Power level selection schemes to improve throughput and stability of slotted ALOHA under heavy load

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Abstract

We propose and analyse three different power selection schemes for slotted ALOHA random access protocol operating under multiple power levels. Through analysis and numerical examples we demonstrate that these schemes can significantly improve the performance of slotted ALOHA, in terms of throughput and stability, under heavy load. The proposed schemes are truly distributive in nature and can be easily implemented in wireless access systems without requiring any centralised control. © 2002 Elsevier Science B.V. All rights reserved.

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1. Introduction

The conventional and emerging wireless networks, such as satellite, cellular, wireless local area, ad hoc and wireless personal area networks, all face the same problem of a set of nodes trying to share a common broadcast wireless channel. Among the protocols for random access through a common wireless channel, slotted ALOHA is one of the most attractive one due to its simplicity and low delay (under light load) for bursty traffic. Slotted ALOHA is frequently used as a component of more complex protocols in wireless systems, e.g. in the capacity/channel request phase of wireless ATM [1] and GSM cellular networks [2,3].

The original slotted ALOHA, which uses a single power level for transmission, has two inherent problems. Firstly, it has a very low capacity (maximum throughput) of only 0.37 which is achieved at a system load of one transmission attempt per slot on average \((G = 1)\). Secondly, it exhibits a throughput-collapse phenomenon whereby the throughput decreases exponentially to zero as the mean number of transmission attempts increases beyond one. In the literature, the throughput-collapse is referred to as the stability problem of slotted ALOHA. There are several algorithms [4–6] to stabilise slotted ALOHA. These algorithms try to dynamically adjust transmissions to keep the system load at the optimal operating point of \(G = 1\). Such adjustments adds complexity to the system, increases delay for bursty traffic, and detracts them from the original appeal (simplicity and low delay) of slotted ALOHA.

The use of multiple power level to boost the capacity of slotted ALOHA was first suggested and analysed by Metzner [7]. With multiple power level, a mobile terminal can transmit a packet using one of the available power levels. The advantage of using multiple power level is that the receiver can still successfully capture a packet even if there is a collision in a given slot. The successful capture is possible when the power level of the captured packet is higher than all other contending packets. For two power levels, Metzner [7] showed that if one class of terminals always used high power level and the other always used low power level, the system capacity could be increased from 0.37 to 0.53.

Since class-based groupings of mobile terminals lead to unfairness, several researchers extended Metzner’s idea of multiple power level to achieve fairer algorithms where every terminal selects a given power level with the same probability. In power level division multiple access (PDMA) [8], a central entity (e.g. base station) assigns the power levels to the mobile terminals at each slot to maintain very high throughput (close to 1.0). However, the involvement of a central entity no longer leaves it as a true
distributed protocol and hence is not so attractive from implementation point of view.

To achieve a truly distributed random access protocol, each mobile host must be able to independently select a given power level without the need to communicate with any central entity and/or know the current network size (e.g. the number of mobile terminals competing for the same wireless channel). One solution to achieve a fair distribution of channel bandwidth among all competing mobile terminals is to randomly select a power level, from a set of pre-determined power levels, with uniform distribution. With uniform distribution, every terminal will select every power level with equal frequency on the long run leading to fair distribution of bandwidth. Such random selection of power levels with uniform probability distribution was studied in Ref. [9].

Later, LaMaire et al. [10] showed that uniform distribution do not lead to optimum throughput for a given number of mobile terminals. The optimisation problem was well defined in Ref. [10], but due to complex recursions, the authors admitted that optimum probability distribution for power level selection is exceedingly difficult for large number of mobile terminals.

Although it is now established that using multiple power level can significantly boost the capacity (maximum throughput) of slotted ALOHA, the throughput-collapse phenomenon at high system load remains a serious problem, especially for uniformly distributed power level selections. The objective of our study was to devise simple distributed algorithms for selecting power levels that would prevent throughput-collapse at high system loads.

To address the throughput-collapse at high load, we propose three truly distributed algorithms for selecting a given power level in a multiple power level system. All our algorithms have the following attractive features:

- they prevent throughput-collapse under heavy load ($G \gg 1$),
- they are truly distributed in the sense that they do not have any central component and they do not require the mobile hosts to know the current network size,
- very simple, no complex estimation,
- no need for dynamic adjustment of retransmission probability to keep the system load near $G = 1$.

