoring vector consistent with the ordinal preferences, finding the total score of each alternative and returning the set of k alternatives with the maximum scores. For instance, on Example 2, the outcome of the rule that outputs the alternatives with the best k Borda scores is $\{a, b\}$.

Theorem 1 *A Pareto optimal committee under the responsive set extension committee can be computed in linear time.*

Next, we present a general polynomial-time and strategyproof algorithm that returns a DL-efficient and UL-efficient and hence RS-efficient committee. A mechanism f is *strategyproof* if reporting truthful preferences is a dominant strategy with respect to the responsive set extension: $f(\succsim) \succsim_i^{RS} f(\succsim_i', \succsim_{-i})$ for all preference profiles $\succsim$ and $(\succsim_i', \succsim_{-i})$. Note that defining strategyproofness in this way with respect to the RS extension is stronger than defining it for any of the other four extensions considered in this paper. Nonetheless, we will present some positive results with respect to strategyproofness.

A naive way of achieving RS-efficiency and Pareto optimality is to enumerate the list of possible winning sets and implement serial dictatorship over the possible outcomes as is done in voting (Aziz et al., 2013b). However, the number of possible outcomes is exponential and responsive preferences result in a partial order over the possible winning sets and not a complete and transitive order. This problem is solved by Algorithm 1 which can be viewed as a computationally efficient serial dictatorship.

Algorithm 1 Committee Voting Serial Dictatorship

Input: $(N, A, \succsim, k,$ permutation π of $N)$
Output: $W \in S_k(A)$.

L (last set to be refined) $\longleftarrow A$
r (number of alternatives yet to be fixed) $\longleftarrow k$; $W \longleftarrow \emptyset$
i' (index of the permutation π) $\longleftarrow 1$
while $r \neq 0$ or $i' \neq n$ **do**
  Agent $i = \pi(i')$ selects first t equivalence classes such that $|\bigcup_{j=1}^{t} E_i^j \cap L| \geq r$ and $|\bigcup_{j=1}^{t-1} E_i^j \cap L| < r$.
  $W \longleftarrow W \cup (\bigcup_{j=1}^{t-1} E_i^j \cap L)$ (we say agent i *fixes* the alternatives in $\bigcup_{j=1}^{t-1} E_i^j$);
  $r \longleftarrow |\bigcup_{j=1}^{t} E_i^j \cap L| - |\bigcup_{j=1}^{t-1} E_i^j \cap L|$
  $L \longleftarrow E_i^t$; $r_{i'} \longleftarrow r$
  Increment i' by one
end while
if $r > 0$ **then**
  pick any r alternatives from L and add them to W
end if
return W

Theorem 2 *There exists a linear-time and strategyproof algorithm that returns a committee that is Pareto optimal under RS, DL, and UL set extensions.*

Proof. Consider Algorithm 1. We show that at each stage i', agent $\pi(i')$, implicitly refines the set of feasible committees to the maximal set of most preferred outcomes from the set by providing additional constraints. This is true for the base case $i' = 1$. Now assume it holds from 1 to i'. Note that L contains all those alternatives that are strictly less preferred by agents in $\{\pi(1), \ldots, \pi(i')\}$ than the ones they respectively *fixed*. Moreover, each agent in $\{1, \ldots, \pi(i')\}$ is indifferent between the alternatives in L. As for $\pi(i'+1)$, he fixes the best $|\bigcup_{j=1}^{t-1} E_{\pi(i'+1)}^j \cap L|$ alternatives in L where t is the value such that $|(\bigcup_{j=1}^{t} E_{\pi(i'+1)}^j) \cap L| \geq r_{i'}$ and $|\bigcup_{j=1}^{t-1} E_{\pi(i'+1)}^j \cap L| < r_{i'}$. For $E_{\pi(i'+1)}^t$, the agent only requires that $r_{i'+1} = |(\bigcup_{j=1}^{t-1} E_{\pi(i'+1)}^j) \cap L| - |(\bigcup_{j=1}^{t-1} E_{\pi(i'+1)}^j) \cap L|$ alternatives are selected from his equivalence class $E_{\pi(i'+1)}^t$ which is ensured by the definition of the algorithm. Thus, each agent in her turn refines the set of possible outcomes to her most preferred subset of outcomes. Each committee is the refined set is at least as preferred with respect to RS (and hence with respect to DL and UL) to all committees in the set of possible outcomes. Thus the outcome is DL-efficient and UL-efficient and hence RS-efficient.

