All-dimensional subwavelength cavities made with metamaterials

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By exploiting the reflection phase properties of metamaterial reflectors, the authors propose a method to break the size restrictions imposed strictly on conventional cavities. They design the all-dimensional subwavelength cavities and perform experiments and simulations to demonstrate their subwavelength functionalities. For the smallest cavity that they fabricated, each dimension is only a quarter of the resonance wavelength. © 2006 American Institute of Physics.

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Resonance cavities play crucial roles in both electromagnetic (EM) theory and applications. Conventional cavities are constructed with metallic reflectors, and the governing mechanism imposes a natural lower limit on the cavity size. Consider the simplest \( a \times a \times a \) cubic cavity; the lowest mode is \( \lambda_{111} = \sqrt{2}a \), implying that a cavity must be large enough in order to create a resonance mode with a definite wavelength.\(^1\) Cavities with other shapes have similar restrictions. These restrictions greatly limit the applications of a conventional cavity in compact environments. Several ideas were proposed to break such size restrictions. The simplest one is to fill the cavity with high-index materials. More recent ideas include the insertion of a resonance surface into the cavity\(^,7\) or filling the cavity with both positive- and negative-index materials.\(^3\) Simulations demonstrated that both ideas lead to a significant reduction of the cavity size.\(^2,3\)

Here, we demonstrate an alternative approach to beat the cavity size restrictions. Distinct from previous methods,\(^2,3\) we do not insert any other material to the cavity. The key idea is to replace the conventional metallic reflectors by metamaterial reflectors.\(^4\) In our previous work, we have successfully fabricated a double-plate (i.e., one-dimensional) cavity,\(^7\) with a thickness much less than \( \lambda/2 \) (see also Refs. 8 and 9). We extend the idea to three dimensions and fabricate cavities that are much smaller than conventional ones working with the same wavelengths. The good agreements among experiments, finite-difference time-domain (FDTD) simulations, and model analysis unambiguously demonstrated the subwavelength functionalities of the constructed cavity.

Consider a \( d_x \times d_y \times d_z \) cuboidal cavity bounded by six metamaterial reflectors, each defined by its normal vector (i.e., \( \pm x, \pm y, \pm z \)). Each reflector reflects EM waves with a definite phase \( \Delta \phi_s(f) \) depending on frequency \( f \), translated to a boundary condition \( E'_x/E'_m = \exp(i\Delta \phi_s) \) at the surface.

Here \( E'_x \) and \( E'_m \) denote the parallel components of \( E \)-field for the reflected and incident waves with respect to the surface. For a transverse-magnetic (TM) mode,\(^10\) the \( E_z \) component takes a general form

\[
E_z(r) = \frac{1}{8} \sum_{j=1}^{8} C(k_j)e^{ik_jr}e^{i\sigma t},
\]

with \( k_j = \pm k_x, \pm k_y, \pm k_z \) and

\[
k^2_x + k^2_y + k^2_z = (\omega/c)^2,
\]

where \( c \) is the speed of light. Other field components can be derived from \( E_z \) through Maxwell equations.\(^1\) The expansion parameters \( C(k) \) are determined by matching boundary conditions. For example, for the reflector located at \( y = 0 \) (with a normal vector \( \hat{y} \)), we can identify the wave component \( C(k_x, k_y, k_z) \) as the effective incident beam on the surface and the component \( C(k_x, -k_y, k_z) \) as the corresponding reflected beam.\(^12\) Therefore, boundary conditions require that

\[
\frac{C(k_x - k_y, k_z)}{C(k_x, k_y, k_z)}e^{i(k_x x + k_y y + k_z z)}|_{y=0} = \exp(i\Delta \phi_{sz}).
\]

Similar arguments lead to

\[
\frac{C(k_x, k_y, k_z)}{C(k_x, -k_y, k_z)}e^{i(k_x x + k_y y + k_z z)}|_{y=d_y} = \exp(i\Delta \phi_{sz}).
\]

at the reflector located at \( y = d_y \) (with a normal vector \( -\hat{y} \)). Combining the above two equations, we get

\[
-2k_x d_x + \Delta \phi_{sz} + \Delta \phi_{sz} = m_x 2\pi,
\]

where \( m_x \) is an integer. Similarly, we have

\[
-2k_y d_y + \Delta \phi_{sz} + \Delta \phi_{sz} = m_y 2\pi,
\]

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mushroom structure

4.5-mm-long monopole antenna inside the cavity, obtained by experiments (open circles) and FDTD simulations (solid line). (b) The \(\delta_{m|}(f)\) curves for different modes specified in the legend, taking values of \(\Delta \phi_{m}(f)\) calculated under normal incidence.

FIG. 1. (Color online) (a) Picture of the metamaterial cavity with side length=21 mm. (b) Top view picture of the metallic mesh layer of the cavity shown in (a). (c) Picture of the cavity with side length=14 mm. The inserted coaxial cable as the feed, its outer diameter is 1.22 mm; that of the inner conductor and the surrounded dielectric insulator are 0.28 and 0.90 mm, respectively.

