# Applied Logic I

# Nonmonotonic Reasoning

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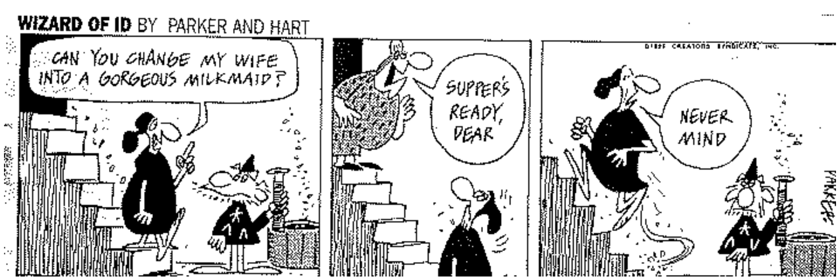
## Nonmonotonic Reasoning








## Overview

- Nonmonotonic reasoning
- The Closed World Assumption
- Predicate Completion
- Reiter's default logic
- KLM approach to nonmonotonic consequence
- Belief change and nonmonotonic consequence
- The Big Picture!
- Summary





## Nonmonotonicity

- Classical logic satisfies the following property
- Monotonicity: If $\Delta \subseteq \Gamma$, then $Cn(\Delta) \subseteq Cn(\Gamma)$ (equivalently, $\Gamma \vdash \phi$ implies $\Gamma \cup \Delta \vdash \phi$)
- However, we often draw conclusions based on 'what is normally the case' or 'true by default'
- More information can lead us to retract previous conclusions
- We shall adopt the following notation
  - $\vdash$ classical consequence relation
  - $\mid\!\sim$ nonmonotonic consequence relation

## Example

Suppose I tell you 'Tweety is a bird'

You might conclude 'Tweety flies'

I then tell you 'Tweety is an emu'

You conclude 'Tweety does not fly'

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$bird(Tweety) \mid\sim flies(Tweety)$

$bird(Tweety) \land emu(Tweety) \mid\sim \neg flies(Tweety)$



## The Closed World Assumption

- A complete theory is one in which for every ground atom in the language, either the atom or its negation appears in the theory
- The closed world assumption (CWA) completes a base (non-closed) set of formulae by including the negation of a ground atom whenever the atom does not follow from the base
- If we have no evidence as to the truth of (ground atom) $P$, we assume that it is false
- Given a base set of formulae $\Delta$ we first calculate the assumption set
  $\neg P \in \Delta_{asm}$ iff for ground atom $P$, $\Delta \not\vdash P$
- $CWA(\Delta) \;=\; Cn\{\Delta \cup \Delta_{asm}\}$



## Example

$\Delta \;=\; \{P(a), P(b), P(a) \to Q(a)\}$

$\Delta_{asm} \;=\; \{\neg Q(b)\}$

**Theorem:** The CWA applied to a consistent set of formulae $\Delta$ is inconsistent iff there are positive ground literals $L_1, \ldots, L_n$ such that $\Delta \models L_1 \lor \ldots \lor L_n$ but $\Delta \not\models L_i$ for $i = 1, \ldots, n$.

- Note that in the example above we limited our attention to the object constants that appeared in $\Delta$ however the language could contain other constants. This is known as the Domain Closure Assumption (DCA)
- Another common assumption is the Unique-Names Assumption (UNA).

  If two ground terms can't be proved equal, assume that they are not.



## Predicate Completion

**Idea:** The only objects that satisfy a predicate are those that must

- For example, suppose we have $P(a)$. Can view this as
  $\forall x.\; x = a \to P(x)$
  the if-half of a definition
- Can add the only if part:
  $\forall x.\; P(x) \to x = a$
- Giving:
  $\forall x.\; P(x) \leftrightarrow x = a$

# Predicate Completion

- **Definition:** A clause is solitary in a predicate $P$ if whenever the clause contains a postivie instance of $P$, it contains only one instance of $P$.
  - For example, $Q(a),\ P(a),\ \neg P(b)$ is not solitary in $P$
    $Q(a),\ R(a),\ P(b)$ is solitary in $P$
- Completion of a predicate is only defined for sets of clauses solitary in that predicate