For strategyproofness, when an agent $\pi(i')$ turn comes, it only has a choice over *fixing* the alternatives in L and requiring $r_{i'}$ alternatives from his equivalence class $E_{\pi(i')}^t$. In this case the algorithm already chooses one of the best possible committees for the agent. $\square$

Note that for $k = 1$, the algorithm is equivalent to serial dictatorship as formalized by Aziz et al. (2013a). Note that a committee that is Pareto optimal under the responsive set extension may not be a result of serial dictatorship. This holds even for $k = 1$ and the basic voting setting.

The problem with the serial dictatorship algorithm formalized is that it overly favours the agent that is the first in the permutation. One way to limit his power is to let him choose only $\lceil k/n \rceil$ alternatives. We note that this attempt at having a fairer extension of serial dictatorship comes at an expense because strategyproofness is compromised. Consider the profile in which 1 has preferences a, b, c and 2 has preferences a, c, b. For $k = 2$, and permutation 12, the outcome is $\{a, c\}$. But if agent 1 reports b, a, c, then the outcome is $\{a, b\}$.

6 Testing Pareto optimality under $RS/UL/DL$ Extension

In many situations, one may already have a status-quo committee and one may want to find a Pareto improvement over it. This problem of testing Pareto optimality and finding a Pareto improvement under the responsive set extension turns out to be a much harder task. Note that if there exists a polynomial-time algorithm to compute a Pareto improvement, then it means that testing Pareto optimality is also polynomial-time solvable.

6.1 Dichotomous Preferences

First, we consider the case of dichotomous preferences. The nice aspect of dichotomous preferences is that RS-efficiency, DL-efficiency, and UL-efficiency coincide under them. Hence, our results in this section apply to all the three concepts. Our central result in the section is a characterization of the conditions under which testing Pareto optimality in coNP-complete. The condition identified is based on the size of the $topwdith$.

Theorem 3 *Checking whether a committee is Pareto optimal under the responsive set extension is coNP-complete even for dichotomous preferences and* $Topwidth(\succsim) \geq 3$.

Proof. We only present the case where $Topwidth(\succsim) = 3$. The reduction is from the NP-complete problem VERTEX COVER Garey and Johnson (1979). Given a simple graph $G = (V, E)$, the MINIMUM VERTEX COVER problem consists in finding a subset $C \subseteq V$ of minimum size such that every edge $e \in E$ is incident to some node of C. Its decision version VERTEX COVER takes as input a simple graph $G = (V, E)$ and an integer k and problem is deciding if there exists a vertex cover $C \subseteq V$ of G with $|C| \leq k$.

Let $\langle (V, E), k \rangle$ be an instance of VERTEX COVER, with $[x, y]$ being one arbitrary edge in E. We build the following instance of Pareto optimality under RS:

- $N = \cup_{e \in E} N_e \cup \{a\}$, where for each edge $e \in E$, N_e is a set of k agents, and a is a special agent.

- $A = V \cup D$, where $D = \{d_1, \ldots, d_k\}$.
- For each $e = [u, v] \in E$, the preferences of agent e^i, for $i = 1, \ldots, k$, and of agent a, are

$$e^i : \{u, v, d_i\}, (D - d_i) \cup (V \setminus \{u, v\})$$
$$a : \{x, y\}, D \cup (V \setminus \{x, y\})$$

The reduction is clearly done within polynomial time and preferences are dichotomous. We claim that that committee D (of size k) is not Pareto optimal under RS if and only if there exists a vertex cover of G of size at most k.

