COMP4415 Assignment 4 – 2003
Due: Fri April 11

1. (EASY) By converting to conjunctive normal form and applying resolution, prove the following:

   (a) $(A \to (B \to C)) \to ((A \to B) \to (A \to C))$ is valid.

   (b) $\{\ \neg((A \leftrightarrow (P \vee Q)) \wedge (\neg P \vee R)),\quad P \leftrightarrow S\ \} \models\ \neg((A \leftrightarrow (S \vee Q)) \wedge (\neg S \vee R))$

2. (HARDER) Suppose we add to the resolution proof procedure the *subsumption rule*: from a clause C derive C', where C' is any clause with $C \subseteq C'$. Write $S \vdash_{SR} C$, where S is a set of clauses and C is a clause, if there exists a proof $C_0, \ldots, C_n = C$ where each C_i is in S, or derived from preceding elements of the sequence using either the resolution or the subsumption rule.

   (a) Show that the subsumption rule is sound.

   (b) Show that resolution plus subsumption is complete for derivation of clauses, i.e., if $S \models C$ then $S \vdash_{SR} C$. Your proof should actually show something stronger: if $S \models C$ then $S \vdash_R C'$ for some clause $C' \subseteq C$, i.e. it suffices to use at most a single step of subsumption at the end of the proof. (Hint: use the fact that $S \models \phi$ iff $S \cup \{\neg\phi\}$ is unsatisfiable and the fact that resolution is refutation complete, i.e., if S is unsatisfiable then $S \vdash_R \Box$, i.e., there exists a resolution proof of $\Box$ (*without* subsumption).