COMP4415 Assignment 9 – 2003
Due: Wed June 11

1. (4 marks) Use the procedure discussed in class to calculate whether the following sets S of propositional Horn clauses are satisfiable, and if so, to provide an assignment in which they are satisfied. Show each step of the computation.

   (a) $\{p,\ p \to q,\ p \wedge q \to r,\ r \to \bot\}$

   (b) $\{p,\ p \to q,\ p \to s,\ q \wedge s \to r,\ u \wedge r \to t,\ t \wedge p \to u,\ t \to \bot\}$

2. In class, we assumed that S and $Prop$ are finite. In the following, we allow these sets to be infinite. (However, Horn clauses are still finite!)

   (a) (2 marks) Show that if $X_0 \subseteq X_1 \subseteq \ldots$ si an infinite sequence of sets, then $T_S(\bigcup_i X_i) \subseteq \bigcup_i T_S(X_i)$

   (b) (2 marks) Let $X_0 = \emptyset$ and define $X_{i+1} = T_S(X_i)$. Let $A = \bigcup_i X_i$. Show that if S is unsatisfiable, then $\bot \in A$, otherwise $A \models S$. (Use the result of the previous part.)

   (c) (2 marks) Use the previous part to show that if S consists of a set of facts and program clauses P and a set of goal clauses G, i.e., $S = P \cup G$, then S is unsatisfiable iff there exists a single goal clause $g \in G$ such that $P \cup \{g\}$ is unsatisfiable.