The condition is sufficient. Let $C \subseteq V$ be a vertex cover of G with a size exactly k (if it is strictly less, then we add arbitrarily vertices). Then set C Pareto dominates D because there is an improvement for agent a because edge e_0 is covered by C and indifference for the others agents.

Conversely, let D' be a Pareto improvement of D. Because, $D' \neq D$, then, agent a has strictly improved its preferences. Then, $D' = C \cup D'$ where $C \subseteq V$, $C \neq \emptyset$ and $D' \subset D$. Let N^i_E be the agents defined by: $N^i_E = \{e^i : e \in E\}$. For any agent $e^i \in N^i_E$ with $i \notin D'$, he is indifferent between alternatives d_i and one vertex of C. This means that C is a vertex cover of G with a size at most k. □

We point out that for dichotomous preferences, the responsive set extension coincides with the downward lexicographic set extension. Hence we get a corollary of our results for responsive preferences:

Corollary 1 *Checking whether a committee is DL-efficient is coNP-complete, even for dichotomous preferences and* $Topwidth(\succsim) \geq 3$.

We point out that for dichotomous preferences, the responsive set extension coincides with the upward lexicographic set extension. Hence we get a corollary of our results for responsive preferences:

Corollary 2 *Checking whether a committee is UL-efficient is coNP-complete, even for dichotomous preferences and* $Topwidth(\succsim) \geq 3$.

Using a similar reduction from the Hitting Set problem, we can also prove Theorem 4 that concerns a parametrized complexity intractability result Downey and Fellows (2013). Hitting Set is defined as follows: given a ground set X of elements, and a collection $\mathcal{C} = \{C_1, \ldots, C_\ell\}$ of subsets of X, does there exist a $H \subset X$ such that $|H| \leq k$ and $H \cap C \neq \emptyset$ for all $C \in \mathcal{C}$?

Theorem 4 *Checking whether a committee is Pareto optimal under the responsive set extension is W[2]-complete under parameter k, even for dichotomous preferences.*

For dichotomous preferences we present a complete characterization of the complexity according to the $Topwidth(\succsim)$ parameter. If $Topwidth(\succsim) = 1$, then in any Pareto improvement over committee D, any alternative in D that is most preferred by some agent needs to be kept selected, and therefore the problem of checking RS-efficiency is easy. If $Topwidth(\succsim) \geq 3$, from Theorem 3, the problem is hard. The only case remaining is $Topwidth(\succsim) = 2$.

Theorem 5 *For dichotomous preferences, a Pareto improvement over a committee with respect to the responsive set extension (or DL or UL set extension) can be computed in polynomial time when $Topwidth(\succsim) \leq 2$.*

Proof. Consider a preference profile $\succsim = (\succsim_1, \ldots, \succsim_n)$ where each $\succsim_i$ is dichotomous and verifies $Topwidth(\succsim) = 2$, and let $D \in S_k(A)$. For each $i \in N$, let (E_i^1, E_i^2) be the partition associated with $\succsim_i$.

First, if for all $i \in N$, $E_i^1 \subseteq D$, then D is obviously RS-efficient. Assume it is not the case, that is, (1) for some $i \in N$, $E_i^1 \setminus D \neq \emptyset$. Let

- $N' = \{i \in N : E_i^1 \cap D = E_i^1\}$, $W' = \cup_{i \in N'} E_i^1$ (by construction, $W' \subseteq D$), and $k' = |W'|$.
- $N'' = \{i \in N \setminus N' : E_i^1 \cap (D \setminus W') \neq \emptyset\}$ and $A'' = \cup_{i \in N''} E_i^1$.

