Belief Change: Introduction to AGM Approach

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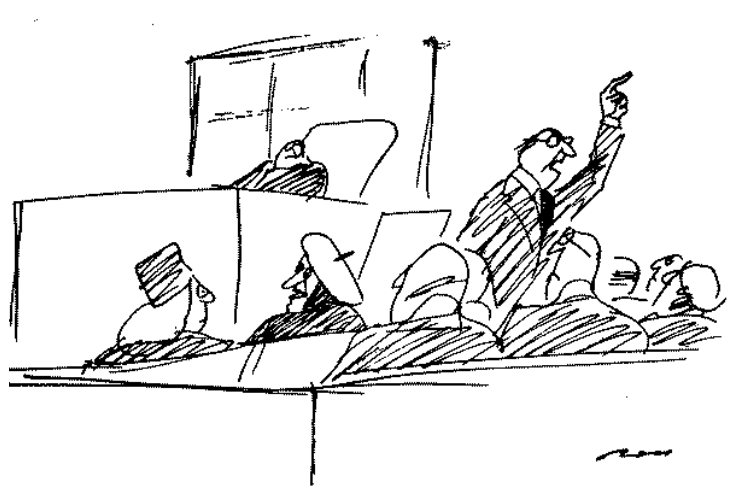
Belief Change



"Logic—the last refuge of a scoundrel."

Overview

Belief contraction

First construction — maximal nonimplying subsets

- Maxichoice contraction
- Full meet contraction
- Partial meet contraction
- Selection functions

Recovery and Withdrawal functions

Saturatable Sets

Belief revision

Summary

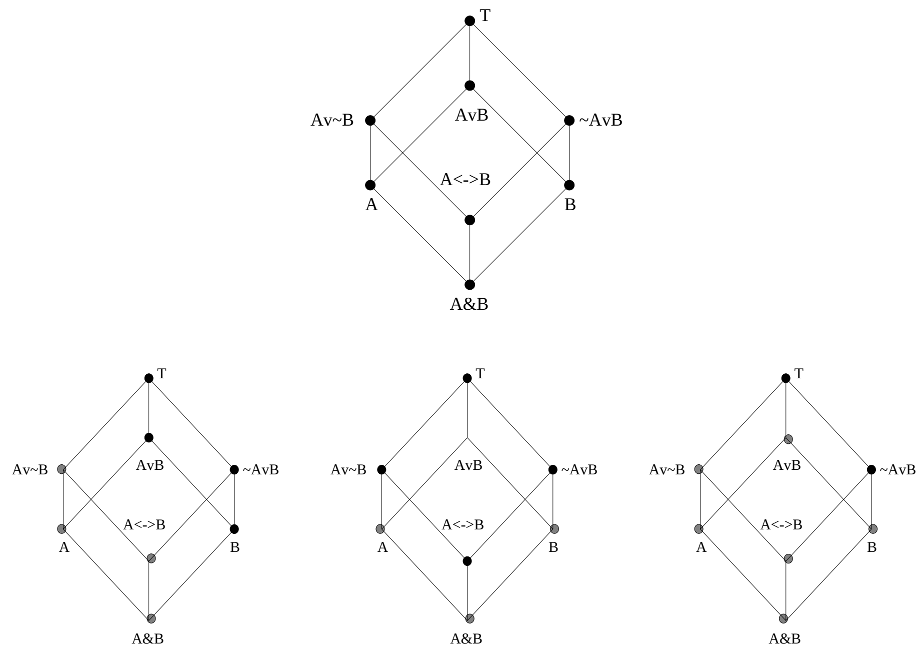
Contraction Example

Suppose K contains the closure of the following formulae:

$a \to b$
a
b
$c \to d$

Consider $K \dot{-} b$:

