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A Strongly Local Contextual Logic

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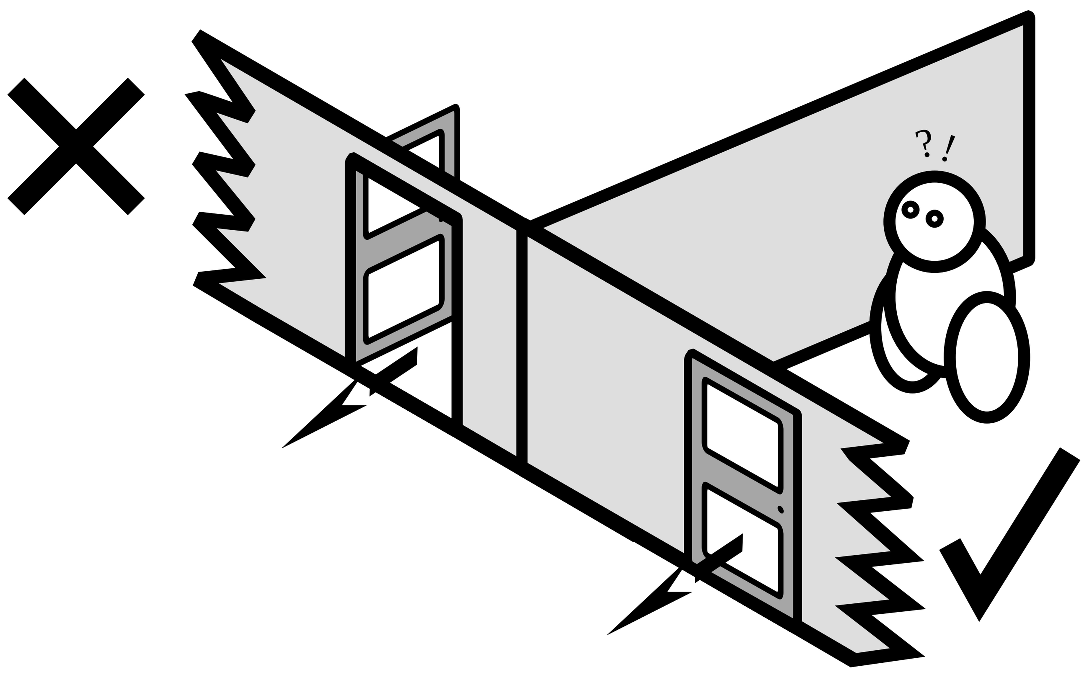
Knowledge Representation for Cognitive Robotics



Issues arising from applying KRR to cognitive robotics:

- Computational: Lifelong learning, open domains
- Representational: Complex, interrelated domains

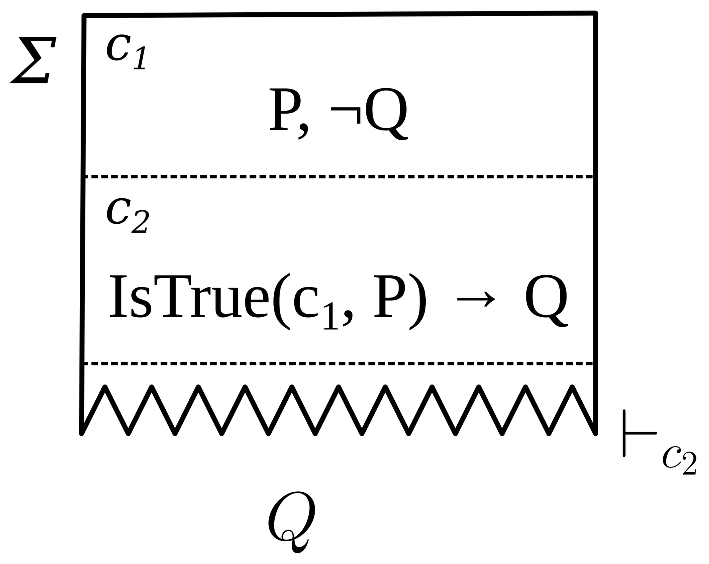
To address these, it is desirable to develop:

- Logic of "mental stuff"
  - Modelling situations, concepts, objects
- Localised reasoning
- Together implicitly focuses reasoning

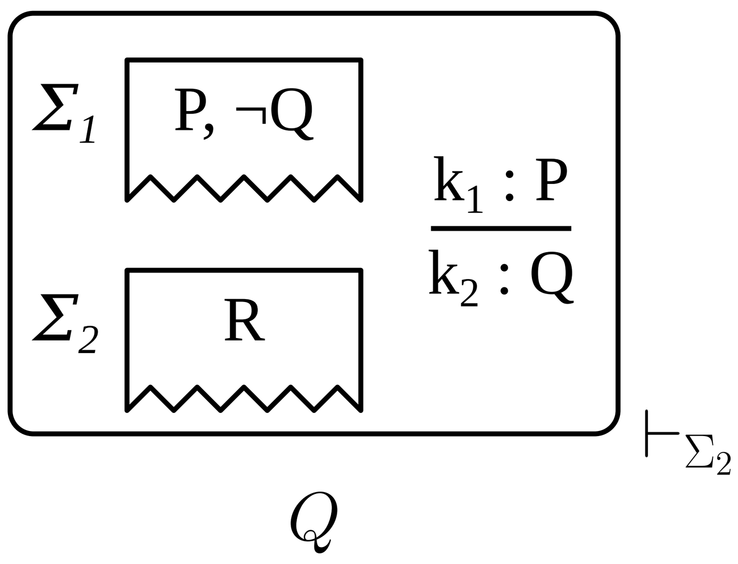
What are contextual logics?

- Formal representation of context
- "Representations of partial states of affairs" (Brézillon, 1999)
- Two main families of contextual logics
  - Logics of Context
  - Multi-context Systems

Logics of context:



Multi-context systems:



By comparison, the logic presented here is:

- A first-order contextual logic
- Combines features of both families:
  - Single system of logic
  - Deductively distinct contexts
  - Contexts and context relations at the object level
- Meets the desiderata for cognitive robotics

The Strongly Local Contextual Logic

Syntax:

- Classical first-order syntax
- "Is-true": $c : \phi$
- Contextual quantifiers: $\forall_c$, $\forall_{\mathcal{C}}$, $\forall_{\mathcal{L}}$

Example

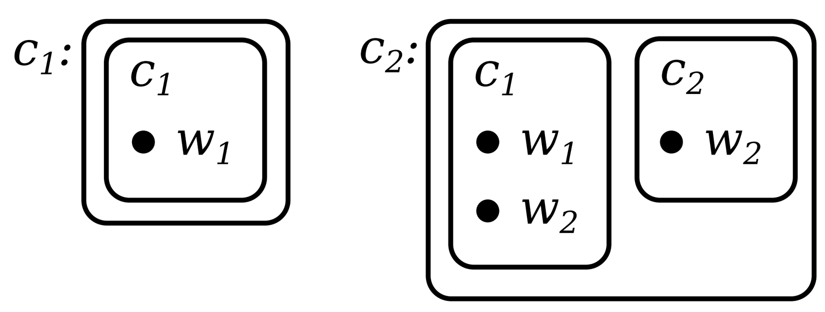
- $\forall_{c_1} x[c_1 : P(x) \rightarrow Q(x)]$

Semantics:

- Classical first-order semantics (locally)
- Extends and refines GLC (Makarios, 2006)
- Contexts are "conditions on possible words"

