

# An Argumentation-Theoretic Characterization of Defeasible Logic

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**Abstract.** Defeasible logic is an efficient non-monotonic logic that is defined only proof-theoretically. It has potential application in some legal domains. We present here an argumentation semantics for defeasible logic that will be useful in these applications. Our development differs at several points from existing argumentation frameworks since there are several features of defeasible logic that have not been addressed in the literature.

## 1 Introduction

Defeasible logic is a practical non-monotonic logic. This logic, and similar logics, have been proposed as the appropriate language for executable regulations [3], contracts [20], and business rules [11]. There are several implementations of defeasible logic, each of which is capable of handling 100,000's of rules [4].

Although defeasible logic can be described informally in terms of arguments, the logic has been formalized in a proof-theoretic setting in which arguments play no role. In this paper we will provide an argumentation-theoretic semantics for defeasible logic.

There are already several different abstract argumentation frameworks [9, 7, 13, 18, 21, 22]. However, defeasible logic provides several challenges that have not yet been addressed by this work:

(1) Defeasible logic has a “directly sceptical” semantics, in the sense of Horty [12], also called “conservative” in Wagner’s classification [23]. Most argumentation-theoretic approaches provide sceptical semantics as the common part of credulous semantics, and so do not address this sort of scepticism. (2) Defeasible logic provides three different kinds of rule, including a rule that cannot support an argument, only defeat one. Most argumentation-theoretic works have addressed a single kind of rule. (3) In defeasible logic, positive conclusions (that a proposition can be proved) are not the only consideration; negative conclusions (that a proposition cannot be proved) are of equal significance. (4) Defeasible logic exhibits “team defeat” [10], in which one collection of arguments may defeat another, although no single argument defeats every argument in the other collection.

Technically, the main modifications we make to conventional argumentation-theoretic frameworks are: the explicit introduction of infinite arguments, the treatment of teams of arguments, rather than considering each argument only individually, and an iterative definition of rejected arguments.

In addition to innovations we make in argument theory, the resulting argumentation-theoretic semantics will be advantageous for defeasible logic. The logic currently has no model theory, and the proof

theory is clumsy. The semantics we provide is considerably more elegant. It will prove useful in the intended applications of defeasible logic mentioned above, where arguments are a natural feature of the problem domain.

This paper is structured as follows. In the next section we provide a brief introduction to defeasible logic. In this short paper there is no room for full details; for those we refer the reader to [15]. We then provide our argumentation-theoretic semantics for defeasible logic in Section 3. We conclude with a discussion of related work.

## 2 Overview of Defeasible Logic

We begin by presenting the basic ingredients of defeasible logic. A defeasible theory contains five different kinds of knowledge: facts, strict rules, defeasible rules, defeaters, and a superiority relation. We consider only essentially propositional rules. Rules containing free variables are interpreted as the set of their variable-free instances.

*Facts* are indisputable statements, for example, “Tweety is an emu”. In the logic, this might be expressed as  $emu(tweety)$ .

*Strict rules* are rules in the classical sense: whenever the premises are indisputable (e.g. facts) then so is the conclusion. An example of a strict rule is “Emus are birds”. Written formally:

$$emu(X) \rightarrow bird(X).$$

*Defeasible rules* are rules that can be defeated by contrary evidence. An example of such a rule is “Birds typically fly”; written formally:

$$bird(X) \Rightarrow flies(X).$$

The idea is that if we know that something is a bird, then we may conclude that it flies, *unless there is other evidence suggesting that it may not fly*.

*Defeaters* are rules that cannot be used to draw any conclusions. Their only use is to prevent some conclusions. In other words, they are used to defeat some defeasible rules by producing evidence to the contrary. An example is “If an animal is heavy then it might not be able to fly”. Formally:

$$heavy(X) \rightsquigarrow \neg flies(X).$$

The main point is that the information that an animal is heavy is not sufficient evidence to conclude that it doesn’t fly. It is only evidence that the animal *may* not be able to fly. In other words, we don’t wish to conclude  $\neg flies$  if *heavy*, we simply want to prevent a conclusion *flies*.