Now, we build a graph $G = (V, E)$ with $V = \{v_1, \ldots, v_r\}$ isomorphic to A'', and $[v_p, v_q] \in E$ iff $E_i^1 = \{a_p, a_q\}$ for some $i \in N''$: each edge of G corresponds to the top two alternatives of some agent, provided one of them is in $D \setminus W'$. Let $\tau(G)$ be the size of an optimal vertex cover of G.

We first claim that there is a Pareto improvement over D if and only if one of follows two conditions is satisfied:

(*i*) $\tau(G) < k - k'$, or
(*ii*) $\tau(G) = k - k'$, and there is an optimal vertex cover of G containing either at least an element of E_i^1 for some $i \notin N' \cup N''$, or two elements of E_i^1 for some $i \in N''$.

We first show that (*i*) and (*ii*) are sufficient. If (*i*) holds then take a committee corresponding to a minimum vertex cover of G, add to it the k' alternatives of W', and add $(k-k')-\tau(G)$ alternatives, with at least one in $\cup_i (E_i^1 \setminus D)$; this is possible because of (1). If (*ii*) holds, then take a committee corresponding to a minimum vertex cover of G, and add to it the k' alternatives of W'. In both cases, the obtained committee contains E_i^1 for all $i \in N'$, contains at least one element of E_i^1 for all $i \in N'$, and contains either two elements of E_i^1 for some $i \in N''$, or an element of E_i^1 for some $i \notin N \cup N''$. Therefore it is a Pareto-improvement over D.

Now, we show that (*i*) and (*ii*) are necessary. Let $W \in S_k(A)$ be a Pareto improvement of D containing a maximum number of alternatives from D. We have the following two properties: $W' \subseteq W$ and $W \setminus W'$ is a vertex cover of G. $W' \subseteq W$ holds, since otherwise there would be an $i \in N'$ such that $W' \succsim_i^{RS} W$ does not hold. For similar reasons, $C' = (W \setminus W') \cap A''$ is a vertex cover of G. If $|(W \setminus W') \cap A''| < \tau(G)$, then by adding to it any set of $D \setminus C'$ of size $k - k' - \tau(G)$ we obtain a set of size k which constitutes a Pareto improvement of D because now, $E_i^1 \subseteq W$ for some $i \in N''$. If $|(W \setminus W') \cap A''| = \tau(G)$, then $(W \setminus W') \cap A'' = W \setminus W'$ and necessarily either $E_i^1 \cap C \neq \emptyset$ for some $i \notin (N' \cup N'')$ or $E_i^1 \subseteq C$ for some $i \in N''$.

It remains to be shown that (*i*) and (*ii*) can be checked in polynomial time. (i) can be done in polynomial-time because G is bipartite: indeed, by construction, G is two-colorable with color sets $A'' \cap D$ and $A'' \setminus D$, and by König's theorem, for bipartite graphs, the problem of finding the minimum vertex cover is equivalent to computing a maximum matching, hence solvable in polynomial time. As for (*ii*), if $\tau(G) = k - k'$, we have to check whether for some optimal vertex cover C of G, either

(ii.1) $E_i^1 \cap C \neq \emptyset$ holds for some $i \notin (N' \cup N'')$, or (ii.2) $E_i^1 \subseteq C$ for some $i \in N''$. In order to check (ii.1), for each $i \notin (N' \cup N'')$ such that there exists $x \in E_i^1 \cap A''$, we transform G into a new bipartite graph $G_{\{x\}}$ where we add a new vertex x' and an edge $[x, x']$. In order to check (ii.2), for each $i \notin N''$, let $E_i^1 = \{x, y\}$; we transform G into a new bipartite graph $G_{\{x,y\}}$ where we add two new vertices x' and y', and two edges $[x, x']$ and $[y, y']$. Finally, we test if $\tau(G) = \tau(G_{\{x\}})$ or if $\tau(G) = \tau(G_{\{x,y\}})$ for one of these graphs, because all optimal vertex covers of $G_{\{x\}}$ (respectively $G_{\{x,y\}}$) must contain x (respectively $\{x, y\}$). □