The rest of the paper is organised as follows. The multiple power system model we use to derive our schemes is described in Section 2. In Section 3, we introduce and analyse our three power selection schemes. Closed-form throughput expressions for each of our schemes are derived in Section 4. Section 5 presents numerical results to demonstrate that our proposed schemes can achieve sustainable throughput even at high system load. In Section 6, we present a more theoretically sound explanation of the stability improvement feature of our schemes, before concluding the paper in Section 7.

2. System model

In this section, we describe the multiple power level model and associated assumptions used in this article. All three schemes, described in Section 3, are based on this system model.

In our model, there is one central receiver and many mobile wireless terminals trying to send packets to the receiver. For a large number of mobile terminals, we make the following conventional assumptions about the traffic:

- new packet generations are Poisson distributed with parameter $\lambda$.
- total number of transmission attempts (new packets plus retransmitted packets) is also a Poisson variable with parameter $G$. This assumption is valid when retransmissions are sufficiently randomised (which is the case with slotted ALOHA).

With the above assumption and notation, $G = 1$ means a system load of one transmission attempt (new or retransmission) per slot on average. $G = 1$ is the optimal operating point for slotted ALOHA with single power level, beyond this load we observe the throughput-collapse phenomenon. As we will see later in the article, we will analyse the performance of our proposed algorithms under a much higher system load of $G (G \gg 1)$.

We assume a ‘perfect capture’ model where a successful capture (reception) of a packet at the receiver occurs when the power level selected for this packet is greater than those of all other packets transmitted in the same slot. More precisely, the successful capture occurs for mobile terminal $x$ if

$$\Delta_x > \Delta_y, \forall y \neq x$$

where $\Delta_x$ is the power received at the central receiving station from mobile terminal $x$. Here, the implicit assumption is that if mobile terminal $x$ selects a higher power level than mobile terminal $y$, the power level received at the central receiver for mobile terminal $x$ will always be higher than that of mobile terminal $y$ irrespective of their distances from the receiver.

In our model, we therefore do not consider the power roll-off effect with distance. This assumption can be justified for the emerging pico-cell environments, such as in densely populated urban areas, for low radius cells for personal area networks, and so on, where all mobile terminals are close to the receiver. Also for large cells in conventional cellular networks, there are now sophisticated power control technologies which can smooth out the power roll-off effect.

3. Power selection schemes

In this section, we propose and analyse three different power selection schemes, linear, annular and circular shell,
for the mobile terminals to address the throughput-collapse phenomenon under high system load ($G \gg 1$). Probability distribution for power level selection is derived for each scheme. All three schemes are based on the following intuitive conjecture.

**Conjecture.** If higher power levels are selected with lower probability, the probability of receiving multiple packets at the highest power level received will decrease, which in turn will increase the probability of successful transmission (in other words, this will increase the system throughput).

### 3.1. Linear scheme

In this scheme, we decrease the selection probability of higher power levels linearly. To achieve this, we first construct a probability-height diagram as shown in Fig. 1, where the height of a power level determines its selection probability. Note that for uniformly distributed selection, each of the $N$ power levels is selected with probability $1/N$ and hence all heights are equal (the middle diagram in Fig. 1) resulting in a horizontal roof-line. Here, 1 is the highest power level and $N$ is the lowest power level.

Now all we have to do to decrease the selection probability of higher power levels linearly is to tilt the roof-line in the direction shown in the top diagram in Fig. 1. The increase of probability height for the lowest power level ($N$) is denoted by positive $h$. Therefore, depending on the sign of $h$, we have the following three cases.

1. If $h > 0$, the probability of transmission at lower power levels is *higher* than that of at higher power levels. We consider this case for our linear scheme.
2. If $h = 0$, the probability of transmission of packets at all $N$ power levels is uniformly distributed.
3. If $h < 0$, the probability of transmission at lower power levels is *lower* than that of at higher power levels. This case is against our conjecture stated in Section 3.