# Predicate Completion

- Each clause can be written:
  $\forall y.\ Q_1 \wedge \ldots \wedge Q_m \rightarrow P(t)$ ($P$ not contained in $Q_i$)
  $\forall y.\ \forall x.\ (x = t) \wedge Q_1 \wedge \ldots \wedge Q_m \rightarrow P(x)$
  $\forall x.(\exists y.\ (x = t) \wedge Q_1 \wedge \ldots \wedge Q_m \rightarrow P(x))$ (normal form of clause)
- Doing this to every clause gives us a set of clauses of the form:
  $\forall x.\ E_1 \rightarrow P(x)$
  $\ldots$
  $\forall x.\ E_n \rightarrow P(x)$
- Grouping these together we get:
  $\forall x.\ E_1 \vee \ldots \vee E_n \rightarrow P(x)$
- Completion becomes: $\forall x.\ P(x) \leftrightarrow E_1 \vee \ldots \vee E_n$ and we can add this to the original set of formulae





# Example

- Suppose $\Delta = \{\forall x.\ Emu(x) \rightarrow Bird(x)$
  $Bird(Tweety)$,
  $\neg Emu(Tweety)\}$
- We can write this as
  $\forall x.\ (Emu(x) \vee x = Tweety) \rightarrow Bird(x)$
- Predicate completion of $P$ in $\Delta$ becomes
  $\Delta \cup \{\forall x.\ Bird(x) \leftrightarrow Emu(x) \vee x = Tweety\}$





# Reiter's Default Logic (1980)

- Add default rules of the form $\frac{\alpha:\beta}{\gamma}$
  - "If $\alpha$ can be proven and consistent to assume $\beta$, then conclude $\gamma$"
- Often consider normal default rules $\frac{\alpha:\beta}{\beta}$
- Example: $\frac{bird(x):flies(x)}{flies(x)}$
- Default theory $\langle D,\ W\rangle$
  $D$ – set of defaults; $W$ – set of facts
- Extension of default theory contains as many default conclusions as possible and must be consistent (and is closed under classical consequence $Cn$)
- Concluding whether formula $\phi$ follows from $\langle D,\ W\rangle$
  - Sceptical inference: $\phi$ occurs in every extension of $\langle D,\ W\rangle$
    Credulous inference: $\phi$ occurs in some extension of $\langle D,\ W\rangle$

# Examples

- $W = \{\}; D = \{\frac{:p}{\neg p}\}$ – no extensions
- $W = \{p \vee r\}; D = \{\frac{p:q}{q}, \frac{r:q}{q}\}$ – one extension $\{p \vee r\}$
- $W = \{p \vee q\}; D = \{\frac{:\neg p}{\neg p}, \frac{:\neg q}{\neg q}\}$ – two extensions $\{\neg p, p \vee q\}$, $\{\neg q, p \vee q\}$
- $W = \{emu(Tweety), emu(x) \rightarrow bird(x)\}; D = \{\frac{bird(x):(flies(x)}{flies(x)}\}$ – one extension
- What if we add $\frac{emu(x):\neg flies(x)}{\neg flies(x)}$?
- Poole (1988) achieves a similar effect (but not quite as general) by changing the way the underlying logic is used rather than introducing a new element into the syntax





# Default Theories—Properties

**Observation:** Every normal default theory (default rules are all normal) has an extension

**Observation:** If a normal default theory has several extensions, they are mutually inconsistent

**Observation:** A default theory has an inconsistent extension iff $D$ is inconsistent

**Theorem:** (Semi-monotonicity)
Given two normal default theories $\langle D, W\rangle$ and $\langle D', W'\rangle$ such that $D \subseteq D'$ then, for any extension $\mathcal{E}(E, W)$ there is an extension $\mathcal{E}(D', W')$ where $\mathcal{E}(E, W) \subseteq \mathcal{E}(D', W')$

(The addition of normal default rules does not lead to the retraction of consequences.)





# Nonmonotonic Consequence

- Abstract study and analysis of nonmonotonic consequence relation $\mid\!\sim$ in terms of general properties Kraus, Lehmann and Magidor (1991)
- Some common properties include:

  **Supraclassicality** If $\phi \vdash \psi$, then $\phi \mid\!\sim \psi$

  **Left Logical Equivalence** If $\vdash \phi \leftrightarrow \psi$ and $\phi \mid\!\sim \chi$, then $\psi \mid\!\sim \chi$