Example 3 We illustrate the algorithm in the proof of Theorem 5. Let $k = 2$ and consider the dichotomous profile, where we specify only the top equivalence class of each agent:

$$1 : \{a, c\} \qquad 2 : \{b, c\} \qquad 3 : \{b, d\}$$
$$4 : \{d, e\} \qquad 5 : \{e, f\}$$

Let $D = \{a, b\}$. We have $N' = W' = \emptyset$, $k' = 0$, $D \setminus W' = \{a, b\}$, $N'' = \{1, 2, 3\}$, and $A'' = \{a, b, c, d\}$. We construct the graph $G = (V, E)$: $V = \{v_a, v_b, v_c, v_d\}$ and $E = \{\{v_a, v_c\}, \{v_b, v_c\}, \{v_b, v_d\}\}$. We have $\tau(G) = 2 = k - k'$. Now we consider the four graphs $G_{\{d\}}$, resulting from the addition to G of a new vertex $v_{d'}$ and edge $[v_d, v_{d'}]$, and $G_{\{a,c\}}$, $G_{\{b,c\}}$ and $G_{\{b,d\}}$: $G_{\{a,c\}}$ results from the addition to G of two new vertices $v_{a'}, v_{c'}$ and edges $[v_a, v_{a'}]$ and $[v_c, v_{c'}]$, etc. Two of these graphs have an optimal cover of size 2: $G_{\{d\}}$, with optimal cover $\{v_c, v_d\}$, and $G_{\{b,c\}}$, with optimal cover $\{v_b, v_c\}$. Therefore, $\{c, d\}$ and $\{b, c\}$ are RS-Pareto-improvements over $\{a, b\}$, and $\{a, b\}$ is not RS-efficient.

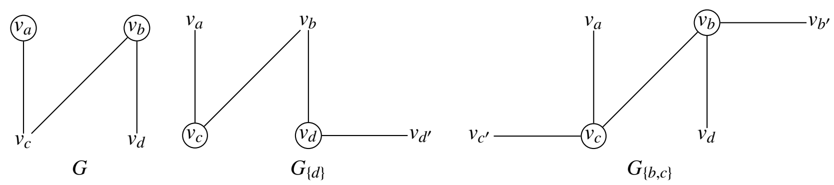

Fig. 2 Graphs corresponding to Example 3

Note that finding an algorithm that computes a Pareto improvement over a committee can be used to decide whether a given a committee D of size k, is Pareto optimal under the responsive set extension.

6.2 Linear preferences

Now, we deal with the case of linear preferences.

Theorem 6 *Checking whether a committee is Pareto optimal under the responsive set extension is coNP-complete for linear preferences.*

Proof. The proof is an adaptation of Theorem 3. The reduction from VERTEX COVER is almost the same, except we do not consider agent a. For any edge $e = [u, v] \in E$ and for $i = 1, \dots, k$, the strict preferences for agent e^i are given by:

$$e^i : \{u, v\}, d_i, (V \setminus \{u, v\}), (D - d_i)$$

where strict preferences over a subset Z are arbitrary given (for instance, $\{u, v\}, d$ means either $u > v > d$ or $v > u > d$).

As previously, we claim that committee D (of size k) is not Pareto optimal under RS if and only if there exists a vertex cover of G of size at most k.