Note that the absolute value of $h$ is bounded by $1/N$ which occurs when either end of the roof-line hits the floor. Let us define $a_i^L$ as the probability of transmitting a packet to the $i$th power level using the linear scheme. This probability can be formulated using simple geometry from Fig. 1 as

$$a_i^L = \frac{2h}{N-1} (i-1) + \frac{1}{N} - h$$

$$= \frac{h}{N-1} (2i - N - 1) + \frac{1}{N} \quad |h| \leq 1/N \quad (2)$$

In Sections 3.2 and 3.3, we consider two different power selection schemes where the probabilities of higher power levels are decreased exponentially. The exponential decrease is motivated by the fact that the throughput of slotted ALOHA decreases exponentially at higher load. Therefore, if the higher power levels are selected with exponentially decreasing probability, the throughput loss may be addressed somewhat evenly.

### 3.2. Annular scheme

Consider a circle of radius $R$. There are $N$ circles within this circle each having a radius $r$ ($R = Nr$). Therefore, we have $N$ annulars as shown in Fig. 2. The fractional area of each annular, which is an exponential expression, determines the probability of a specific power level. The innermost annular has the lowest area and the outermost annular has the largest area. Therefore, to achieve decreasing probability for higher power levels, any mobile terminal selects the first power level with the probability equal to the fractional area of the innermost annular, the second power level with the probability equal to the fractional area of the second annular and so on. In general, the packet transmission at the $i$th power level is the probability equal to the fractional area of the $i$th annular.

Referring to Fig. 2, the area of the $i$th annular and hence the probability of transmitting a packet to the $i$th power level is given by

$$a_i^A = \frac{(i-1)r}{R}$$

where $R = Nr$ is the radius of the outermost circle and $r$ is the radius of the innermost circle.
the probability of selecting \( i \)th power level is obtained as

\[
\alpha_i^w = \int_{(i-1)r}^{ir} \frac{2\pi x \, dx}{\pi R^2} = \frac{2i - 1}{(R/r)^2} = \frac{2i - 1}{N^2}
\]  

(3)

3.3. Circular shell scheme

In the circular shell scheme, we consider a rigid ball of radius \( R \) composed of \( N \) circular shells. Each circular shell has a radius \( r \) (\( R = Nr \)). The packet transmission into the different power levels corresponds to the fractional volume of the circular shell.

A packet of a mobile terminal selects the \( i \)th power level with the probability equal to the fractional volume of the \( i \)th circular shell. The fractional volume of the \( i \)th circular shell (equivalent to the selection probability of the \( i \)th power level) is obtained as:

\[
\alpha_i^c = \int_{(i-1)r}^{ir} \frac{4\pi x^2 \, dx}{4\pi R^3} = \frac{(3i^2 - 3i + 1)r^3}{R^3} = \frac{3i^2 - 3i + 1}{N^3}
\]  

(4)

4. Throughput of different power selection schemes

In this section, we first derive a generic closed-form solution for computing the throughput of slotted aloha with multiple power level without worrying about the actual probability distribution for power selections. Later, the throughput of each power selection scheme will be obtained from this expression by replacing the generic power selection probability with specific probabilities as obtained in Sections 3.1–3.3.

Let us consider a set of \( N \) available power levels, where \( N \) is the lowest and 1 is the highest power level. To obtain the generic throughput expression, we make use the following notations:

\( \alpha_i \) probability that a mobile terminal selects power level \( i \)

\( \lambda_i \) mean packet arrivals at power level \( i \)

\( P_i(X = k) = (\lambda_i^k/k!) \exp(-\lambda_i) \) probability that there are a total of (from all mobile terminals) \( k \) packets transmitted at power level \( i \)

\( \varphi_i \) probability of a successful capture of a packet at power level \( i \)

The packet arrival from all mobile terminals are Poisson with a mean \( G \) and thus the mean arrival at power level \( i \) is \( \lambda_i = G\alpha_i \). A packet transmitted at power level \( i \) can be successfully captured by the receiver if and only if there is no other packets transmitted at the same slot in the upper power levels (i.e. from \( 1 \) to \( i-1 \)) and exactly one packet transmitted in the power level \( i \). Therefore, we obtain:

\[
\varphi_i = P_i(X = 1) \prod_{k=1}^{i-1} P_i(X = 0)
\]

\[
= G\alpha_i \exp(-G\alpha_i) \prod_{k=1}^{i-1} \exp(-G\alpha_k) = G\alpha_i \prod_{k=1}^{i} \exp(-G\alpha_k)
\]

\[
= G\alpha_i \exp\left(-G \sum_{k=1}^{i} \alpha_k \right)
\]

