Applied Logic I

Introduction to AGM Approach II

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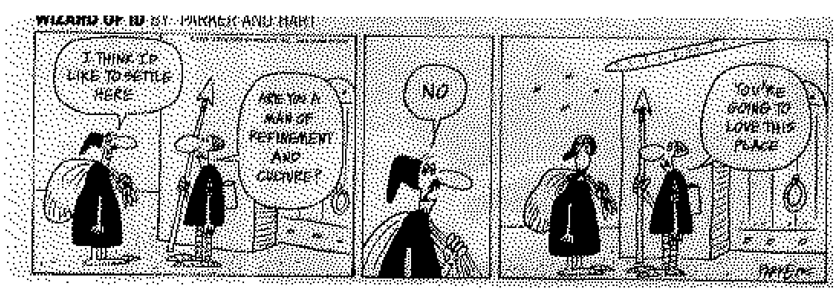


Overview

- Belief Revision
- Second Construction: Systems of Spheres
- Third Construction: Epistemic Entrenchment
- Summary
- Areas of Study in Belief Change
- Belief Change Summary

Belief Revision

Want to incorporate a belief in a consistent fashion

(K*1)	For any sentence ϕ and any belief set K, $K * \phi$ is a belief set	(closure)
(K*2)	$\phi \in K * \phi$	(success)
(K*3)	$K * \phi \subseteq K + \phi$	(inclusion)
(K*4)	If $\neg\phi \notin K$, then $K + \phi \subseteq K * \phi$	(preservation)
(K*5)	$K * \phi = K_{\perp}$ if and only if $\vdash \neg\phi$	(vacuity)
(K*6)	If $\vdash \phi \leftrightarrow \psi$, then $K * \phi = K * \psi$	(extensionality)
(K*7)	$K * \phi \wedge \psi \subseteq (K * \phi) + \psi$	(super expansion)
(K*8)	If $\neg\psi \notin K * \phi$, then $(K * \phi) + \psi \subseteq K * (\phi \wedge \psi)$	(sub expansion)

Additional Properties

1. If $\phi \in K$, then $K * \phi = K$
2. $K * \phi = (K \cap K * \phi) + \phi$
3. $K * \phi = K * \psi$ if and only if $\psi \in K * \phi$ and $\phi \in K * \psi$
4. $K * \phi \cap K * \psi \subseteq K * (\phi \vee \psi)$
5. If $\neg\psi \notin K * (\phi \vee \psi)$, then $K * (\phi \vee \psi) \subseteq K * \psi$
6. $K * (\phi \vee \psi) = K * \phi$ or $K * (\phi \vee \psi) = K * \psi$ or $K * (\phi \vee \psi) = K * \phi \cap K * \psi$

Epistemic Entrenchment

- Ordering over formulae in $\mathcal{L}$
- Certain beliefs about the world are more important than others when planning future actions, etc.
- $\phi \leq \psi$: ψ is at least as epistemically entrenched as ϕ
- In contraction, sentences in K with lower entrenchment given up
- Tautologies maximally entrenched, non-beliefs minimally entrenched

- But what does an epistemic entrenchment relation look like?

Epistemic Entrenchment

(EE1)	If $\phi \leq \psi$ and $\psi \leq \gamma$ then $\phi \leq \gamma$	(transitivity)
(EE2)	If $\{\phi\} \vdash \psi$ then $\phi \leq \psi$	(dominance)
(EE3)	For any ϕ and ψ, $\phi \leq \phi \wedge \psi$ or $\psi \leq \phi \wedge \psi$	(conjunctiveness)
(EE4)	When $K \neq K_\perp$, $\phi \in K$ iff $\phi \leq \psi$ for all ψ	(minimality)
(EE5)	If $\phi \leq \psi$ for all ϕ then $\vdash \psi$	(maximality)

- Essentially we have a series of "ranks" or levels containing formulae of equal entrenchment. Moreover, the tautologies are maximally entrenched (we cannot give these up!) and non-beliefs are minimally entrenched (we don't care about these!)