The *superiority relation* among rules is used to define priorities among rules, that is, where one rule may override the conclusion of another rule. For example, given the defeasible rules

$$\begin{aligned} r : \quad & \text{bird} \Rightarrow \text{flies} \\ r' : \quad & \text{brokenWing} \Rightarrow \neg \text{flies} \end{aligned}$$

which contradict one another, no conclusive decision can be made about whether a bird with a broken wing can fly. But if we introduce a superiority relation  $>$  with  $r' > r$ , then we can indeed conclude that the bird cannot fly. The superiority relation is required to be acyclic. It turns out that we only need to define the superiority relation over rules with contradictory conclusions.

It is not possible in this short paper to give a complete formal description of the logic. However, we hope to give enough information about the logic to make the discussion intelligible. We refer the reader to [17, 6, 15] for more thorough treatments.

A rule  $r$  consists of its *antecedent* (or *body*)  $A(r)$  which is a finite set of literals, an arrow, and its *consequent* (or *head*)  $C(r)$  which is a literal. Given a set  $R$  of rules, we denote the set of all strict rules in  $R$  by  $R_s$ , the set of strict and defeasible rules in  $R$  by  $R_{sd}$ , the set of defeasible rules in  $R$  by  $R_d$ , and the set of defeaters in  $R$  by  $R_{df}$ .  $R[q]$  denotes the set of rules in  $R$  with consequent  $q$ . If  $q$  is a literal,  $\sim q$  denotes the complementary literal (if  $q$  is a positive literal  $p$  then  $\sim q$  is  $\neg p$ ; and if  $q$  is  $\neg p$ , then  $\sim q$  is  $p$ ).

A *defeasible theory*  $D$  is a triple  $(F, R, >)$  where  $F$  is a finite set of facts,  $R$  a finite set of rules, and  $>$  a superiority relation on  $R$ .

A *conclusion* of  $D$  is a tagged literal and can have one of the following four forms:

- $+\Delta q$ , which is intended to mean that  $q$  is definitely provable in  $D$  (i.e., using only facts and strict rules).
- $-\Delta q$ , which is intended to mean that we have proved that  $q$  is not definitely provable in  $D$ .
- $+\partial q$ , which is intended to mean that  $q$  is defeasibly provable in  $D$ .
- $-\partial q$  which is intended to mean that we have proved that  $q$  is not defeasibly provable in  $D$ .

Provability is based on the concept of a *derivation* (or proof) in  $D = (F, R, >)$ . A derivation is a finite sequence  $P = (P(1), \dots, P(n))$  of tagged literals satisfying four conditions (which correspond to inference rules for each of the four kinds of conclusion). Here we briefly state the condition for positive defeasible conclusions [6]. ( $P(1..i)$  denotes the initial part of the sequence  $P$  of length  $i$ ):

- $+\partial$ : If  $P(i+1) = +\partial q$  then either
  - (1)  $+\Delta q \in P(1..i)$  or
  - (2) (2.1)  $\exists r \in R_{sd}[q] \forall a \in A(r) : +\partial a \in P(1..i)$  and
    - (2.2)  $-\Delta \sim q \in P(1..i)$  and
    - (2.3)  $\forall s \in R[\sim q]$  either
      - (2.3.1)  $\exists a \in A(s) : -\partial a \in P(1..i)$  or
      - (2.3.2)  $\exists t \in R_{sd}[q]$  such that
        - (2.3.2.1)  $\forall a \in A(t) : +\partial a \in P(1..i)$  and  $t > s$

Let us work through this condition. To show that  $q$  is provable defeasibly we have two choices: (1) We show that  $q$  is already definitely provable; or (2) we need to argue using the defeasible part of  $D$  as well. In particular, we require that there must be a strict or defeasible rule with head  $q$  which can be applied (2.1). But now we need to consider possible “attacks”, that is, reasoning chains in support of  $\sim q$ .

