COMP4415 Assignment 7 – 2003
Due: Wed May 21

Each part is worth 2 marks:

1. (EASY) Use tableau proofs to show that the following are valid

   (a) $\forall x(P(x) \to P(x))$

   (b) $\exists x(\phi \to \psi(x)) \to (\phi \to \exists x(\psi(x)))$, where x is not free in ϕ

   (c) $\forall x(P(x) \vee Q(x)) \wedge \forall x(P(x) \to R(x)) \wedge \forall x(Q(x) \to R(x)) \to \forall x(R(x))$

2. (HARDER) A binary relation R is

   - *reflexive* if it satisfies $\forall x(R(x,x))$
   - *symmetric* if it satisfies $\forall x \forall y(R(x,y) \to R(y,x))$
   - *transitive* if it satisfies $\forall x \forall y \forall z((R(x,y) \wedge R(y,z)) \to R(x,z))$
   - *euclidean* if it satisfies $\forall x \forall y \forall z((R(x,y) \wedge R(x,z)) \to R(y,z))$

   Use tableau proofs to show:

   (a) If R is reflexive and euclidean then R is symmetric.

   (b) If R is reflexive and euclidean then R is transitive.

   Hint: first draw some pictures to understand informally why this should be the case.