  **Right Weakening** If $\vdash \psi \rightarrow \chi$ and $\phi \mid\!\sim \psi$, then $\phi \mid\!\sim \chi$

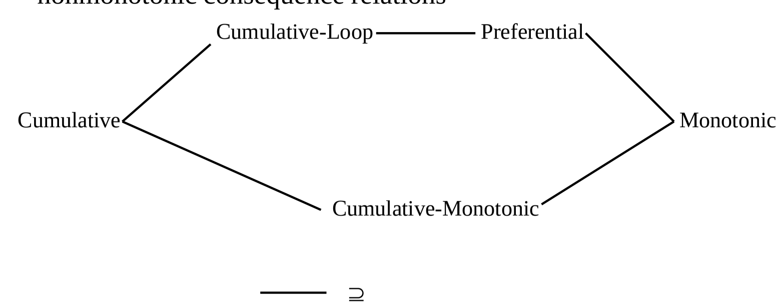
  **And** If $\phi \mid\!\sim \psi$ and $\phi \mid\!\sim \chi$, then $\phi \mid\!\sim \psi \wedge \chi$
- Plus many more!





# KLM Systems

- Kraus, Lehman and Magidor (1991) study various classes of nonmonotonic consequence relations




- This has been extended since. A good reference for this line of work is Schlechta (1997)

# Belief Change and NMR

- Can define $\phi \mid\!\sim_K \psi$ iff $\psi \in K * \phi$ (cf. Ramsey test)
- Translation of AGM revision postulates:
  (K*1) = If $\phi \mid\!\sim \psi_i$ for all $\psi_i \in K$ and $K \vdash \chi$, then $\phi \mid\!\sim \chi$ (Closure)
  (K*2) = $\phi \mid\!\sim \phi$ (Reflexivity)
  (K*3) = If $\phi \mid\!\sim \psi$, then $\top \mid\!\sim \phi \to \psi$ (Weak Conditionalisation)
  (K*4) = If $\top \mid\!\not\sim \neg\phi$ and $\top \mid\!\sim \phi \to \psi$, then $\phi \mid\!\sim \psi$ (Weak Rational Monotony)
  (K*5) = If $\phi \mid\!\sim \bot$, then $\phi \vdash \bot$ (Consistency Preservation)
  (K*6) = Left Logical Equivalence
  (K*7) = If $\phi \wedge \psi \mid\!\sim \chi$, then $\phi \mid\!\sim \psi \to \chi$ (Conditionalisation)
  (K*8) = If $\phi \mid\!\not\sim \neg\psi$ and $\phi \mid\!\sim \chi$, then $\phi \wedge \psi \mid\!\sim \chi$ (Rational Monotony)
- Note that $\mid\!\sim$ is with respect to a particular $K$
- All these postulates are known in the nonmonotonic consequence literature





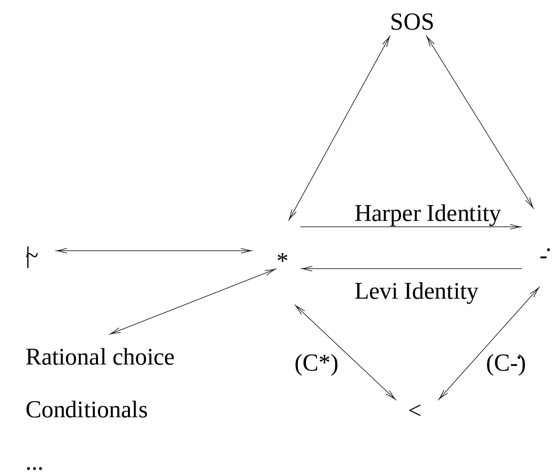
# Belief Change and Defaults

- Can view normal defaults $\frac{\alpha:\beta}{\beta}$ as supernormal defaults $\frac{\top:\alpha\to\beta}{\alpha\to\beta}$
- Can encode such defaults in epistemic entrenchment as
  $\alpha \to \neg\beta < \alpha \to \beta$
- 'Given information $\alpha$, strictly prefer $\beta$ to $\neg\beta$'
- $bird(Tweety) \to \neg flies(Tweety) < bird(Tweety) \to flies(Tweety)$
- It is consistent to have
  $bird(Tweety) \wedge emu(Tweety) \to flies(Tweety) <$
  $bird(Tweety) \wedge emu(Tweety) \to \neg flies(Tweety)$





# The big picture!








# Summary

Nonmonotonic reasoning attempts to capture a form of commonsense reasoning

Nonmonotonic reasoning often deals with inferences based on defaults or 'what is usually the case'

Belief change and nonmonotonic reasoning: two sides of the same coin?

Can introduce abstract study of nonmonotonic consequence relations in same way as we study classical consequence relations

Similar links exist with conditionals

Nonmonotonic reasoning is a vast area of study

An important form of nonmonotonic reasoning, especially for reasoning about action, is circumscription but we do not have time to cover it here