Let $C \subseteq V$ be a vertex cover of G with $|C| = k$, where $v_e \in C$ is a vertex covering edge $e \in E$. Consider an arbitrary agent e^i for some $i \in \{1, \dots, k\}$ and $e = [u, v] \in E$. By construction, $v_e \succ^{RS}_{e^i} d_i$ and $(C \setminus \{v_e\}) \succsim^{RS}_{e^i} (D \setminus \{d_i\})$ since $D \setminus \{d_i\}$ are the least preferred candidates for agent e^i. Hence, $C \succ^{RS} D$. Conversely, the proof is similar to the one given in Theorem 3. Let D' be a Pareto improvement of D, that is $D' \succ^{RS} D$; then, there is $d_i \in D \setminus D'$ since $|D'| = |D| = k$. We will prove that $C = D' \setminus D$ is a vertex cover of G. Consider any agents e^i for $e = [u, v] \in E$. Since $C \succ^{RS}_{e^i} D \setminus D'$, then $f(d_i) \succ_{e^i} d_i$, or equivalently $f(d_i) \in \{u, v\}$. □

Theorem 7 *Checking whether a committee is Pareto optimal under DL is coNP-complete for linear preferences.*

Proof. The proof is exactly the same as that of Theorem 6 because in that reduction for any $D' \in S_k(A)$, we have $D' \succ^{RS} D$ if and only if $D' \succ^{DL} D$. □

We end the subsection with the case of upward lexicographic set extension.

Theorem 8 *Checking whether a committee is Pareto optimal under UL is coNP-complete for linear preferences.*

Proof. The reduction is given from VERTEX COVER. Let $\langle G = (V, E), k\rangle$ be an instance of VERTEX COVER where $|V| = n$ and $|E| = m$. We build the following instance of Pareto optimality under UL where the size of committee is n. There is $m(n - k)$ agents e^i for $e \in E$ and $i \in \{1, \dots, n - k\}$ and a set of $2n - k$ alternatives $A = V \cup D$ where $D = \{d_1, \dots, d_{n-k}\}$. The strict preferences of agent e^i for $e = [u, v] \in E$ and $i = 1, \dots, n - k$ are given by:

$$e^i : \ (D \setminus \{d_i\}), (V \setminus \{u, v\}), d_i, \{u, v\}$$

where strict preferences over a subset Z are arbitrary given as explained in Theorem 6. The reduction is clearly done within polynomial time and the set of preferences are strict. We claim that G has a vertex cover of size k iff committee V of size n is not Pareto optimal under UL.

Let $C \subseteq V$ be a vertex cover of G with $|C| = k$, where $f(e) \in \{u, v\} \subseteq C$ is a vertex incident to edge $e = [u, v] \in E$. The committee $C \cup D$ of size n satisfies $(C \cup D) \succ^{UL} V$. Actually, it is the case $(C \cup D) \succ^{UL}_{e^i} V$ for every agent e^i with $e \in E$ since alternative $f(e)$ is missing in the committee. Conversely, let V' be a Pareto improvement of V under UL, ie., $V' \succ^{UL} V$; then, there is $d_i \in V'$ for some $i \in \{1, \dots, n - k\}$. We will prove $C = V \setminus V'$ is a vertex cover of G. By contradiction, assume C is not a vertex cover of G; then some edge $e = [u, v] \in E$ is not covered by C, or equivalently $\{d_i, u, v\} \subseteq V'$. In this case $V >^{UL}_{e^i} V'$ (because $d_i \notin V$) which is a contradiction. In conclusion, C is a vertex cover and the proof is complete. □

7 'Best' Set Extension

Next, we consider Pareto optimality with respect to $\mathcal{B}$, which has been used for defining many rules (see Section 2).

