

## COMP 2411 Midterm Exam, Session 1, 2000

Answer all questions. Select exactly one item from each question. Incorrect answers will not be given negative marks.

**Note:** Read each question carefully. Some questions ask you to select which choice is **false**!

1. Consider the formulae (1)  $p \longrightarrow (\neg q \vee r)$  and (2)  $q \longleftrightarrow (r \wedge p)$ . On a line of the truth table where  $p$  is true and  $q$  and  $r$  are both false,
  - (a) both (1) and (2) are true.
  - (b) (1) is true and (2) is false.
  - (c) (1) is false and (2) is true.
  - (d) both (1) and (2) are false.
2. Which of the following sentence structures in English does *not* correspond to the formula  $p \longrightarrow q$ ?
  - (a) If  $p$  then  $q$ .
  - (b)  $q$  if  $p$ .
  - (c)  $p$  only if  $q$ .
  - (d)  $q$  only if  $p$ .
3. A formula of propositional logic is valid if
  - (a) it is true on some line of the truth table .
  - (b) it is true on every line of the truth table.
  - (c) it is false on some line of the truth table.
  - (d) it is false on every line of the truth table.
4. which of the following is a valid logical equivalence?
  - (a)  $p \wedge (q \vee r) \equiv (p \vee q) \wedge (p \vee r)$
  - (b)  $p \longrightarrow (q \longrightarrow r) \equiv (p \longrightarrow q) \longrightarrow r$
  - (c)  $p \longrightarrow q \equiv \neg p \vee q$
  - (d)  $p \longleftrightarrow (q \wedge r) \equiv (p \longleftrightarrow q) \wedge (p \longleftrightarrow r)$

5. Consider the following two arguments: (1)  $p \longrightarrow (q \longrightarrow r)$ ,  $q \models p \longrightarrow r$  and (2)  $p \longrightarrow (q \longrightarrow r)$ ,  $\neg r \models \neg q \longrightarrow \neg p$ . Which of the following is true?
- (a) Both (1) and (2) are valid.
  - (b) (1) is valid but (2) is not.
  - (c) (2) is valid but (1) is not.
  - (d) Both (1) and (2) are invalid.
6. Consider the following set of clauses.

$$\begin{aligned}
 p &\longrightarrow q \\
 q \wedge r &\longrightarrow p \\
 s &\longrightarrow r \\
 \top &\longrightarrow s \\
 p &\longrightarrow \perp
 \end{aligned}$$

Which of the following is true?

- (a) The algorithm HORN cannot be applied to this set of clauses because they are not all Horn clauses.
  - (b) The algorithm HORN can be applied to this set of clauses, but it does not terminate.
  - (c) The algorithm HORN can be applied to this set of clauses, and it returns "satisfiable".
  - (d) The algorithm HORN can be applied to this set of clauses, and it returns "unsatisfiable".
7. Which of the following is NOT known to be true?
- (a) There exists an efficient algorithm for determining if a set of Horn clauses is satisfiable.
  - (b) There exists an efficient algorithm for determining if a set of Horn clauses is unsatisfiable.
  - (c) There exists an efficient algorithm for determining if the conjunction of a set of clauses is satisfiable.
  - (d) There exists an efficient algorithm for determining if the conjunction of a set of clauses is valid.
8. The result of converting the formula  $\neg((p \vee q) \wedge (r \wedge s))$  to conjunctive normal form
- (a) contains exactly one clause.
  - (b) contains exactly two clauses.
  - (c) contains exactly three clauses.
  - (d) contains more than three clauses.

9. A conjunctive normal form formula is valid if
- (a) it is possible to derive  $\perp$  by resolution from the set of clauses in the formula.
  - (b) it is not possible to derive  $\perp$  by resolution from the set of clauses in the formula.
  - (c) some clause in the formula contains a pair of complementary literals.
  - (d) every clause in the formula contains a pair of complementary literals.

10. Consider the clauses

$$\begin{aligned} p \vee q \vee \neg r \\ \neg p \vee r \\ \neg q \vee \neg r \end{aligned}$$

Which of the following clauses **cannot** be produced from these clauses by a single step of resolution?

