

Solution for Q.3 Tutorial 4

Let $G = (V, E)$ be the given graph, where $V = \{u, v, w\}$ and $E = \{(u, v), (v, w), (w, u)\}$.

We can translate the two-colourable problem, with two colours R and Y , to a set of clauses

$$S(G) = \{ \\ 1 : R_u \vee Y_u, \\ 2 : R_v \vee Y_v, \\ 3 : R_w \vee Y_w, \\ 4 : \neg R_u \vee \neg Y_u, \\ 5 : \neg R_v \vee \neg Y_v, \\ 6 : \neg R_w \vee \neg Y_w, \\ 7 : \neg R_u \vee \neg R_v, \\ 8 : \neg R_v \vee \neg R_w, \\ 9 : \neg R_w \vee \neg R_u, \\ 10 : \neg Y_u \vee \neg Y_v, \\ 11 : \neg Y_v \vee \neg Y_w, \\ 12 : \neg Y_w \vee \neg Y_u\}$$

We prove that G is not two-colourable, i.e. $S(G)$ is not satisfiable:

1 – 12	<i>Premises</i>
13 : $\neg Y_u \vee R_v$	<i>Res, 2, 10</i>
14 : $\neg Y_u \vee R_w$	<i>Res, 3, 12</i>
15 : $\neg Y_u \vee \neg R_v$	<i>Res, 8, 14</i>
16 : $\neg Y_u$	<i>Res, 13, 15</i>
17 : $\neg R_u \vee Y_v$	<i>Res, 2, 7</i>
18 : $\neg R_u \vee Y_w$	<i>Res, 3, 9</i>
19 : $\neg R_u \vee \neg Y_v$	<i>Res, 11, 18</i>
20 : $\neg R_u$	<i>Res, 17, 19</i>
21 : Y_u	<i>Res, 1, 20</i>
22 : \perp	<i>Res, 16, 21</i>

Thus, $S(G) \vdash_{Res} \perp$, or G is not two-colourable.