(5)

By adding the successful capture probabilities of all power levels, we obtain the throughput of slotted ALOHA with the multiple power level as:

\[
S = \sum_{i=1}^{N} \varphi_i = G \sum_{i=1}^{N} \alpha_i \exp\left(-G \sum_{k=1}^{i} \alpha_k \right)
\]

(6)

Eq. (6) provides us the generic expression for computing the throughput of slotted ALOHA with multiple power level. \( \alpha_i \) will be different for different power selection schemes. Hence by replacing \( \alpha_i \) in Eq. (6) with appropriate values, we
can obtain the closed-form solution for throughput for different power level selection schemes as follows:

\[
S = \begin{cases} 
\frac{G}{N} \exp\left(-\frac{G}{N}\right) + \cdots + \frac{G}{N} \exp\left(-\frac{G}{N}\right) + \cdots + \frac{G}{N} \exp(-G) & \text{for uniform} \\
\frac{G}{N} \exp\left(-\frac{G}{N}\right) + \cdots + \frac{G}{N} \exp\left(-\frac{G}{N}\right) + \cdots + \frac{G}{N} \exp(-G) & \text{for linear} \\
\frac{G}{N^2} \exp\left(-\frac{G}{N}\right) + \cdots + \frac{G}{N^2} \exp\left(-\frac{G}{N}\right) + \cdots + \frac{G}{N^2} \exp(-G) & \text{for annular} \\
\frac{G}{N^3} \exp\left(-\frac{G}{N}\right) + \cdots + \frac{G}{N^3} \exp\left(-\frac{G}{N}\right) + \cdots + \frac{G}{N^3} \exp(-G) & \text{for circular shell}
\end{cases}
\]

(7)

5. Numerical examples for throughput curves

In this section, we analyse the throughput curves of our proposed power selection schemes and compare them with the uniformly distributed scheme. The throughput versus offered load curves for the linear scheme is depicted in Fig. 3. The curve for \( h = 0 \) shows the throughput dynamics of the uniform selection scheme. With \( h < 0 \), the performance is worse than uniform distribution as expected. With \( h > 0 \), we can see that the linear scheme achieves higher throughput than the uniform distribution under heavy load as suggested by our conjecture.

Similarly, the higher throughputs of the annular and circular shell schemes are evidenced in Figs. 4 and 5, respectively, especially under heavy load. We see that the annular scheme achieves higher throughput than that of uniform distribution under heavy load without sacrificing maximum throughput, whereas the circular shell approach achieves higher throughput under heavy load at the expense of lower maximum throughput at light load. The question, therefore, arises whether the circular shell scheme has anything to offer over the annular scheme.

The annular and the circular shell schemes are compared in Fig. 6. The trade-off is very clear. The circular shell scheme outperforms the annular scheme at larger load at the expense of lower maximum throughput at lighter load. The point at which the circular shell outperforms the annular scheme is a function of the number of the available power levels. The higher the number of power levels, the larger the system load required for the circular shell scheme to outperform the annular scheme. Depending on the expected system load, the operator of the wireless access system can choose one or the other.

6. Stability analysis

The purpose of this section is to mathematically analyse the stability of the slotted ALOHA channel operated under our proposed power selection schemes. In a stable channel, equilibrium throughput-delay results are achievable over an infinite time horizon. However, if the channel is unstable, the equilibrium throughput-results are achieved only for a finite time before the channel goes into saturation and succumbs to throughput-collapse discussed earlier. Through numerical examples, we show that all our proposed power selection schemes pass some selective stability test, whereas the uniform power selection scheme fails the test.