$a \to b$	a	c
c	c	$c \to d$
$c \to d$	$c \to d$	
c		



Belief Contraction

(K$\dot{-}$1)	For any sentence ϕ and any belief set K,	
	$K\dot{-}\phi$ is a belief set	(closure)
(K$\dot{-}$2)	$K\dot{-}\phi \subseteq K$	(inclusion)
(K$\dot{-}$3)	If $\phi \notin K$, then $K\dot{-}\phi = K$	(vacuity)
(K$\dot{-}$4)	If $\nvdash \phi$ then $\phi \notin K\dot{-}\phi$	(success)
(K$\dot{-}$5)	If $\phi \in K$, $K \subseteq (K\dot{-}\phi) + \phi$	(recovery)
(K$\dot{-}$6)	If $\vdash \phi \leftrightarrow \psi$, then $K\dot{-}\phi = K\dot{-}\psi$	(preservation)
(K$\dot{-}$7)	$K\dot{-}\phi \cap K\dot{-}\psi \subseteq K\dot{-}(\phi \wedge \psi)$	(conj. overlap)
(K$\dot{-}$8)	If $\phi \notin K\dot{-}(\phi \wedge \psi)$, then $K\dot{-}(\phi \wedge \psi) \subseteq K\dot{-}\phi$	(conj. inclusion)

Other properties

1. If $\phi \in K$, then $(K \dot{-} \phi) + \phi \subseteq K$
2. $K \dot{-} \phi = K \cap (K \dot{-} \phi) + \neg\phi$
3. If $\vdash \psi \rightarrow \phi$, then $K \dot{-} \phi \subseteq K \dot{-} \psi$.
4. $K \dot{-} (\phi \wedge \psi) = K \dot{-} \phi \cap K \dot{-} \psi$
5. $K \dot{-} \phi \cap Cn(\{\phi\}) \subseteq K \dot{-} (\phi \wedge \psi)$
6. Either $K \dot{-} (\phi \wedge \psi) \subseteq K \dot{-} \phi$ or $K \dot{-} (\phi \wedge \psi) \subseteq K \dot{-} \psi$
7. Either $K \dot{-} (\phi \wedge \psi) = K \dot{-} \phi$ or $K \dot{-} (\phi \wedge \psi) = K \dot{-} \psi$ or $K \dot{-} (\phi \wedge \psi) = K \dot{-} \phi \cap K \dot{-} \psi$
8. If $\psi \rightarrow \phi \in K \dot{-} \phi$ and $\phi \rightarrow \psi \in K \dot{-} \psi$, then $K \dot{-} \phi = K \dot{-} \psi$

Maximal Non-implying Subsets

Definition: K' is a maximal subset of K that fails to imply ϕ (a *ϕ-remainder*) iff

(i) $K' \subseteq K$

(ii) $\phi \notin K'$

(iii) for any $\psi \in K$ such that $\psi \notin K'$, $\psi \rightarrow \phi \in K'$

We denote by $K \perp \phi$ the set of all such maximal non-implying subsets.

Definition: A *selection function* $\gamma : 2^{\mathcal{K}} \rightarrow \mathcal{K}$ is a function such that for any $K \in \mathcal{K}$ and $\phi \in \mathcal{L}$, $\emptyset \neq \gamma(K \perp \phi) \subseteq K \perp \phi$ whenever $K \perp \phi \neq \emptyset$ and K otherwise. If γ always returns a singleton whenever $K \perp \phi \neq \emptyset$, then γ is referred to as an *opinionated selection function.*

Maxichoice Contraction

Idea: select the "best" element from $K \bot \alpha$ (minimal change).

Definition: Let γ be an opinionated selection function. A *maxichoice contraction function over* K may be defined as follows

(Def Max) $$K \dot{-} \phi = \begin{cases} \gamma(K \bot \phi) & \text{whenever } K \bot \phi \neq \emptyset \\ K & \text{otherwise} \end{cases}$$

Theorem: If $\dot{-}$ is a maxichoice contraction function over K, then it satisfies (K$\dot{-}$1) – (K$\dot{-}$6).

Theorem: If a revision function $*$ is obtained from a maxichoice contraction function $\dot{-}$ via the Levi Identity, then for any ϕ such that $\neg\phi \in K$, $K * \phi$ is *complete*.

Full Meet Contraction

Idea: All or nothing!

Definition: A *full meet contraction over* K may be defined as follows (Def Max) $$K \dot{-} \phi = \begin{cases} \bigcap(K \bot \phi) & \text{whenever } K \bot \phi \neq \emptyset \\ K & \text{otherwise} \end{cases}$$

Theorem: Any full meet contraction function satisfies (K$\dot{-}$1) – (K$\dot{-}$6)

Theorem: If a revision function $*$ is obtained from a full meet contraction function $\dot{-}$ via the Levi Identity, then for any ϕ such that $\neg\phi \in K$, $K * \phi = Cn(\phi)$.

Partial Meet Contraction

Idea: Compromise!