- (a)  $q$
  - (b)  $\neg r \vee p \vee \neg r$
  - (c)  $r \vee q \vee \neg r$
  - (d)  $\neg p \vee \neg q$
11. The formula  $p \vee q \vee \neg r \vee \neg s$
- (a) is a definite Horn clause.
  - (b) is not a clause.
  - (c) is Horn clause but not a definite Horn clause.
  - (d) is a clause but not a Horn clause.

12. Consider the following set of clauses:

$$\begin{aligned} p \longrightarrow q \\ q \wedge r \longrightarrow p \\ r \wedge s \longrightarrow p \\ \top \longrightarrow r \\ p \longrightarrow \perp \end{aligned}$$

Construct the proof tree produced by SLD resolution with leftmost literal selection and top to bottom choice of clause. Which of the following statements about this tree is **false**?

- (a) The tree has an infinite branch.
- (b) The tree has infinitely many branches.
- (c) A depth first search of this tree will terminate.
- (d) The tree does not contain branch with  $\perp$  at the leaf.

13. Consider the following incomplete natural deduction proof.

1 :	$p \longrightarrow (q \longrightarrow r)$	Premise												
2 :	$q$	Premise												
3 :	$r \longrightarrow t$	Premise												
4 :	$((p \longrightarrow t) \wedge r) \longrightarrow s$	Premise												
<table style="width: 100%; border-collapse: collapse;"> <tr> <td style="padding-right: 10px;">5 :</td> <td style="padding-right: 20px;">...</td> <td style="padding-left: 20px;">Assumption</td> </tr> <tr> <td>6 :</td> <td><math>q \longrightarrow r</math></td> <td>...</td> </tr> <tr> <td>7 :</td> <td><math>r</math></td> <td>2, 6, <math>\longrightarrow e</math></td> </tr> <tr> <td>8 :</td> <td><math>t</math></td> <td>...</td> </tr> </table>			5 :	...	Assumption	6 :	$q \longrightarrow r$	...	7 :	$r$	2, 6, $\longrightarrow e$	8 :	$t$	...
5 :	...	Assumption												
6 :	$q \longrightarrow r$	...												
7 :	$r$	2, 6, $\longrightarrow e$												
8 :	$t$	...												
9 :	$p \longrightarrow t$	...												
10 :	$(p \longrightarrow t) \wedge r$	...												
11 :	$s$	4, 10, $\longrightarrow e$												

Which of the following assertions is true?

- (a) This cannot be completed to make up a correct proof, because nothing can be found to fill the dots at step 5 that will make step 9 correct.
  - (b) This cannot be completed to make up a correct proof, because step 10 cannot be justified.
  - (c) If we put a  $p$  at step 5 then we can fill in all the missing justifications so as to make up a correct proof.
  - (d) The justification for step 11 is incorrect.
14. Suppose that  $\phi_1, \dots, \phi_n \models \psi$ . Which if the following is guaranteed to be true?
- (a) Provided  $\psi$  is a clause, we can convert  $\phi_1, \dots, \phi_n$  into a set of clauses, and use resolution to prove  $\psi$ .
  - (b) If we convert  $\phi_1, \dots, \phi_n$  and  $\psi$  into a set of clauses, we can use resolution to prove  $\perp$ .
  - (c) If we convert  $\phi_1, \dots, \phi_n$  and  $\neg\psi$  into a set of clauses, we can use resolution to prove  $\perp$ .
  - (d) We can convert  $\phi_1, \dots, \phi_n$  and  $\neg\psi$  into a set of Horn clauses, and use the algorithm HORN to prove that this set is unsatisfiable.
15. Consider the following formula:

$$p(X, Y) \longrightarrow \forall Z(q(X, Z) \wedge R(Y, Z) \longrightarrow \exists Y(P(X, Y)))$$

- (a) This formula has exactly one free occurrence of  $X$ .
- (b) This formula has exactly two free occurrences of  $X$ .
- (c) All the occurrences of  $X$  in this formula are free.
- (d) This formula is a sentence.

16. Consider the chess position depicted on the final page of this exam. Suppose we represent this position by a structure in which the universe is the set of all pieces and squares in this position. Define

- $\text{pawn}(X)$  to be true just when  $X$  is a pawn, and
- $\text{adjacent}(X, Y)$  to be true when  $X$  and  $Y$  are two distinct pieces that are on adjacent squares.

Which of the following formulae is true in this structure?