Additional Properties

1. $\phi \leq \psi$ or $\psi \leq \phi$ (connectedness)
2. If $\psi \wedge \chi \leq \phi$, then $\phi \leq \phi$ or $\chi \leq \phi$
3. $\phi < \psi$ iff $\phi \wedge \psi < \psi$
4. If $\chi \leq \phi$ and $\chi \leq \psi$, then $\chi \leq \phi \wedge \psi$
5. If $\phi \leq \psi$, then $\phi \leq \phi \wedge \psi$
6. $\phi \wedge \psi = min(\phi, \ \psi)$
7. $\phi \vee \psi \geq max(\phi, \ \psi)$

(NB: $\phi < \psi \equiv \psi \not\leq \phi$)

Belief Change via Entrenchment

(Gärdenfors and Makinson 1988)

$(C\leq)$ $\phi \leq \psi$ iff $\phi \notin K\dot{-}(\phi \wedge \psi)$ or $\vdash \phi \wedge \psi$

• Prefer ψ to ϕ if we would give up ϕ when given a choice between giving up ϕ or ψ or if it's not possible to give up either formula

$(C\dot{-})$ $\psi \in K\dot{-}\phi$ iff $\psi \in K$ and either $\phi < \phi \vee \psi$ or $\vdash \phi$

• Retain belief if it was originally believed and there is "independent evidence" for maintaining it or if it is not possible to remove ϕ

$(C*)$ $\psi \in K * \phi$ iff either $\neg\phi \leq \neg\phi \vee \psi$ or $\vdash \neg\phi$

Theorem: If an ordering $\leq$ satisfies (EE1) – (EE5), then the contraction function which is uniquely determined by $(C\dot{-})$ satisfies $(K\dot{-}1)$ – $(K\dot{-}8)$ as well as condition $(C\leq)$.

Theorem: If a contraction function $\dot{-}$ satisfies $(K\dot{-}1)$ – $(K\dot{-}8)$, then the ordering that is uniquely determined by $(C\leq)$ satisfies (EE1) – (EE8) as well as condition $(C\dot{-})$.

Systems of Spheres

Definition: Let $\mathcal{S}$ be any collection of subsets of $\mathcal{M}_{\mathcal{L}}$. We call $\mathcal{S}$ a system of spheres, centred on $X \subseteq \mathcal{M}_{\mathcal{L}}$, if it satisfies the following conditions:

($\mathcal{S}$1) $\mathcal{S}$ is totally ordered by $\subseteq$; that is, if $U, V \in \mathcal{S}$, then $U \subseteq V$ or $V \subseteq U$

($\mathcal{S}$2) X is the $\subseteq$-minimum of $\mathcal{S}$

($\mathcal{S}$3) $\mathcal{M}_{\mathcal{L}}$ is the $\subseteq$-maximum of $\mathcal{S}$

($\mathcal{S}$4) If $\phi \in \mathcal{L}$ and $\not\vdash \neg\phi$, then there is a smallest sphere in $\mathcal{S}$ intersecting $[\phi]$ (i.e., there is a sphere $U \in \mathcal{S}$ such that $U \cap [\phi] \neq \emptyset$, and $V \cap [\phi] \neq \emptyset$ implies $U \subseteq V$ for all $V \in \mathcal{S}$)
(Lewis' *Limit Assumption)*

Grove's Spheres

- Ordering over "possible worlds" (maximally consistent sets of formulae)
- Motivated by Lewis' sphere semantics for counterfactuals
- System of spheres: sets of possible worlds nested one within the other
- The set of all worlds $\mathcal{M}_{\mathcal{L}}$ is the outermost (largest) sphere
- $[K]$ is the set of worlds consistent with K; these worlds form the innermost (smallest) sphere
- System of spheres centred on $[K]$

Belief Change via SOSs

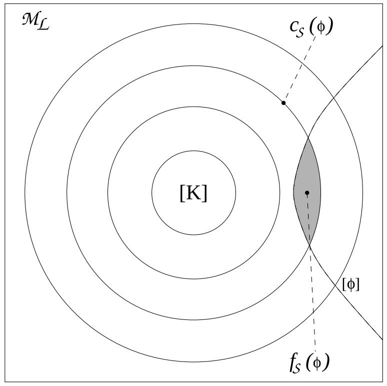
$c_{\mathcal{S}}(\phi)$ — the smallest sphere interesecting $[\phi]$

$f_{\mathcal{S}}(\phi) = c_{\mathcal{S}}(\phi) \cap [\phi]$ — innermost ϕ-worlds

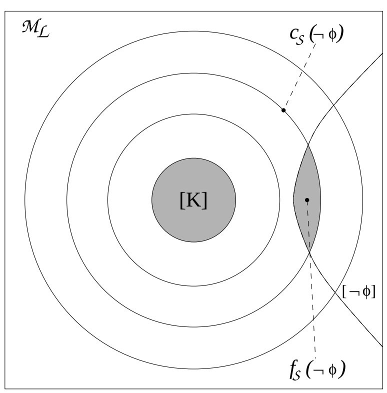
Theorem: Let $\mathcal{S}$ be any system of spheres in $\mathcal{M}_{\mathcal{L}}$ centred on $[K]$ for some theory $K \in \mathcal{K}$. If one defines, for any $\phi \in \mathcal{L}$, $K * \phi$ to be $th(f_{\mathcal{S}}(\phi))$, then the axioms (K*1) – (K*8) are satisifed.

