COMP4415 Assignment 2 – 2003
Due: Fri March 28

1. (EASY) An (undirected) graph is a pair $G = (V, E)$ where V is a set of vertices and E is a set of two-element sets $\{v, w\}$ with $v \neq w$ vertices in V. An n-colouring of G is a function $c : V \to \{1, \ldots, n\}$ such that for all $\{v, w\} \in E$ we have $c(v) \neq c(w)$. A subgraph of G is a graph $G' = (V', E')$ such that $V' \subseteq V$ and for all $v, w \in V'$, we have $\{v, w\} \in E'$ iff $\{v, w\} \in E$.

   Consider an infinite graph G with vertices $V = \{v_0, v_1, \ldots\}$. Show that G is n-colourable iff every finite subgraph of G is n-colourable. (Hint: use Konig's Lemma. Consider the infinite tree containing a vertex of the form of a finite sequence $\langle c_0 \ldots c_n \rangle$ if the subgraph of G with vertices $\{v_0, \ldots, v_n\}$ is n-colourable.)

2. (HARDER) Consider the following language. The language is based on an infinite set of variables $Vars = \{x, y, z, \ldots\}$. The formulas of the language are of the form '$path(x, y)$', where x and y are variables. The semantics of the language is defined with respect to models of the form $M = (V, E, \pi)$, where V is a set, E is a subset of $V \times V$, and π is a function from $Vars$ to V. Intuitively, a model consists of a *directed* graph (V, E) together with a function that interprets variables as vertices of the graph. (We allow $\pi(x) = \pi(y)$ for $X \neq y$). Define satisfaction of the formulas of the language as follows: $(V, E, \pi) \models path(x, y)$ if there exists a finite sequence $v_0, \ldots, v_n$ in V such that $\pi(x) = v_0$, $\pi(y) = v_n$ and for all $i = 0 \ldots n - 1$ we have $(v_i, v_{i+1}) \in E$. As usual, define logical consequence by $S \models \phi$ (where S is a set of formulas and ϕ is a formula), when for all models M, if $M \models \psi$ for all $\psi \in S$, then $M \models \phi$. Prove or disprove: this language is compact.