To be more specific: to prove  $q$  defeasibly we must show that  $\sim q$  is not definitely provable (2.2). Also (2.3) we must consider the set of all rules which are not known to be inapplicable and which have head  $\sim q$  (note that here we consider defeaters, too, whereas they could not be used to support the conclusion  $q$ ; this is in line with the motivation of defeaters given earlier). Essentially each such rule  $s$  attacks the conclusion  $q$ . For  $q$  to be provable, each such rule  $s$  must be counterattacked by a rule  $t$  with head  $q$  with the following properties: (i)  $t$  must be applicable at this point, and (ii)  $t$  must be stronger than  $s$ . Thus each attack on the conclusion  $q$  must be counterattacked by a stronger rule. In other words,  $r$  and the rules  $t$  form a team (for  $q$ ) that defeats the rules  $s$ .

### 3 Argumentation for Defeasible Logic

Argumentation systems usually contain the following basic elements: an underlying logical language, and the definitions of: argument, conflict between arguments, and the status of arguments. The latter elements are often used to define a consequence relation. In what follows we present an argumentation system containing the above elements in a way appropriate for defeasible logic.

Obviously, the underlying logical language we use is the language of defeasible logic; however, we consider facts to be strict rules with empty bodies.

Arguments are often defined to be either proof trees or monotonic derivations in the underlying logic. However, defeasible logic requires a more refined definition: as we have seen in the previous section, rules form teams to support conclusions. Thus we extend the simpler notion of argument and we allow arguments to be sets of proof trees (see example 2 for a more detailed explanation). Defeasible logic also requires a more general notion of proof tree that admits infinite trees, so that the distinction is kept between an unrefuted, but infinite, chain of reasoning and a refuted chain.

A *proof tree* for a literal  $p$  based on a set of rules  $R$  is a (possibly infinite) tree with nodes labelled by literals such that the root is labelled by  $p$  and for every node  $h$ :

- If  $b_1, \dots, b_n$  label the children of  $h$  then there is a ground instance of a rule in  $R$  with body  $b_1, \dots, b_n$  and head  $h$ .
- If, in addition,  $h$  is not the root of the tree then the rule must be a strict or defeasible rule.

If the rule at the root of a proof tree is strict or defeasible and the proof tree is finite we say it is a *supportive proof tree*. If all the rules in a proof tree are strict then we say that it is a *strict proof tree*.

An *argument* for a literal  $p$  is a set of proof trees for  $p$ . We say that an argument  $A$  is *finite* if every proof tree in  $A$  is finite. An argument  $A$  is *strict* if every proof tree in  $A$  is strict. If an argument is not strict it is *defeasible*.

An argument  $A$  for  $p$  is a *supportive argument* if every proof tree for  $p$  in it is supportive.

Defeasible logic has three kinds of rules and only two of them can be used to support the derivation of a conclusion. Defeaters can only block derivations. Intuitively a supportive argument is an argument from which a conclusion can be drawn, but if we changed its definition, replacing “every” with “some”, then we would have the following scenario: let  $A$  and  $B$  be, respectively, the arguments  $\{r_1 : \Rightarrow p, r_2 : \sim p\}$  and  $\{r_3 : \Rightarrow \neg p\}$  where  $r_1 < r_3$  and  $r_3 < r_2$ .  $A$  would be a supportive argument, and its conclusion,  $p$ , derivable, but defeasible logic is not able to derive  $+\partial p$ .

At this stage we can characterize the definite conclusions of defeasible logic in argumentation-theoretic terms.

**Proposition 1** Let  $P$  be a defeasible theory and  $p$  be a literal.

- $P \vdash +\Delta p$  iff there is a strict supportive argument for  $p$  in  $Args_P$
- $P \vdash -\Delta p$  iff there is no (finite or infinite) strict argument for  $p$  in  $Args_P$

This characterization is straightforward, since strict rules are the monotonic subset of defeasible logic. Characterizing defeasible provability requires more definitions.

A (proper) subargument of an argument  $A$  is a subtree of a proof tree in  $A$ .

An argument is based on an ordered theory  $(R, <)$  if every rule in the argument is a ground instance of a rule in  $R$ . Clearly, a defeasible theory  $(F, R, <)$  can be considered an ordered theory  $(F \cup R, <)$ .  $Args_P$  is the set of arguments based on the ordered theory  $P$ .

An argument  $A$  attacks an argument  $B$  if a conclusion of  $A$  is the complement of a conclusion of  $B$ .