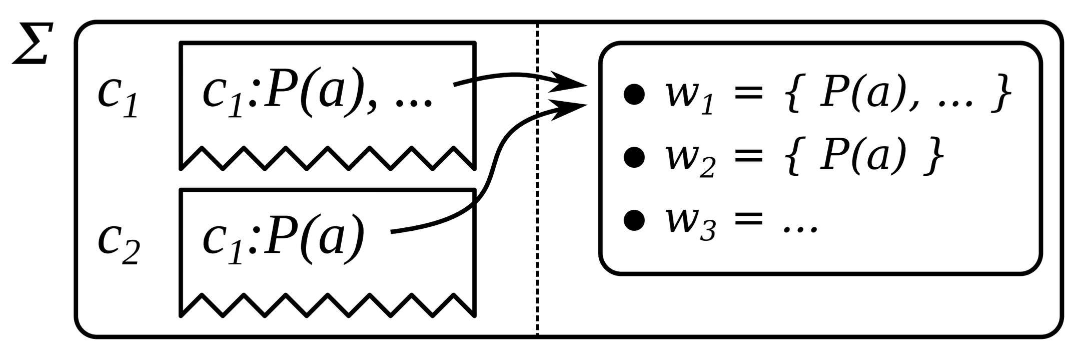
Example

- $w_1 = \{P(a), P(b), \ldots\}$
- $w_2 = \{P(a)\}$



Contextual derivations:

- Classical derivations and rules of inference locally
- Derivations defined with respect to a context
- Axiomatic cross-context inference
- Local deductions "interact" via global model



Example

- Simple planning context *plan*:

  $\forall_{\mathcal{C}} c.\forall a.\ [c : Poss(a) \rightarrow Poss(a)]$

  $s_0 : Door(d1)$

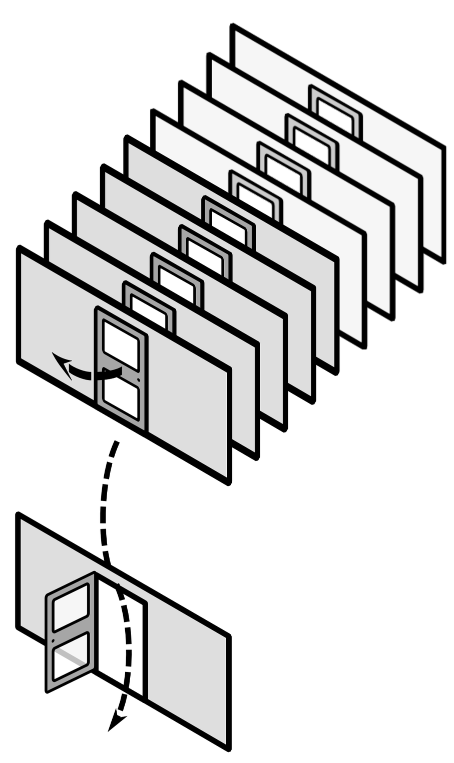
- Simple concept context *doors*:

  $doors : Door(d1) \wedge Door(d2) \wedge \ldots$

  $\forall_{\mathcal{C}} s.\forall x.\ [s : Poss(do(move, x)) \equiv s : Door(x)]$

- Licensed to conclude in *plan*:

  $Poss(do(move, d1))$ but not: $Poss(do(move, d2))$



Formal results:

- Sound:
  - Rules of inference valid for local derivations
  - Globally sound since also valid for all models
- Complete:
  - Set of Henkin sets, for each permutation of context pairs
  - Local proof generates partial global model
  - Globally complete when consistent with partial model:
- Properties, e.g. weak distributivity of is-true:
  - $\vDash c : [\phi \wedge \psi] \leftrightarrow [c : \phi \wedge c : \psi]$
  - $\vDash c : [\phi \vee \psi] \rightarrow [c : \phi \vee c : \psi]$

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References

P. Brézillon. Context In Problem Solving: A Survey. *The Knowledge Engineering Review*, 14(1):47–80, 1999.

S. Makarios. A Model Theory for a Quantified Generalized Logic of Contexts. Technical Report KSL-06-08, Stanford University, 2006.