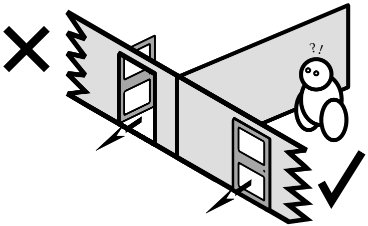
A Strongly Local Contextual Logic

Michael James Gratton
School of Computer Science and Engineering

IJCAI 2013
August 7, 2013

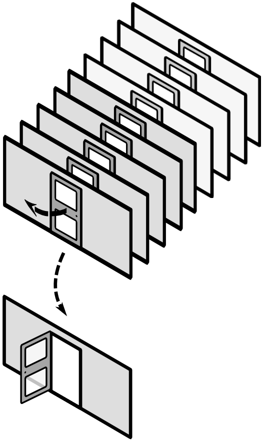
Motivation

- KRR for cognitive robotics
  - Computational issues: Lifelong learning, open domains
  - Representational issues: Complex, interrelated domains



Motivation

- Logic of “mental stuff”
  - Modelling situations, concepts, objects
- Localised reasoning
- Together implicitly focuses reasoning



Outline

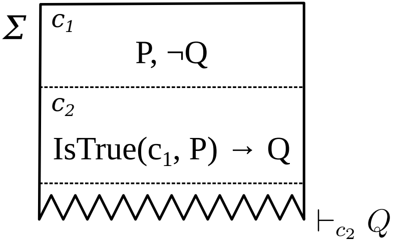
- Introduction to contextual logics
- Overview of SLCL (syntax, semantics, deductive system)
- Worked example
- Formal results
- Conclude

What are contextual logics?

- Formal representation of context
- "Representations of partial states of affairs" (Brézillon, 1999)
- Two main families of contextual logics:
    - Logics of Context
    - Multi-context Systems

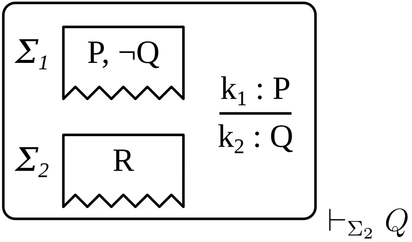
Logics of Context

- Unified system of logic
- Contexts partition a knowledge base
- Object-level relations between contexts



Multi-context Systems

- Multiple systems of logic
- Contexts represented by individual deductive systems
- Meta-level relations between contexts



Contextual Reasoning for Robotics

Desiderata

- Local reasoning?
- "Lightweight" notion of context?
- Introspection of beliefs?

The Strongly Local Contextual Logic

- A first-order contextual logic
- Combines features of both families:
  - Single system of logic
  - Deductively distinct contexts
  - Contexts and context relations at the object level
- Meets the desiderata for cognitive robotics

The SLCL Deductive System

Syntax

- Classical first-order syntax
- “Is-true”: $c : \phi$
- Contextual quantifiers: $\forall_c$, $\forall_{\mathcal{C}}$, $\forall_{\mathcal{L}}$

Example

- $\forall_{c_1} x[c_1 : P(x) \rightarrow Q(x)]$

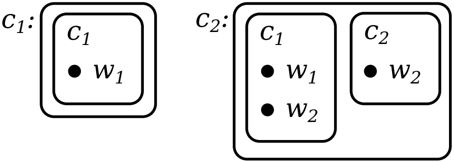
The SLCL Deductive System

Semantics

- Classical first-order semantics (locally)
- Extends and refines GLC (Makarios, 2006)
- Contexts are "conditions on possible words"

Example

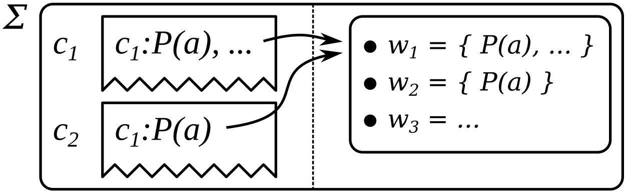
- $w_1 = \{P(a), P(b), \dots\}$
- $w_2 = \{P(a)\}$



The SLCL Deductive System

Contextual Derivations

- Classical derivations and rules of inference locally
- Derivations defined with respect to a context
- Axiomatic cross-context inference
- Local deductions "interact" via global model:



Example

- Modelling an agent's current situation in some planning context $plan$:

$$\forall_{\mathcal{C}} c. \forall a.\ [c : Poss(a) \rightarrow Poss(a)]$$
$$s_0 : Door(d1)$$

and the concept of doors in the context $doors$:

$$doors : Door(d1) \wedge Door(d2) \wedge \ldots$$
$$\forall_{\mathcal{C}} s. \forall x.\ [s : Poss(do(move, x)) \equiv s : Door(x)]$$

- We are licensed to conclude in $plan$:

$$Poss(do(move, d1)) \quad \text{but not:} \quad Poss(do(move, d2))$$

Example

- Modelling an agent's current situation in some planning context $plan$:

$$\forall_{\mathcal{C}} c. \forall a. \, [c : Poss(a) \rightarrow Poss(a)]$$

$$s_0 : Door(d1)$$

and the concept of doors in the context $doors$:

$$doors : Door(d1) \wedge Door(d2) \wedge \ldots$$

$$\forall_{\mathcal{C}} s. \forall x. \, [s : Poss(do(move, x)) \equiv s : Door(x)]$$

- We are licensed to conclude in $plan$:

$Poss(do(move, d1))$ but not: $Poss(do(move, d2))$

Example

- Modelling an agent's current situation in some planning context $plan$:

$$\forall_{\mathcal{C}} c.\forall a.\ [c : Poss(a) \rightarrow Poss(a)]$$
$$s_0 : Door(d1)$$

  and the concept of doors in the context $doors$:

$$doors : Door(d1) \wedge Door(d2) \wedge \ldots$$
$$\forall_{\mathcal{C}} s.\forall x.\ [s : Poss(do(move, x)) \equiv s : Door(x)]$$

- We are licensed to conclude in $plan$:

$$Poss(do(move, d1)) \quad \text{but not:} \quad Poss(do(move, d2))$$

Formal Results

- Sound:
  - Rules of inference valid for local derivations
  - Globally sound since also valid for all models
- Complete:
  - Set of Henkin sets, for each permutation of context pairs
  - Local proof generates partial global model
  - Globally complete when consistent with partial model
- Properties, e.g. weak distributivity of is-true:
  - $\models c : [\phi \wedge \psi] \;\leftrightarrow\; [c : \phi \wedge c : \psi]$
  - $\models c : [\phi \vee \psi] \;\rightarrow\; [c : \phi \vee c : \psi]$

Formal Results

- Sound:
  - Rules of inference valid for local derivations
  - Globally sound since also valid for all models
- Complete:
  - Set of Henkin sets, for each permutation of context pairs
  - Local proof generates partial global model
  - Globally complete when consistent with partial model
- Properties, e.g. weak distributivity of is-true:
  - $\models c : [\phi \wedge \psi] \ \leftrightarrow \ [c : \phi \wedge c : \psi]$
  - $\models c : [\phi \vee \psi] \ \rightarrow \ [c : \phi \vee c : \psi]$

Formal Results

- Sound:
  - Rules of inference valid for local derivations
  - Globally sound since also valid for all models
- Complete:
  - Set of Henkin sets, for each permutation of context pairs
  - Local proof generates partial global model
  - Globally complete when consistent with partial model
- Properties, e.g. weak distributivity of is-true:
  - $\models c : [\phi \wedge \psi] \;\leftrightarrow\; [c : \phi \wedge c : \psi]$
  - $\models c : [\phi \vee \psi] \;\rightarrow\; [c : \phi \vee c : \psi]$

In Conclusion

- Developed a new contextual logic:
  - Implicitly focused reasoning
  - Modular, introspectable knowledge representation
- Desirable features for cognitive robotics
- Demonstrated soundness, completeness, with initial properties

Future Work

- Examine algebra of contexts
- Elicit and explicate semantic properties
- Develop automated contextual reasoner
- Develop contextual theories for cognitive robotics

Thanks

Contact

- **mikeg@cse.unsw.edu.au**
- **http://www.cse.unsw.edu.au/~mikeg/**

References

P. Brézillon. Context In Problem Solving: A Survey. *The Knowledge Engineering Review*, 14(1):47–80, 1999.

S. Makarios. A Model Theory for a Quantified Generalized Logic of Contexts. Technical Report KSL-06-08, Stanford University, 2006.