On the Incompatibility of Efficiency and Strategyproofness in Randomized Social Choice

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Randomized Social Choice

A social decision scheme (SDS) maps a preference profile to a lottery over alternatives. Two desirable properties of SDSs are

• **Efficiency**: no agent can be made better off without making another agent worse off, and

• **Strategyproofness**: no agent can benefit from misrepresenting his preferences.

Related Work

Gibbard (1973) & Satterthwaite (1975): Every single-valued, Pareto-optimal, and strategyproof social choice function is a dictatorship.

Gibbard & Sonnenschein (1977): Every Pareto optimal and strongly SD-strategyproof SDS is a probability mixture of dictatorships.

Bogomolnaia & Moulin (2001): There is no anonymous, neutral, SD-efficient, and strongly SD-strategyproof SDS.

Preliminaries

Each agent $i$ has a complete and transitive preference relation $R_i$ over a set of alternatives $A$. A social decision scheme (SDS) maps a preference profile $(R_1,\ldots,R_n)$ to a lottery $p$ over $A$. An SDS is

• anonymous if it is symmetric with respect to voters.

• neutral if it is symmetric with respect to alternatives.

• pairwise if the outcome depends on the pairwise majority relation only.

Lottery Extensions

There are many reasonable ways to extend preferences over alternatives to preferences over lotteries.

Bilinear dominance (BD): $p R_i^{BD} q$ if $p(x)q(y) \geq p(y)q(x)$ for all $x,y$ where $x \neq P_i y$.

Stochastic dominance (SD): $p R_i^{SD} q$ if $\sum_{x \in A} p(x)q(x) \geq \sum_{x \in A} q(x)p(x)$ for all $x$.

Pairwise comparison (PC): $p R_i^{PC} q$ if $\sum_{x \in A} p(x)q(y) \geq \sum_{x \in A} q(x)p(y)$ for all $x,y$.

Downward/Upward lexicographic (DL/UL): $p R_i^{DL} q$ if $p = q$ or there is $x$ such that $p(x) > q(x)$ and $p(y) = q(y)$ for all $y \neq P_i x$, $p R_i^{UL} q$ if $p = q$ or there is $x$ such that $p(x) < q(x)$ and $p(y) = q(y)$ for all $x \neq P_i y$.

Efficiency and Strategyproofness

• In a preference profile $R$, a lottery $p$ is $\epsilon$-efficient if there is no $q$ such that $q R_i^\epsilon p$ for all $i$. An SDS $f$ is $\epsilon$-efficient if $f(R)$ is $\epsilon$-efficient for every preference profile $R$.

• An SDS $f$ is **Pareto-optimal** if $f(R)$ does only support Pareto-optimal alternatives for every $R$.

• An SDS $f$ is **$\epsilon$-strategyproof** if there are no $R$, $R'$, $i$ such that $R_i = R_i'$ for all $j \neq i$ and $f(R') R_i^\epsilon f(R)$.

• An SDS $f$ is **strongly $\epsilon$-strategyproof** if there are no $R$, $R'$, $i$ such that $R_i = R_i'$ for all $j \neq i$ and **not** $f(R) R_i^\epsilon f(R')$.

Impossibility Theorems

• There is no anonymous, neutral, **PC**-efficient, and **PC**-strategyproof SDS.

• There is no anonymous, UL-efficient, and UL-strategyproof SDS.

• There is no pairwise, Pareto-optimal, and BD-strategyproof SDS.

• There is no anonymous, neutral, Pareto-optimal, and BD

**Conjecture**: There is no anonymous, SD-efficient, and SD-strategyproof SDS.

Anonymity is required for all our results as serial dictatorship satisfies all considered notions of efficiency and strategyproofness.

1Interestingly, random dictatorship satisfies anonymity, neutrality, DL-efficiency, and DL-strategyproofness.

2This strengthens a result by Aziz, Brandt, and Brill (2013).

3This strengthens a result by Bogomolnaia, Moulin, and Stong (2005).