On the Incompatibility of Efficiency and Strategyproofness in Randomized Social Choice

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Randomized Social Choice

A **social decision scheme (SDS)** maps a preference profile to a lottery over alternatives. Two desirable properties of SDSs are

- **Efficiency:** no agent can be made better off without making another agent worse off, and
- **Strategyproofness:** no agent can benefit from misrepresenting his preferences.

Related Work

Gibbard (1973) & Satterthwaite (1975): Every single-valued, Pareto-optimal, and strategyproof social choice function is a dictatorship.

Gibbard & Sonnenschein (1977): Every Pareto optimal and strongly *SD*-strategyproof SDS is a probability mixture of dictatorships.

Bogomolnaia & Moulin (2001): There is no anonymous, neutral, *SD*-efficient, and strongly *SD*-strategyproof SDS.

Preliminaries

Each **agent** i has a complete and transitive **preference relation** R_i over a set of **alternatives** A. A **social decision scheme (SDS)** maps a preference profile $(R_1,\ldots,R_n)$ to a **lottery** p over A. An SDS is

- **anonymous** if it is symmetric with respect to voters.
- **neutral** if it is symmetric with respect to alternatives.
- **pairwise** if the outcome depends on the pairwise majority relation only.

Lottery Extensions

There are many reasonable ways to extend preferences over alternatives to preferences over lotteries.

Bilinear dominance (*BD*): $p\ R_i^{BD}\ q$ if $p(x)q(y) \geq p(y)q(x)$ for all x,y where $x\ P_i\ y$.

Stochastic dominance (*SD*): $p\ R_i^{SD}\ q$ if $\sum_{yRx} p(x) \geqq \sum_{yRx} q(x)$ for all x.

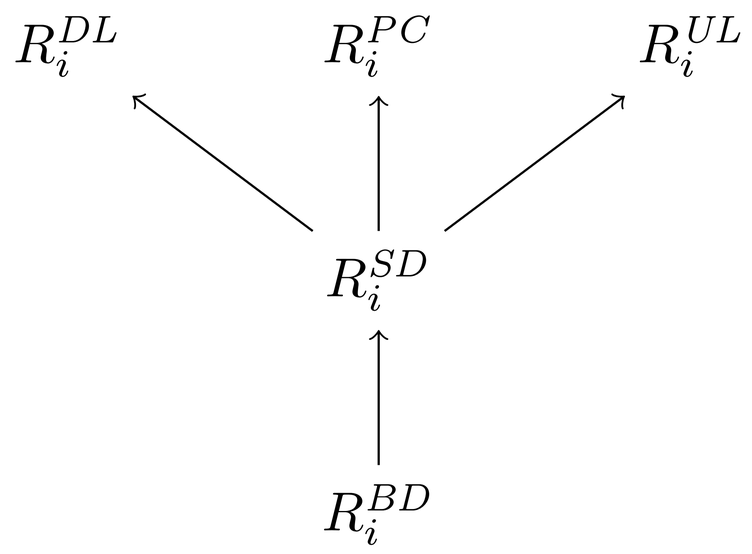
Pairwise comparison (*PC*): $p\ R_i^{PC}\ q$ if $\sum_{xRy} p(x)q(y) \geqq \sum_{xRy} q(x)p(y)$ for all x,y.

Downward/Upward lexicographic (*DL*/*UL*): $p\ R_i^{DL}\ q$ if $p = q$ or there is x such that $p(x) > q(x)$ and $p(y) = q(y)$ for all $y\ P_i\ x$. $p\ R_i^{UL}\ q$ if $p = q$ or there is x such that $p(x) < q(x)$ and $p(y) = q(y)$ for all $x\ P_i\ y$.

Example: $a\ P_i\ b\ P_i\ c$;

$p = \frac{2}{3}a + \frac{1}{3}c$; $q = b$; and $r = \frac{2}{3}b + \frac{1}{3}c$.

Then $p\ P_i^{PC}\ q$; $p\ P_i^{DL}\ q$; $p\ P_i^{SD}\ r$; $q\ P_i^{BD}\ r$; and $q\ P_i^{UL}\ p$.



Efficiency and Strategyproofness

- In a preference profile R, a lottery p is **ε-efficient** if there is no q such that $q\ R_i^{\varepsilon}\ p$ for all i. An SDS f is ε-efficient if $f(R)$ is ε-efficient for every preference profile R.
- An SDS f is **Pareto-optimal** if $f(R)$ does only support Pareto-optimal alternatives for every R.
- An SDS f is **ε-strategyproof** if there are no R, R', i such that $R_j = R'_j$ for all $j \neq i$ and $f(R')\ P_i^{\varepsilon}\ f(R)$.
- An SDS f is **strongly ε-strategyproof** if there are no R, R', i such that $R_j = R'_j$ for all $j \neq i$ and **not** $f(R)\ R_i^{\varepsilon}\ f(R')$.

Impossibility Theorems

- There is no anonymous, neutral, *PC*-efficient, and *PC*-strategyproof SDS.
- There is no anonymous, *UL*-efficient, and *UL*-strategyproof SDS.
- There is no pairwise, Pareto-optimal, and *BD*-strategyproof SDS.
- There is no anonymous, neutral, Pareto-optimal, and *BD*

Conjecture: There is no anonymous, *SD*-efficient, and *SD*-strategyproof SDS.

Anonymity is required for all our results as **serial dictatorship** satisfies all considered notions of efficiency and strategyproofness.

[1]Interestingly, random dictatorship satisfies anonymity, neutrality, *DL*-efficiency, and *DL*-strategyproofness.

[2]This strengthens a result by Aziz, Brandt, and Brill (2013).

[3]This strengthens a result by Bogomolnaia, Moulin, and Stong (2005).

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