

COMP2411 Tutorial 5

1. Which of the following are Horn clauses (in either the book's notation or as a disjunction of literals)? Which are definite Horn clauses? Which are equivalent to a set (i.e., conjunction) of Horn Clauses?

(a) $(A \rightarrow B) \rightarrow C$

(b) $(A \vee B) \rightarrow C$

(c) $A \vee B \vee \neg C$

(d) \perp

(e) $A \vee A$

(f) $A \wedge A \rightarrow A$

(g) $\neg(A \wedge B)$

2. For each of the following sets of Horn clauses, apply the two techniques for determining unsatisfiability discussed in lectures in Week 5: the algorithmic technique and SLD resolution with leftmost selection rule and a depth first search. When there exists a choice of which clause to use, try the possibilities from top to bottom in the following lists. Comment on the outcome.

(a)

$$\begin{aligned} \top &\rightarrow p \\ p &\rightarrow r \\ p \wedge r &\rightarrow s \\ s \wedge r &\rightarrow t \\ a &\rightarrow c \\ c &\rightarrow f \\ c &\rightarrow h \\ c &\rightarrow k \\ t &\rightarrow \perp \end{aligned}$$

(b)

$$\begin{aligned} \top &\rightarrow b \\ a \wedge z &\rightarrow x \\ c \wedge d &\rightarrow a \\ c \wedge f &\rightarrow a \\ b &\rightarrow a \\ r &\rightarrow c \\ b \wedge q &\rightarrow z \\ r &\rightarrow q \\ x &\rightarrow \perp \end{aligned}$$

- (c) The clauses of the previous problem, plus $q \rightarrow r$ (at the bottom of the list.)

(d) The clauses of the problem (b), plus

$$\begin{array}{l} q \longrightarrow r \\ b \wedge f \longrightarrow z \\ \top \longrightarrow f \end{array}$$

(add these clauses at the bottom of the list.)

Optional Advanced Questions

1. A two-clause is a clause containing at most two literals. (Some two-clauses are not Horn clauses, since there could be two positive literals.) Show that there is an efficient (non-exponential) algorithm that computes whether a set of two-clauses is satisfiable. Does your approach work for clauses with up to three literals?