An argument  $A$  defeats a defeasible argument  $B$  at  $q$  if there exists  $r_A \in A$  and  $r_B \in B$  with conclusions  $\sim q$  and  $q$ , respectively, such that  $r_A \prec r_B$ . A set of arguments  $S$  defeats a defeasible argument  $B$  if there is  $A \in S$  that defeats  $B$ .

**Example 1** Let  $D$  be a defeasible theory containing the rules

$$\begin{array}{ll} r_1 : a \Rightarrow p & r_3 : b \Rightarrow \neg p \\ r_2 : p \Rightarrow q & r_4 : \neg p \Rightarrow \neg q \end{array}$$

the facts  $a, b$ , and the superiority relation is  $r_4 < r_2$ . We consider the arguments

$$\begin{array}{l} A : a \Rightarrow p \Rightarrow q \\ B : b \Rightarrow \neg p \Rightarrow \neg q \end{array}$$

$A$  defeats  $B$  both at  $\neg p$ , because  $r_4 < r_2$ , and at  $\neg q$ , because there is no superiority relation between  $r_1$  and  $r_3$ .  $B$  defeats  $A$  at  $p$  for the same reason  $A$  defeats  $B$  at  $\neg p$ .  $\square$

An argument  $A$  team defeats a defeasible argument  $B$  at  $q$  if for every  $r_B \in B$  with conclusion  $q$  there exists a supportive rule  $r_A \in A$  with conclusion  $\sim q$  such that  $r_B < r_A$ .

**Example 2** Let  $D$  be a defeasible theory containing the rules

$$\begin{array}{ll} r_1 : a_1 \Rightarrow p & r_3 : b_1 \Rightarrow \neg p \\ r_2 : a_2 \Rightarrow p & r_4 : b_2 \Rightarrow \neg p \end{array}$$

the facts

$$a_1, a_2, b_1, b_2$$

and the superiority relation is  $r_3 < r_1, r_4 < r_2$ .

Consider the argument  $A$

$$\begin{array}{l} a_1 \\ \Downarrow \\ a_2 \end{array} \Rightarrow p$$

containing two proof trees.  $A$  team defeats:

- the argument  $B_1 : b_1 \Rightarrow \neg p$  since  $r_3 < r_1$ ;
- the argument  $B_2 : b_2 \Rightarrow \neg p$  since  $r_4 < r_2$ ;
- the argument  $B_3 : \begin{array}{l} b_1 \\ \Downarrow \\ b_2 \end{array} \Rightarrow \neg p$  since  $r_3 < r_1$  and  $r_4 < r_2$ .  $\square$

**Example 3** Some explanation is due to justify the exclusion of arguments ending with a defeaters from the notion of team defeat (see also the comment about supportive arguments above). First of all one

of the main aims of such a notion is to help establishing conclusive arguments (that is, arguments that can be used to draw positive conclusions). Let us consider a defeasible theory  $D'$  obtained from the defeasible theory of example 2 by replacing the rule  $r_2$  by the defeater  $r_2 : a_2 \rightsquigarrow p$ . Let  $A$  be the argument

$$\begin{array}{l} a_1 \\ \Downarrow \\ a_2 \end{array} \rightsquigarrow p$$

Since  $A$  contains a defeater, it cannot team defeat the argument  $B_3$  of the previous example. Let us compare this situation with the definition of  $+\partial$ .  $r_2$  cannot be used to derive  $p$ : it is a defeater. On the other hand  $r_1$  could be used to derive  $p$  if there is no applicable rule for  $\neg p$ . But, in this case, we have  $r_3$  and  $r_4$ , and, when  $r_4$  is applicable, we have a conflict between  $r_1$  and  $r_4$ . However,  $p$  could be reinstated if there is an applicable supportive rule stronger than  $r_4$  (2.3.2), but in this case the only rule stronger than  $r_4$  is the defeater  $r_2$ , and so  $p$  cannot be concluded from  $r_1$  and  $r_2$ .  $\square$

In an ordered theory  $P$ , let  $strong_P(S)$  be the set of arguments of  $P$ , all of whose proper subarguments are supported by  $S$ . Obviously  $S \subseteq strong_P(S)$ . Also note that, if  $A_1, A_2 \in strong_P(S)$  are arguments for a literal  $q$ , then  $A_1 \cup A_2 \in strong_P(S)$ . Thus there is a maximal argument for  $q$  in  $strong_P(S)$ , which we denote by  $max(q, S)$ . A defeasible argument  $A$  is undercut by a set of arguments  $S$  if there is a literal  $q$  such that  $strong_P(S)$  defeats a proper subargument of  $A$  at  $q$ , and  $A$  does not team defeat  $max(\neg q, S)$  at  $\neg q$ .

