Belief Change: Introduction I

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Belief Change

Lecture Series Outline

Lecture 1: Belief Change: Introduction and Motivation

Lecture 2: Belief Change: Introduction to AGM Approach

Lecture 3: Belief Change via Preference

Lecture 4: Belief Change and Nonmonotonic Reasoning

Lecture 5: Current Research and Advanced Topics

Overview

What is belief change?

Rationality criteria

AGM framework

Belief Expansion

Belief Contraction

Summary

Belief Change — An example

"...suppose that on a public holiday you are standing in the street in a town that has two hamburger restaurants. ...When you meet me, eating a hamburger, you draw the conclusion that at least one of the two restaurants is open. ...Further, seeing from a distance that one of the two restaurants has its lights on, you believe that this particular restaurant is open. ...

When you have reached the restaurant, however, you find a sign saying that it is closed all day. The lights are only turned on for the purpose of cleaning. ...

In contrast, suppose you had not met me or anyone else eating a hamburger. Then your only clue would have been the lights from the restaurant." (Hansson 1989)

Belief Change — An example

(Gärdenfors & Rott 1995)

Beliefs

The bird caught in the trap is a swan
The bird caught in the trap comes from Sweden
Sweden is part of Europe
All Europen swans are white

Consequences

The bird caught in the trap is white

New information

The bird caught in the trap is black

Which sentences do you give up?

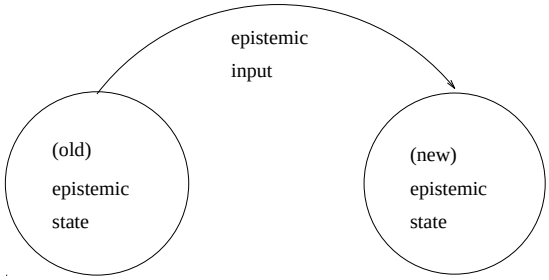
Logical considerations alone are not sufficient to answer this question.

One possibility

The bird caught in the trap is a swan
The bird caught in the trap comes from Sweden
Sweden is part of Europe
All Europen swans, except for some of the Swedish, are white
The bird caught in the trap is black

Belief Change

Dynamics of epistemic states



Rationality Criteria

Principle of Categorical Matching The representation of a belief state after change should be of the same format as that prior to change

Consistency Beliefs in belief state should be consistent

Deductive Closure If the beliefs in a belief state logically entail a sentence ϕ, then ϕ should be included in the state

Principle of Informational Economy The amount of information lost during change should be kept to a minimum

Preference Beliefs considered more important or entrenched should be retained in favour of less important ones.

Technical Preliminaries

Fixed finite language $\mathcal{L}$

Underlying logic $(\vdash, Cn)$

- Includes classical propositional calculus
- Satisfies compactness
- Satisfies deduction theorem

$\phi, \psi, \ldots$ formulae $(\in \mathcal{L})$

$\Delta, \Gamma, \ldots$ sets of formulae

$H, K, \ldots$ belief sets $(K \in \mathcal{K}$ iff $K = Cn(K)$; $K_\perp = \mathcal{L})$

AGM Framework

(Alchourrón, Gärdenfors and Makinson)

Epistemic states — belief sets (closed under Cn)

Epistemic input — formula

Operators — belief expansion, contraction, revision

Principles of rationality (e.g., minimal change)

Postulates for *rational* belief change

Constructions

- Selection functions
- Grove's spheres
- Epistemic entrenchment

AGM Belief Change Operations

belief expansion $(+)$ epistemic input added to the current belief state without removal of any existing beliefs

belief contraction $(\dot{-})$ beliefs removed from the current belief state in order to effect removal of the epistemic input

belief revision $(*)$ epistemic input is incorporated into the current belief state but some existing beliefs may also need to be removed to maintain consistency

Belief change function $+,\ \dot{-},\ *\ : \mathcal{K} \times \mathcal{L} \to \mathcal{K}$.

Commensurability Thesis (Levi 1991)

"Given an initial state of full belief K_1 and another state of full belief K_2, there is always a sequence of expansions and contractions, beginning with K_1, remaining within the state of potential states of full belief and terminating with K_2."

Levi Identity:

$K * \phi = (K \dot{-} \neg\phi) + \phi$

Harper Identity:

$K \dot{-} \phi = K \cap K * \neg\phi$

Belief Expansion

(K+1)	For any sentence α and any belief set K, $K^+\alpha$ is a belief set	(closure)
(K+2)	$\alpha \in K + \alpha$	(success)
(K+3)	$K \subseteq K + \alpha$	(inclusion)
(K+4)	If $\alpha \in K$, then $K + \alpha = K$	(vacuity)
(K+5)	If $K \subseteq H$, then $K + \alpha \subseteq H + \alpha$	(monotonicity)
(K+6)	For all belief sets K and all sentences α, $K + \alpha$ is the smallest belief set that satisfies (K+1) — (K+5)	(minimality)

Theorem:

The expansion function + satisfies $(K + 1)$ — $(K + 6)$ iff $K + \alpha = Cn(K \cup \{\alpha\})$.

Belief Contraction

(K$\dot{-}$1)	For any sentence ϕ and any belief set K, $K\dot{-}\phi$ is a belief set	(closure)
(K$\dot{-}$2)	$K\dot{-}\phi \subseteq K$	(inclusion)
(K$\dot{-}$3)	If $\phi \notin K$, then $K\dot{-}\phi = K$	(vacuity)
(K$\dot{-}$4)	If $\not\vdash \phi$ then $\phi \notin K\dot{-}\phi$	(success)
(K$\dot{-}$5)	If $\phi \in K$, $K \subseteq (K\dot{-}\phi) + \phi$	(recovery)
(K$\dot{-}$6)	If $\vdash \phi \leftrightarrow \psi$, then $K\dot{-}\phi = K\dot{-}\psi$	(preservation)
(K$\dot{-}$7)	$K\dot{-}\phi \cap K\dot{-}\psi \subseteq K\dot{-}(\phi \wedge \psi)$	(conj. overlap)
(K$\dot{-}$8)	If $\phi \notin K\dot{-}(\phi \wedge \psi)$, then $K\dot{-}(\phi \wedge \psi) \subseteq K\dot{-}\phi$	(conj. inclusion)

Other properties

1. If $\phi \in K$, then $(K\dot{-}\phi) + \phi \subseteq K$
2. $K\dot{-}\phi = K \cap (K\dot{-}\phi) + \neg\phi$
3. $K\dot{-}\phi \cap Cn(\{\phi\}) \subseteq K\dot{-}(\phi \wedge \psi)$
4. Either $K\dot{-}(\phi \wedge \psi) \subseteq K\dot{-}\phi$ or $K\dot{-}(\phi \wedge \psi) \subseteq K\dot{-}\psi$
5. Either $K\dot{-}(\phi \wedge \psi) = K\dot{-}\phi$ or $K\dot{-}(\phi \wedge \psi) = K\dot{-}\psi$ or $K\dot{-}(\phi \wedge \psi) = K\dot{-}\phi \cap K\dot{-}\psi$
6. If $\psi \rightarrow \phi \in K\dot{-}\phi$ and $\phi \rightarrow \psi \in K\dot{-}\psi$, then $K\dot{-}\phi = K\dot{-}\psi$

Summary

Belief change concerns the dynamics of belief states

Interested in rational belief change

Characterisation in terms of (intuitive) postulates and constructions

Operations: expansion, contraction and revision