A Strongly Local Contextual Logic

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Motivation

- KRR for cognitive robotics
  - Computational issues: Lifelong learning, open domains
  - Representational issues: Complex, interrelated domains
Motivation

- Logic of “mental stuff”
  - Modelling situations, concepts, objects
- Localised reasoning
- Together implicitly focuses reasoning
Outline

- Introduction to contextual logics
- Overview of SLCL (syntax, semantics, deductive system)
- Worked example
- Formal results
- Conclude
What are contextual logics?

- Formal representation of context
- “Representations of partial states of affairs” (Brézillon, 1999)
- Two main families of contextual logics:
  - Logics of Context
  - Multi-context Systems
Logics of Context

- Unified system of logic
- Contexts partition a knowledge base
- Object-level relations between contexts

\[ \sum_{c_1} \quad \text{IsTrue}(c_1, P) \rightarrow Q \]
\[ \vdash_{c_2} Q \]
Multi-context Systems

- Multiple systems of logic
- Contexts represented by individual deductive systems
- Meta-level relations between contexts

\[ \Sigma_1 \quad P, \neg Q \]

\[ \Sigma_2 \quad R \]

\[ k_1 : P \]

\[ k_2 : Q \]

\[ \vdash \Sigma_2 \quad Q \]
Contextual Reasoning for Robotics

Desiderata

- Local reasoning?
- “Lightweight” notion of context?
- Introspection of beliefs?
The Strongly Local Contextual Logic

- A first-order contextual logic
- Combines features of both families:
  - Single system of logic
  - Deductively distinct contexts
  - Contexts and context relations at the object level
- Meets the desiderata for cognitive robotics
The SLCL Deductive System

Syntax

- Classical first-order syntax
- “Is-true”: \( c : \phi \)
- Contextual quantifiers: \( \forall_c, \forall_C, \forall_L \)

Example

- \( \forall_{c_1} x [c_1 : P(x) \rightarrow Q(x)] \)
The SLCL Deductive System

Semantics

- Classical first-order semantics (locally)
- Extends and refines GLC (Makarios, 2006)
- Contexts are “conditions on possible words”

Example

- \( w_1 = \{ P(a), P(b), \ldots \} \)
- \( w_2 = \{ P(a) \} \)
The SLCL Deductive System

Contextual Derivations

▶ Classical derivations and rules of inference locally
▶ Derivations defined with respect to a context
▶ Axiomatic cross-context inference
▶ Local deductions “interact” via global model:

\[ \Sigma \]

- \[ w_1 = \{ P(a), \ldots \} \]
- \[ w_2 = \{ P(a) \} \]
- \[ w_3 = \ldots \]
Example

Modelling an agent’s current situation in some planning context \textit{plan}:

\[
\forall c. c. \forall a. [c : \text{Poss}(a) \rightarrow \text{Poss}(a)]
\]

\[
s_0 : \text{Door}(d1)
\]

and the concept of doors in the context \textit{doors}:

\[
doors : \text{Door}(d1) \land \text{Door}(d2) \land \ldots
\]

\[
\forall c. s. \forall x. [s : \text{Poss}(\text{do(move, } x)) \equiv s : \text{Door}(x)]
\]

We are licensed to conclude in \textit{plan}:

\[
\text{Poss(do(move, } d1)) \quad \text{but not:} \quad \text{Poss(do(move, } d2))
\]
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- Modelling an agent’s current situation in some planning context *plan*:

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- We are licensed to conclude in *plan*:

  \[ \text{Poss}(\text{do}(\text{move}, d1)) \] but not: \[ \text{Poss}(\text{do}(\text{move}, d2)) \]
Formal Results

▶ Sound:
  ▶ Rules of inference valid for local derivations
  ▶ Globally sound since also valid for all models

▶ Complete:
  ▶ Set of Henkin sets, for each permutation of context pairs
  ▶ Local proof generates partial global model
  ▶ Globally complete when consistent with partial model

▶ Properties, e.g. weak distributivity of is-true:
  ▶ $\models c : [\phi \land \psi] \leftrightarrow [c : \phi \land c : \psi]$
  ▶ $\models c : [\phi \lor \psi] \rightarrow [c : \phi \lor c : \psi]$
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In Conclusion

- Developed a new contextual logic:
  - Implicitly focused reasoning
  - Modular, introspectable knowledge representation
- Desirable features for cognitive robotics
- Demonstrated soundness, completeness, with initial properties
Future Work

- Examine algebra of contexts
- Elicit and explicate semantic properties
- Develop automated contextual reasoner
- Develop contextual theories for cognitive robotics
Thanks

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References