Theorem 9 *Checking whether a committee is $\mathcal{B}$-efficient is coNP-complete, even for dichotomous preferences and $Topwidth(\succsim) \geq 2$.*

Proof. We only present the case where $Topwidth(\succsim) = 2$. and as previously the reduction is from VERTEX COVER. Let $\langle G = (V, E), k\rangle$ be an instance of VERTEX COVER where $|V| = n$ and $|E| = m$. We build the following instance of Pareto optimality under $\mathcal{B}$ where the size of committee is n. We construct the following profile P:

- The set of $m + (n + 1)(n - k)$ agents is $N = E \cup \{v^i, i : v \in V, i = 1, \ldots, n - k\}$.
- The set of $2n - k$ alternatives is $A = V \cup D$ where $D = \{d_1, \ldots, d_{n-k}\}$.
- Let $e = [u, v] \in E$ be an edge of G; the preferences of agent e for $e \in E$ are:

$$e : \{u, v\}, D \cup (V \setminus \{u, v\}).$$

- The preferences of the $n(n - k)$ agents v^i are given by: for $i = 1, \ldots, n - k$, $v \in V$,

$$v^i : \{v, d_i\}, (V \setminus \{v\}) \cup (D \setminus \{d_i\}).$$

- The preferences of the last $n - k$ agents i are given by: for $i = 1, \ldots, n - k$,

$$i : d_i, V \cup (D \setminus \{d_i\}).$$

The reduction is clearly done within polynomial time and the set of preferences given by $\succsim$ are dichotomous with $Topwidth(\succsim) = 2$. We claim that G has a vertex cover of size k iff committee V of size n is not Pareto optimal under $\mathcal{B}$.

Let $C \subseteq V$ be a vertex cover of G with $|C| = k$, where $v_e \in C$ is a vertex covering edge $e \in E$. The committee $C \cup D$ of size n satisfies $(C \cup D) \succsim^{\mathcal{B}} V$. Actually, for agent $e \in E$ or agent v^i $i \leq n - k$, and $v \in V$, there is an indifference between both committees V and $C \cup D$ while $(C \cup D)$ is strictly preferred to V under $\mathcal{B}$ for all agents $i = 1, \ldots, n-k,$. Conversely, let V' be a Pareto improvement of V under $\mathcal{B}$; then, there is $v \in V \setminus V'$ since $|V'| = |V| = n$. We will prove $C = V \setminus V'$ is a vertex cover of G. Consider, the $n - k$ agents v^i for $i = 1, \ldots, n - k$. Since $V' \succsim^{\mathcal{B}}_{v^i} V$, then $d_i \in V'$ for every $i \leq n - k$; Hence, $D \subseteq V'$. Now, $V' \succsim^{\mathcal{B}}_e V$ implies edge e is covered by $V' \setminus D$ and the proof is complete. □

The problem becomes easy to solve if the topwidth is less than or equal to 1.

Theorem 10 *Checking whether a committee is $\mathcal{B}$-efficient is polynomial-time solvable for dichotomous preferences and $Topwidth(\succsim) \leq 1$.*

Proof. Under the conditions, a given committee W is $\mathcal{B}$-efficient, if either (1) each agent has a most preferred candidate in W or (2) each candidate in W is a most preferred candidate of some agent. We prove as follows. If (1) holds, then W is clearly $\mathcal{B}$-efficient. If (2) holds, then no candidate from W can be replaced without making at least one agent less happy. Hence W is $\mathcal{B}$-efficient. Now suppose that neither (1) nor (2) hold. Then this means that there exists some candidate that is not the most preferred of any agent. Such a candidate can be replaced by the most preferred candidate of that agent who has no top candidate in W. Such a replacement leads to a Pareto improvement with respect to the 'best' set extension. □

Theorem 11 *Computing a $\mathcal{B}$-efficient committee is NP-hard, even for dichotomous preferences.*

Proof. We give a reduction from Hitting Set. Let $N = \{1, \ldots, \ell\}$, $A = X$ and for each $i \in N$, i's dichotomous preferences are $i : C_i, (X \setminus C_i)$. If there exists a polynomial-time algorithm to compute a $\mathcal{B}$-efficient committee, it will return a committee in which each agent gets a most preferred alternative if such a committee exists. But such a committee corresponds to a hitting set of size k. □

Dealing with linear preferences, in contrast to Theorem 11, a $\mathcal{B}$-efficient committee can be computed in polynomial time.