For our stability analysis, we use the ‘load-line’ theorem proposed by Kleinrock [11]. Before we explain the load-line theorem and present our stability analysis, we define the following notations:

\[
M = \text{total number of mobile terminals}
\]
Given the above notations, the load-line of a slotted ALOHA channel can be defined as follows. Consider the $S$ versus $n$ graph. The load-line is the straight line $S = (M - n)p_n$ which intercepts the $n$-axis at $n = M$ and has a slope equal to $-p_n$. Given the above definition of the channel load-line, the load-line theorem for testing channel stability is given in Ref. [11]:

The stability definition. A slotted ALOHA channel is said to be stable if its load-line intersects (nontangentially) the equilibrium contour in exactly one place. Otherwise the channel is said to be unstable.

In the above stability definition, the equilibrium contour refers to the $S$ versus $n$ curve. Therefore, before we can proceed with the load-line test, we must derive the expressions for $S$ for all the proposed power selection schemes. Accounting for the retransmissions, we get:

$$G = (M - n)p_n + np_t$$  \hspace{1cm} (8)

In equilibrium, the incoming traffic is equal to the outgoing traffic, so $S = (M - n)p_n$. Using this value in Eq. (8), we obtain

$$G = S + np_t$$  \hspace{1cm} (9)

Combining Eqs. (7) and (9), we obtain the system throughput as:

$$S = \begin{cases} 
\frac{S + np_t}{N} \sum_{i=1}^{N} \exp\left\{-\frac{(S + np_t)}{N}\right\} & \text{for uniform} \\
\frac{S + np_t}{N} \sum_{i=1}^{N} \left[\frac{h}{N - 1} \cdot (2i - N - 1) + \frac{1}{N}\right] \exp\left\{\frac{ih(N - i)}{N - 1} - \frac{i}{N}\right\} & \text{for linear} \\
\frac{S + np_t}{N^2} \sum_{i=1}^{N} \frac{2i - 1}{N^2} \exp\left\{\frac{(S + np_t)}{N^2}\right\} & \text{for annular} \\
\frac{S + np_t}{N^3} \sum_{i=1}^{N} \frac{3i^3 - 3i + 1}{N^3} \exp\left\{\frac{(S + np_t)}{N^3}\right\} & \text{for circular shell} 
\end{cases}$$  \hspace{1cm} (10)

With the above expressions for $S$, we can now conduct the stability test using the load-line theorem. To conduct the stability tests for the annular scheme (AS) and circular shell (CS), we show (in Fig. 7) a numerical example of the $S$ versus $n$ graph (using MathCad) for the proposed AS and CS schemes along with the uniform scheme. It can be clearly seen that both AS and the CS schemes pass the stability test as the load-line (nontangentially) intersects the $S$ versus $n$ curves at exactly one point. We also find that the uniform scheme do not yield a stable slotted ALOHA channel, as its load-line intersects its $S$ versus $n$ curve at more than one point.

The stability of the linear scheme is numerically investigated in Fig. 8. We plotted several $S$ versus $n$ curves for different values of $h$. We find that for the linear scheme yields a stable channel for positive values of $h$, but as $h$ approaches zero, the schemes gets closer to unstable situation. At $h = 0$, the system is clearly unstable (the load-line intersects the curve at more than one points).

Let us summarise the major conclusions from Figs. 7 and 8.

- If power levels are selected randomly with uniform probability distribution, the slotted ALOHA channel will be unstable and susceptible to throughput-collapse. To prevent throughput-collapse and make the channel stable for the uniform selection scheme, it would be necessary to dynamically control the retransmission probability according to the system load to keep the total system load at optimum operating point. However, such dynamic adjustment requires estimation of current load and increases implementation complexity (not very attractive for small handheld devices with scarce system resources).
- Our proposed annular and circular shell schemes will guarantee channel stability without requiring any dynamic adjustment of the retransmission probability. These schemes, therefore, are very attractive from implementation point of view for the emerging mobile networking environment.
7. Conclusions

Randomly selecting a power level with uniform probability distribution in multiple power level slotted ALOHA leads to instability and throughput-collapse under heavy load. We have proposed three power selection schemes where the probability of selecting a higher power level is lower than the probability of selecting a lower power level. We have shown that all three schemes can improve the system stability and throughput of the slotted ALOHA under heavy load. The trade-off between the schemes is higher throughput at high load at the expense of lower maximum throughput at light load. All proposed schemes are distributive in nature and can be easily implemented in mobile terminals without requiring any central control.

References

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