We performed FDTD simulations to understand the observed phenomena. In our simulations, we discretized the realistic structure by a basic cell sized \(0.5 \times 0.5 \times 0.5 \text{ mm}^3\) and adopted perfect metal boundary conditions at each metal surface. Finer meshes were adopted whenever necessary. The calculated S11 spectrum is shown in Fig. 2(a) as a solid line. Fairly good agreement is found compared with the experimental result.

We employed Eq. (5) to identify the indices of these modes. We first calculated the reflection phases \(\Delta \phi_{m}(f)\) for every reflector under normal incidence, and then inserted \(\Delta \phi_{m}(f)\) into Eq. (5) to calculate the function \(\delta_{m|}\) with different mode indices. The \(\delta_{m|}\) curves depicted in Fig. 2(b) indicated that the resonance frequencies are 7.45, 8.75, and 8.69 GHz, for the lowest three modes indexed, respectively, by (110), (101) [degenerate with (011)], and (111). However, these results are in apparent disagreement with the experimental and the FDTD simulation ones. Such discrepancies stimulate us to reexamine the validity of the approximation to calculate \(\Delta \phi_{m}(f)\) under normal incidence. While \(\Delta \phi_{m}(f)\) has no dependence on the incidence angle for a metal, and a weak dependence for a metallic mesh, it does exhibit a strong incidence-angle dependence for the metamaterial reflector, as illustrated in Fig. 3(a) where \(\Delta \phi(f) \sim f\)
curves are shown for different incidence angles. For the lowest (110) mode, symmetry requires strictly that \( k_x = k_y \), so that the reflection at the metamaterial surface corresponds to an incidence angle of 45°. For (101) and (111) modes, we have no such symmetry properties. We incorporate the incidence-angle corrections in an iterative way. We first use the normal incidence \( \Delta \phi_{\text{inc}}(f) \) to solve Eqs. (1)-(5) and estimate the corresponding incidence angles (with respect to the metamaterial reflectors) from the values of \( k_x, k_y, k_z \) obtained for these modes. Calculations show that the results are 54.7° for the (110) mode and 66.3° for the (111) mode. We then calculated \( \Delta \phi_{\text{inc}}(f) \) under these incidence angles and used the corrected \( \Delta \phi_{\text{inc}}(f) \) results [as shown in Fig. 3(a)] to recalculate the function \( \delta_{m} \) and depicted the corrected \( \delta_{m} \sim f \) curves in Fig. 3(b). The resonance frequencies obtained following these procedures are 8.0, 9.08, and 9.63 GHz for these modes, which are in excellent agreements with both simulations and experiments. In particular, we find that the (110) mode becomes lower than the (111), after considering the incidence angle corrections. Such incidence angle dependence is a unique feature of the present metamaterial cavity, distinct from a conventional cavity.

We also measured the radiation patterns of the antenna placed inside the cavity, for the mode at \( f = 8.0 \text{ GHz} \) (Fig. 4). The \( H \)-plane pattern is symmetrical with respect to the 45° axis, consistent with the symmetry properties of the (011) mode. However, the \( E \)-plane pattern is not entirely symmetrical, due to the intrinsic asymmetry of the monopole antenna itself. FDTD simulation results are shown in the same figures as solid lines. Good agreement between theory and experiment is noted.

In principle, there is no restriction on the cavity size if we can freely tune \( \Delta \phi_{\text{inc}}(f) \) as desired. The cavity can be achieved via tuning the magnetic resonance frequencies of the metamaterial reflectors. To illustrate this point, we fabricated two new metamaterial reflectors in which \( e_z \), of the inner PCB layer changes to 10.2 and applied them to construct a \( 14 \times 14 \times 14 \text{ mm}^3 \) cavity [picture shown in Fig. 1(c)] and redo the same measurements and simulations. These metamaterials possess even lower magnetic resonance frequencies, so that the eigenmodes inside the cavity are in a much smaller subwavelength size. Shown in Fig. 5 are the S11 spectra of a 4-mm-long antenna inside such a cavity, obtained by measurements (open circles) and FDTD simulations (solid line). Although the quantitative agreement between theory and experiment is not perfect, the positions of the resonance modes indicated in two spectra accurately agree with each other. In particular, we note that the lowest mode of the present cavity is at \( f = 5.5 \text{ GHz} \), which is roughly 1/3 of that for a conventional cavity with the same size (\( \sim 15.15 \text{ GHz} \)). Meanwhile, the side length of the cavity is only (roughly) 1/4 of the working wavelength \( \sim 54.5 \text{ mm} \).

In short, we demonstrated a mechanism to construct all-dimensional subwavelength cavities based on metamaterials. The present idea may be useful in applications such as a subwavelength high-Q filter and subwavelength cavity antenna.

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