It is worth noting that the above definitions concern only defeasible arguments; for strict arguments we stipulate that they cannot be undercut or defeated.

**Example 4** We consider again the defeasible theory  $D$  of example 1. Let  $S = \{a, b\}$  be a set of arguments. The argument

$$A : a \Rightarrow p \Rightarrow q$$

is undercut by  $S$  since the argument  $B : b \Rightarrow \neg p$  is in  $strong_D(S)$  and it is the maximal argument for  $\neg p$ . Moreover  $B$  defeats a proper subargument of  $A$  at  $p$ , but it is not team defeated by  $A$  at  $p$ .  $\square$

That an argument  $A$  is undercut by  $S$  means that we can show that some premises of  $A$  cannot be proved; the next example explains the reason for the use of team defeat in the definition of undercut.

**Example 5** Let  $D'$  be the defeasible theory obtained from the defeasible theory of example 2 by adding the rule  $p \Rightarrow q$ . And let  $S = \{a_1, a_2, b_1, b_2\}$  be a set of arguments. Let  $A$  be the argument

$$\begin{array}{l} a_1 \\ \Downarrow \\ a_2 \end{array} \Rightarrow p \Rightarrow q$$

Notice that each of the arguments  $B_1, B_2$ , and  $B_3$  of example 2 defeats  $A$  at  $p$ , but  $A$  is not undercut by  $S$  at  $p$  since the argument  $A$  team defeats the  $max(\neg p, S)$  in  $strong_{D'}(S)$ . Here  $max(\neg p, S)$  is the argument  $B_3$  of example 2. Thus team defeat in the definition of undercut is necessary to be consistent with the use of team defeat at the top level of arguments.  $\square$

An argument  $A$  is supported by a set of arguments  $S$  if every conclusion in  $A$  is also the conclusion of a supportive argument in  $S$ .

An argument  $A$  for  $p$  is acceptable w.r.t a set of arguments  $S$  if

- 1  $A$  is strict, or
- 2a every proper subargument of  $A$  is supported by  $S$ , and
- 2b every argument attacking  $A$  is either undercut by  $S$  or team defeated by  $A$ .

Let  $P$  be an ordered theory. We define  $J_i^P$  as follows.

- $J_0^P = \emptyset$
- $J_{i+1}^P = \{a \in \text{Args}_P \mid a \text{ is acceptable w.r.t. } J_i^P\}$

The set of *justified arguments* in an ordered theory  $P$  is  $J\text{Args}^P = \bigcup_{i=1}^{\infty} J_i^P$ .

A literal  $p$  is *justified* if it is the conclusion of a supportive argument in  $J\text{Arg}^P$ .

**Theorem 2** *Let  $P$  be a defeasible theory. Let  $p$  be a literal.*  
 $P \vdash +\partial p$  iff  $p$  is justified.

This theorem provides a characterization of positive defeasible conclusions in defeasible logic by means of justified arguments.

That an argument  $A$  is justified means that it resists every reasonable refutation. However, defeasible logic is more expressive since it is able to say when a conclusion is demonstrably non provable ( $-\partial$ ). Briefly, that a conclusion is demonstrably non provable means that every possible conclusive argument has been refuted. In the following we show how to capture this notion in our argumentation system by assigning the status rejected to arguments that are refuted. Roughly speaking, an argument is rejected if it has a rejected subargument or it cannot overcome an attack from a justified argument.

An argument  $A$  is *rejected* by sets of arguments  $S$  and  $T$  when

- 1  $A$  is not strict, and either
- 2a a proper subargument of  $A$  is in  $S$ , or
- 2b there exists an argument  $B$  attacking  $A$ , such that:  $B$  is supported by  $T$ , and  $A$  does not team defeat  $B$ .