- (a)  $\forall X(\text{pawn}(X) \longrightarrow \exists Y(\text{pawn}(Y) \wedge \text{adjacent}(X, Y)))$
- (b)  $\exists X(\text{pawn}(X) \wedge \forall Y(\text{pawn}(Y) \longrightarrow \text{adjacent}(X, Y)))$
- (c)  $\forall X(\text{pawn}(X) \longrightarrow \forall Y(\text{pawn}(Y) \wedge \text{adjacent}(X, Y)))$
- (d)  $\forall X(\text{pawn}(X) \wedge \exists Y(\text{pawn}(Y) \wedge \text{adjacent}(X, Y)))$

17. Consider the same structure as in the previous question. Which of the following formulae is true in this structure?

- (a)  $\exists X \exists Y \exists Z(\text{pawn}(X) \wedge \text{pawn}(Y) \wedge \text{pawn}(Z) \wedge \text{adjacent}(X, Y) \wedge \text{adjacent}(Y, Z))$
- (b)  $\exists X \exists Y(\neg \text{pawn}(X) \wedge \neg \text{pawn}(Y) \wedge \text{adjacent}(X, Y))$
- (c)  $\forall X \forall Y((\text{pawn}(X) \wedge \text{pawn}(Y)) \longrightarrow \text{adjacent}(X, Y))$
- (d)  $\forall X \forall Y((\text{pawn}(X) \wedge \text{pawn}(Y)) \longrightarrow \neg \text{adjacent}(X, Y))$

18. Which of the following formulas best expresses “As a general rule of chess, if two rooks of opposite colour are on adjacent squares then they can take each other.”

- (a)  $\exists X \exists Y(\text{rook}(X) \wedge \text{rook}(Y) \wedge \text{adjacent}(X, Y) \wedge \neg \text{colour}(X) = \text{colour}(Y)) \longrightarrow (\text{can - take}(X, Y) \wedge \text{can - take}(Y, X))$
- (b)  $\exists X \exists Y([\text{rook}(X) \wedge \text{rook}(Y) \wedge \text{adjacent}(X, Y) \wedge \neg \text{colour}(X) = \text{colour}(Y)]) \longrightarrow [\text{can - take}(X, Y) \wedge \text{can - take}(Y, X)]$
- (c)  $\forall X \forall Y([\text{rook}(X) \wedge \text{rook}(Y) \wedge \text{adjacent}(X, Y) \wedge \neg \text{colour}(X) = \text{colour}(Y)]) \longrightarrow [\text{can - take}(X, Y) \wedge \text{can - take}(Y, X)]$
- (d)  $\forall X \forall Y(\text{rook}(X) \wedge \text{rook}(Y) \wedge \text{adjacent}(X, Y) \wedge [(\neg \text{colour}(X) = \text{colour}(Y)) \longrightarrow (\text{can - take}(X, Y) \wedge \text{can - take}(Y, X))])$

19. Suppose that  $f$  is a function symbol of arity 2,  $g$  is a function symbol of arity one,  $c$  is a constant symbol and  $X$  is a variable. Let  $\mathcal{M}$  be a structure with

- universe  $A^{\mathcal{M}}$  equal to the set of natural numbers  $\mathbf{N}$ ,
- $c^{\mathcal{M}} = 3$ ,
- $f^{\mathcal{M}}(m, n) = m + n$  for all  $m, n \in \mathbf{N}$ ,
- $g^{\mathcal{M}}(n) = n + 1$  for all  $n \in \mathbf{N}$ .

and let  $\sigma$  be an environment with  $\sigma(X) = 2$ . Then

- (a)  $\mathcal{M} \models_{\sigma} f(g(X), c) = c$
- (b)  $\mathcal{M} \models_{\sigma} f(g(X), c) = g(c)$
- (c)  $\mathcal{M} \models_{\sigma} f(g(X), c) = f(c, c)$
- (d)  $\mathcal{M} \models_{\sigma} f(g(X), c) = f(g(c), c)$

20. Using the convention that variables are in upper case and constants in lower case, which of the following formulas of predicate logic is a sentence?

- (a)  $\forall X(f(Z) = c \longrightarrow \exists Y(p(X, Y)))$
- (b)  $\forall X(f(X) = c \longrightarrow \exists Y(p(Z, Y)))$
- (c)  $\forall X(f(X) = c \longrightarrow \exists Y(p(Z, Y)))$
- (d)  $\forall X(f(c) = c \longrightarrow \exists Y(p(X, Y)))$