COMP2411 Tutorial 1

1. You are planning another party for your difficult friends.

   (a) If John comes, Sarah or Kim should be there.

   (b) Kim will only come if Sarah and John both do.

   (c) If Sarah comes then John will not come.

   Translate the constraints to propositional logic. Using truth tables, figure out what combinations of people you can invite.

2. Consider the following formula:

   $$((X \land (Y \land Z)) \lor ((X \land \neg Y) \land \neg Z)) \lor ((\neg X \land Y) \land Z) \lor (((\neg X \land \neg Y) \land Z) \lor (X \land (\neg Y \land Z)))$$

   Draw its parse tree. Using logical equivalences, try to find an equivalent formula using only the operators $\land, \lor, \neg$ that is as small as you can make it. Draw its parse tree also. If you had to make 1 million copies of a circuit for the above formula using only "and", "or" and "not" gates, each costing 1c, how much would you save by using your reduced form?

3. A 3-bit adder is a circuit that takes has binary inputs, X, Y and Z, S (sum) and C (carry), such that the binary number $C\ S$ is the two-digit binary representation of $X + Y + Z$. Draw the truth tables corresponding to S and C. Using these, find formulas representing S and C.

Optional Advanced Questions

1. Suppose $\mathcal{I}$ is a set of propositions called *input propositions* and $\mathcal{O}$ is a set of propositions called *output* propositions. A specification ϕ expressed as a formula of propositional logic over the set of propositions $\mathcal{I} \cup \mathcal{O}$ is said to be *realizable* if for *every* possible combination of truth values for the input variables $\mathcal{I}$, there exists an assignment of truth value to the output variables $\mathcal{O}$ such that ϕ evaluates to true.

   Consider the following requirements R, concerning the propositions

   - S = The alarm sounds.
   - A = The system has been armed.
   - D = The door has been disturbed.
   - F = There is a fire.

   Here the input variables are $\mathcal{I} = \{A, D, F\}$ and the output variables are $\mathcal{O} = \{S\}$. Translate these requirements into a formula of propositional logic. Is this formula realizable? (Hint: group the rows of the truth table into adjacent pairs, with the values of A, D, F fixed within each pair and only the value of S changing within a pair.)

Requirements R: A burglar alarm system for a house is to operate as follows. The alarm should not sound unless the system has been armed. If the system has been armed and a door is disturbed, the alarm should ring. If there is a fire, the alarm should always go off.

2. We've been assuming that we can always tell whether a sentence of English is a proposition or not, but life was not meant to be that easy. Consider the following:

   (a) Is the following a proposition: "Santa peroxides his beard." Taking into account that Santa does not exist (notwithstanding a legion of imposters), is it true or false?

   (b) Is the following a proposition: "This sentence is true."? If so, is it true or false? How about "This sentence is false."?

   (c) Is the following sentence, found on one side of a white card, a proposition: "The sentence on the other side is false."? What if the sentence on the other side says "The sentence on the other side is true."?

3. Can you strengthen justification of the truth table for $\longrightarrow$ by showing that it is the *only* truth table that (given that the truth tables for the other connectives are fixed) makes some set of intuitively desirable formulae come out as valid (true in all rows of the truth table). Here are some candidate formulae to consider:

   - $(A \longrightarrow B) \longrightarrow \neg(A \wedge \neg B)$
   - $((A \longrightarrow B) \wedge (B \longrightarrow C)) \longrightarrow (A \longrightarrow C)$
   - $(A \longrightarrow B) \longrightarrow (\neg B \longrightarrow \neg A)$