Theorem: Let $* : \mathcal{K} \times \mathcal{L} \to \mathcal{K}$ be any function satisfying axioms (K*1) – (K*8). Then for any (fixed) theory K there is a system of spheres on $\mathcal{M}_{\mathcal{L}}$, $\mathcal{S}$ say, centred on $[K]$ and satisfying $K * \phi = th(f_{\mathcal{S}}(\phi))$ for all $\phi \in \mathcal{L}$.

AGM Revision



AGM Partial Meet Contraction



Properties of th (Grove 1988)

$th : 2^{\mathcal{M}_{\mathcal{L}}} \rightarrow \mathcal{K}$

(i) $th([K]) = K$ for all belief sets (i.e., theories) K if the underlying logic is compact

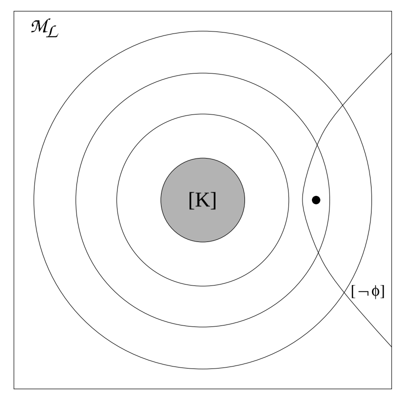
(ii) $th(X) \neq K_\perp$ if and only if X is nonempty

(iii) For any sentence $\phi \in \mathcal{L}$ and $X \subseteq \mathcal{M}_{\mathcal{L}}$, $th(X \cap [\phi]) = Cn(th(X) \cup \{\phi\})$

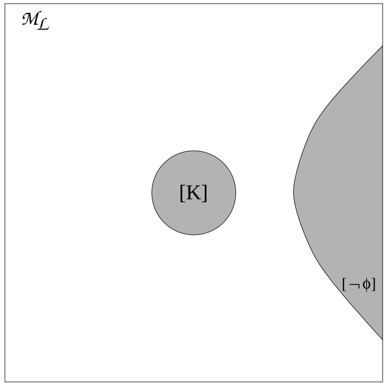
(iv) For $X,\ X' \subseteq \mathcal{M}_{\mathcal{L}}$, if $X \subseteq X'$, then $th(X') \subseteq th(X)$

(v) For $K,\ K' \in \mathcal{K}$, if $K \subseteq K'$, then $[K'] \subseteq [K]$

AGM Maxichoice Contraction



AGM Full Meet Contraction



SOS ⇔ EE

(Gärdenfors 1988)

Can translate back and forth from a Systems of Spheres $\mathcal{S}$ and an epistemic entrenchment relation $\leq$ using the following condition:

$$\phi \leq \psi \text{ iff } c_{\mathcal{S}}(\neg\phi) \subseteq c_{\mathcal{S}}(\neg\psi)$$

Summary

- Postulates for AGM belief revision
- Revision can be motivated separately or via contraction using the Levi Identity (can you visualise the Levi Identity using system of spheres?)
- We now have two further constructions:
  - Construction in terms of ordering over formulae
  - Construction in terms of ordering over possible worlds
- Both provide further insight into the belief change process

Areas of Study in Belief Change

- Belief bases and computational approaches
- Coherence (how do beliefs ‘cohere’)
- Contraction proposals
- Iterated revision
- Relationships between belief change, nonmonotonic reasoning, conditionals, rational choice, …
- Non-prioritised belief change
- Abductive belief change
- Reasoning about action and belief change
- Dealing with uncertainty (Spohn, Bayesian networks, …)

Belief Change Summary

- Many belief change areas still to explore
- Can have richer structure than AGM (e.g., Spohn, probabilistic approaches)
- How practical is it to implement belief change? (To use?)
- Many interconnections with other areas in reasoning (as we shall see)
- Belief change provides but one way of viewing reasoning; we shall see others in the subsequent lectures