Definition: Let γ be a selection function. A *partial meet contraction over K* may be defined as follows

(Def Max) $$K\dot{-}\phi = \begin{cases} \bigcap \gamma(K\bot\phi) & \text{whenever } K\bot\phi \neq \emptyset \\ K & \text{otherwise} \end{cases}$$

Theorem: For every belief set K, $\dot{-}$ is a partial meet contraction function *iff* $\dot{-}$ satisfies (K$\dot{-}$1) – (K$\dot{-}$6).

Selection Functions — more details

Marking-off identitiy $\leq$
$\gamma(K\bot\phi) = \{K' \in K\bot\phi : K'' \leq K' \text{ for all } K'' \in K\bot\phi\}$

Definition: γ is a *transitively relational* iff it can be defined via a marking-off identity $\leq$ which is transitive.

Theorem:
For every belief set K, $\dot{-}$ is a transitively relational partial meet contraction function *iff* $\dot{-}$ satisfies (K$\dot{-}$1) – (K$\dot{-}$8).

Recovery

If $\phi \in K$, then $K = (K \dot{-} \phi) + \phi$

Counterexample?: (Hansson 1991)

George is a murderer (m)
George is a law breaker (b)
George is a tax evader (t)

$K = Cn(\{m\} \cup \{b\})$
$m \notin K \dot{-} b$
$\neg t \notin K \supseteq K \dot{-} b$
$K \subseteq Cn(K \dot{-} b \cup \{b\}) \subseteq Cn(K \dot{-} b \cup \{t\})$
$m \in Cn(K \dot{-} b \cup \{t\})$

Withdrawals (Makinson 1986)

Definition: A function $\dot{-}$ is a withdrawal function iff it satisfies postulates (K$\dot{-}$1) – (K$\dot{-}$4) and (K$\dot{-}$6) for contraction over K.

Definition: Two withdrawal functions $-$ and $\dot{-}$ are *revision equivalent* iff they generate the same revision function via the Levi Identity.

Theorem: Let K be any belief set. Then for each withdrawal operation $-$ on K, there is a unique contraction function $\dot{-}$ on K that is revision equivalent to $-$ and this $\dot{-}$ is the greatest element of $[-]$.

Saturatable Sets (Levi 1991)

Definition: K' is a saturatable contraction of K by ϕ iff

(i) $K' = Cn(K)$

(ii) $K' \subseteq K$

(iii) $Cn(K' \cup \{\neg\phi\})$ is maximally consistent in $\mathcal{L}$.

We denote by $K \perp\!\!\!\perp \phi$ the set of all such saturatable sets.

Why saturatable sets?

Levi: (Partial) meets of such saturatable sets characterise all ways of removing ϕ from a belief set K.

Belief Revision

(K*1)	For any sentence ϕ and any belief set K, $K * \phi$ is a belief set	(closure)
(K*2)	$\phi \in K * \phi$	(success)
(K*3)	$K * \phi \subseteq K + \phi$	(inclusion)
(K*4)	If $\neg\phi \notin K$, then $K + \phi \subseteq K * \phi$	(preservation)
(K*5)	$K * \phi = K_\perp$ if and only if $\vdash \neg\phi$	(vacuity)
(K*6)	If $\vdash \phi \leftrightarrow \psi$, then $K * \phi = K * \psi$	(extensionaliity)
(K*7)	$K * \phi \wedge \psi \subseteq (K * \phi) + \psi$	(super expansion)
(K*8)	If $\neg\psi \notin K * \phi$, then $(K * \phi) + \psi \subseteq K * (\phi \wedge \psi)$	(sub expansion)

Further Properties

1. If $\phi \in K$, then $K * \phi = K$
2. $K * \phi = (K \cap K * \phi) + \phi$
3. $K * \phi = K * \psi$ if and only if $\psi \in K * \phi$ and $\phi \in K * \psi$
4. $K * \phi \cap K * \psi \subseteq K * (\phi \vee \psi)$
5. If $\neg\psi \notin K * (\phi \vee \psi)$, then $K * (\phi \vee \psi) \subseteq K * \psi$
6. $K * (\phi \vee \psi) = K * \phi$ or $K * (\phi \vee \psi) = K * \psi$ or $K * (\phi \vee \psi) = K * \phi \cap K * \psi$

Summary

Belief contraction

First construction (maximal non-implying subsets)

Recovery postulate

Withdrawal functions

Belief revision