We define  $R_i^P$  as follows.

- $R_0^P = \emptyset$
- $R_{i+1}^P = \{a \in \text{Args}_P \mid a \text{ is rejected by } R_i^P \text{ and } J\text{Args}^P\}$

The set of *rejected arguments* in an ordered theory  $P$  is  $R\text{Args}^P = \bigcup_{i=1}^{\infty} R_i^P$ .

A literal  $p$  is *rejected* if there is no argument in  $\text{Args}_P - R\text{Args}^P$  that ends with a supportive rule for  $p$ .

**Theorem 3** *Let  $P$  be a defeasible theory. Let  $p$  be a literal.*  
 $P \vdash -\partial p$  iff  $p$  is rejected.

**Example 6** The following defeasible logic theory illustrates why  $R\text{Args}^P$  needs to be constructed iteratively, even after all the justified literals have been identified.

There are the following rules, for  $i = 1, \dots, n$ :

$$\begin{aligned} \text{true} &\Rightarrow b_i \\ a_i &\Rightarrow \neg b_i \\ b_{i-1} &\Rightarrow a_i \\ \text{true} &\Rightarrow \neg a_i \end{aligned}$$

and the fact  $b_0$ . The superiority relation is empty.

This theory produces the following conclusions:  $-\partial a_i, -\partial \neg a_i, +\partial b_i, -\partial \neg b_i$ , for  $i = 1, \dots, n$ .

The arguments defined by this theory are, for each  $i$ :

$$\begin{aligned} A_i : & \text{true} \Rightarrow \neg a_i \\ B_i : & \text{true} \Rightarrow b_{i-1} \Rightarrow a_i \Rightarrow \neg b_i \end{aligned}$$

and their subarguments. Notice that

- each argument  $A_i$  is attacked by  $B_i$  at  $a_i$ .
- each argument  $B_i$  is attacked by  $B_{i-1}$  at  $b_{i-1}$ .

Eventually, both  $A_i$  and  $B_i$  will be rejected, since neither can team defeat the other, but this cannot be done until the status of  $b_{i-1}$  is determined. As noted above, this depends on  $B_{i-1}$ . Thus the situation incorporates some sequentiality, where  $B_{i-1}$  must be resolved before resolving  $B_i$ , and this suggests that a characterization of  $R\text{Args}^P$  must be iterative, even after all the justified literals have been identified.  $\square$

We conclude this section with examples demonstrating how two traditionally problematic features of argumentation are handled by our semantics.

**Example 7** (Self-defeating arguments) In this example we show how our framework deals with the so called self-defeating arguments. Consider the defeasible theory with no facts, an empty superiority relation and the following rules:

$$\begin{aligned} \text{true} &\Rightarrow p \\ p &\Rightarrow \neg p \end{aligned}$$

This defeasible theory produces the following conclusion  $-\partial \neg p$ . The arguments that can be built from the theory are:

$$\begin{aligned} A_1 : & \text{true} \Rightarrow p \\ A_2 : & \text{true} \Rightarrow p \Rightarrow \neg p \end{aligned}$$

Here  $A_2$  is a self-defeating arguments. Since the superiority relation is empty there is no team defeat.  $A_1$ , although supported by  $J_0^P$ , is not acceptable in  $J_0^P$  since there is an attacking argument,  $A_2$ , which is not undercut by  $J_0^P$ : no proper subargument of  $A_2$  is defeated by an argument supported by  $J_0^P$ . For the same reason  $A_2$  is not acceptable in  $J_0^P$ . Consequently  $J_1^P = J_0^P$ , and therefore  $J\text{Arg}_P$  is empty. Furthermore,  $A_2 \in R\text{Arg}^P$ . The reason why  $A_2$  is rejected is the following: although  $A_1$  is not justified, it is supported by  $J\text{Args}^P$ , and so it can be used to stop the validity of another argument, since we have no means of deciding which one is to be preferred. On the other hand,  $A_1$  cannot be rejected since the argument attacking it ( $A_2$ ) is not supported by  $J\text{Args}^P$ : as we have already seen  $\text{true} \Rightarrow p$  is not a justified argument.  $\square$

**Example 8** (Circular arguments) Here we examine circular arguments. Very often circular arguments are not considered to be true arguments since they represent a very well known fallacy, and they are excluded from the set of arguments using syntactical definitions. Briefly an argument is circular if a conclusion depends on itself as a premise.