Theorem 12 *Under linear preferences, there is a linear-time algorithm to compute a $\mathcal{B}$-efficient committee.*

Proof. The algorithm works as follows. We go agent by agent and let her pick her most preferred candidate if it has not already been chosen. We stop when k candidates have been selected or all agents have been exhausted. If all the agents have been exhausted but k candidates have not been selected, we can fill the remaining slots by choosing arbitrary candidates. The algorithm chooses a most preferred committee for the agents who have their most preferred candidate in the committee. For other other agents, there simply is not enough space to get an improvement for them without affecting the agents who who have their most preferred candidate in the committee. Hence the outcome is □

Finally, note the same proof as the one given in Theorem 6 allow to conclude the following.

Theorem 13 *Checking whether a committee is Pareto optimal under $\mathcal{B}$ is coNP-complete for linear preferences.*

Proof. The proof is completely similar to Theorem 6 because in that reduction for any $D' \in S_k(A)$, we have $D' \succ^{RS} D$ if and only if $D' \succ^{\mathcal{B}} D$. □

8 'Worst' Set Extension

In contrast to all the other set extensions considered in the paper, Pareto optimality with respect to the 'worst' set extension can be checked in polynomial time.

Theorem 14 *There exists a polynomial-time algorithm that checks whether a committee is $\mathcal{W}$-efficient and computes a Pareto improvement over it if possible.*

Proof. Let $W \in S_k(A)$. For each $i \in N$, let $E_i^{t_i}$ be the least preferred equivalence class such that $E_i^{t_i} \cap W \neq \emptyset$. We want to check whether there is a k-set D of alternatives in which at least some agent $i \in N$ gets a strictly better outcome and all the other agents get at least as preferred an outcome. We check this as follows. For $i \in N$, let $B_i = A \setminus ((\bigcup_{\ell=t_i}^{k_i} E_i^{\ell}) \cup \bigcup_{j\in N\setminus\{i\}} \bigcup_{\ell=t_j+1}^{k_j} E_j^{\ell}))$. We check whether $|B_i| \geq k$ or not. If $|B_i| \geq k$, we know that there exists a subset of B_i, that is strictly more preferred by $i \in N$ and at least as preferred by each agent. The reason is that B_i contains a more preferred worst alternative for agent i than D and contains at least as preferred worst alternative for other agents j than D. If $|B_i| < k$, then this means that a Pareto improvement with i strictly improving is only possible if the size of the winning set is less than k which is not feasible. □

We now consider strategyproofness together with $\mathcal{W}$-efficiency. We first note that Algorithm 1 may not return a $\mathcal{W}$-efficient outcome. However, we construct a suitable strategyproof and $\mathcal{W}$-efficient by formalising an appropriate serial dictatorship algorithm for the worst set extension.

Theorem 15 *There exists a linear-time and strategyproof algorithm that returns a $\mathcal{W}$-efficient committee.*

Proof. Consider the agents in a permutation π. The set of alternatives A' is initialized to A. We reduce the set A' while ensuring that it of size at least k. The next agent i in the permutation comes and deletes the maximum number of least preferred equivalence classes from his preferences and the corresponding alternatives in A' while ensuring that $|A'| \geq k$. Each successive agent in the permutation gets a most preferred outcome while ensuring that agents before him in the permutation get at least as preferred an outcome as before. Thus the algorithm is strategyproof and Pareto optimal with respect to the 'worst' set extension. □

9 Conclusions

We considered Pareto optimality in multi-winner voting with respect to a number of prominent set extensions. We presented results on the relations between the notions as well as complexity of computing and verifying Pareto optimal outcomes. Another direction is consider the compatibility of Pareto optimality concepts with other axioms. Finally, we remark that our serial dictatorship algorithm can be used to define a multiwinner generalization of random serial dictatorship, which is worth investigating and raises interesting computational problems.

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