In our approach, circular arguments correspond to infinite arguments, and they are not justified. At the same time, however, they are not automatically rejected. Moreover, such an argument can be used to attack (and defeat) other arguments.

Let us first consider the defeasible theory  $D_1$  consisting of the rules

$$\begin{aligned} p &\Rightarrow q \\ q &\Rightarrow p \end{aligned}$$

It is immediate to see that the only possible arguments here are the infinite arguments

$$\begin{aligned} A_1 \dots p &\Rightarrow q \Rightarrow p \Rightarrow q \\ A_2 \dots q &\Rightarrow p \Rightarrow q \Rightarrow p \end{aligned}$$

They are not justified since no proper subargument is justified, and they are not rejected since no proper subargument is rejected and there is no argument attacking them.

The meaning of the theory at hand is that if something is  $p$ , then normally is  $q$ , and if something is  $q$ , then normally is  $p$ . Thus this amounts to say that normally  $p$  and  $q$  are equivalent properties.

We add to  $D_1$  the following rules:

$$\begin{array}{l} q \Rightarrow r \\ true \Rightarrow \neg r \end{array}$$

obtaining the defeasible theory  $D_2$ . In this scenario each argument for  $r$  is infinite, circular, and rejected since there is a supported argument for  $\neg r$ . However, the argument  $A_3 : true \Rightarrow \neg r$  is not justified, since each argument for  $r$  attacks it and is not undercut (no argument attacks a proper subargument of an argument for  $r$ ).

Finally  $D_3$  is obtained from  $D_2$  by adding the rule  $true \Rightarrow \neg p$ . Now  $A_3$  becomes justified since, trivially, the argument  $A_4 : true \Rightarrow p$  is supported by  $J_0^{D_3}$ ,  $A_3$  attacks  $A_2$ , and therefore each argument for  $r$  is undercut.  $\square$

## 4 Related Works

[14] proposes an abstract defeasible reasoning framework that is achieved by mapping elements of defeasible reasoning into the default reasoning framework of [7]. While this framework is suitable for developing new defeasible reasoning languages, it is not appropriate for characterizing defeasible logic because:

- [14, 7] do not address direct scepticism.
- [7] does not address Kunen's semantics of logic programs which provides a characterization of failure-to-prove in defeasible logic [16].
- The correctness of the mapping needs to be established if [14] is to be applied to an existing language like defeasible logic. In fact the representation of priorities is inappropriate for defeasible logic, although results of [2, 1] might be adapted to remedy this point.

The abstract argumentation framework of [22] addresses both strict and defeasible rules, but not defeaters. However, the treatment of strict rules in defeasible arguments is different from that of defeasible logic, and there is no concept of team defeat. There are structural similarities between the definitions of inductive warrant and warrant in [22] and  $J_i^F$  and  $J_{Args_P}$ , but they differ in that acceptability is monotonic in  $S$  whereas the corresponding definitions in [22] are antitone. The semantics that results is not sceptical, and more related to stable semantics than Kunen semantics. The framework does have a notion of *ultimately defeated argument* similar to our rejected arguments, but the definition is not iterative, possibly because the framework does not have a directly sceptical semantics.

Prakken and Sartor [19, 18], motivated by legal reasoning, have proposed an argumentation system that combines the language of extended logic programming with a well-founded semantics. The use of this semantics makes Prakken and Sartor's system not directly sceptical. It is worth noting that our definition of defeat is the same as that of rebut in [19, 18], but the systems differ on the notion of acceptability of arguments. Moreover, Prakken and Sartor do not address the question of teams of rules.

Among other contributions, [8] provides a sceptical argumentation theoretic semantics and shows that LPwNF – which is weaker, but very similar to defeasible logic [5] – is sound with respect to this semantics. However, both LPwNF and defeasible logic are not complete with respect to this semantics.

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