Belief Change via Preference

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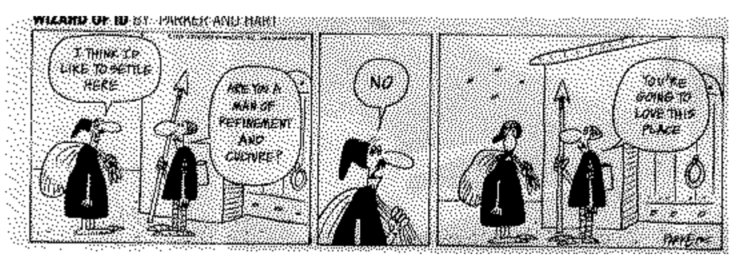


Overview

Epistemic Entrenchment

Belief change via entrenchment

Systems of Spheres

Belief change via systems of spheres

Alternative forms of belief contraction

Summary

Epistemic Entrenchment

Ordering over formulae in $\mathcal{L}$

Certain beliefs about world are more important than others when planning future actions, etc.

$\phi \leq \psi$: ψ is at least as *epistemically entrenched* as ϕ

In contraction, sentences in K with lower entrenchment given up

Tautologies maximally entrenched, non-beliefs minimally ent.

But what does an epistemic entrenchment relation loook like?

Epistemic Entrenchment

(EE1)	If $\phi \leq \psi$ and $\psi \leq \gamma$ then $\phi \leq \gamma$	(transitivity)
(EE2)	If $\{\phi\} \vdash \psi$ then $\phi \leq \psi$	(dominance)
(EE3)	For any ϕ and ψ, $\phi \leq \phi \wedge \psi$ or $\psi \leq \phi \wedge \psi$	(conjunctiveness)
(EE4)	When $K \neq K_{\perp}$, $\phi \in K$ iff $\phi \leq \psi$ for all ψ	(minimality)
(EE5)	If $\phi \leq \psi$ for all ϕ then $\vdash \psi$	(maximality)

Further Properties

1. $\phi \leq \psi$ or $\psi \leq \phi$ (connectedness)
2. If $\psi \wedge \chi \leq \phi$, then $\phi \leq \phi$ or $\chi \leq \phi$
3. $\phi < \psi$ iff $\phi \wedge \psi < \psi$
4. If $\chi \leq \phi$ and $\chi \leq \psi$, then $\chi \leq \phi \wedge \psi$
5. If $\phi \leq \psi$, then $\phi \leq \phi \wedge \psi$
6. $\phi \wedge \psi = min(\phi,\ \psi)$
7. $\phi \vee \psi \geq max(\phi,\ \psi)$

(NB: $\phi < \psi \equiv \psi \not\leq \phi$)

Belief Change via Entrenchment

(Gärdenfors and Makinson 1988)

$(C \leq)$ $\phi \leq \psi$ iff $\phi \notin K \dot{-} (\phi \wedge \psi)$ or $\vdash \phi \wedge \psi$

$(C \dot{-})$ $\psi \in K \dot{-} \phi$ iff $\psi \in K$ and either $\phi < \phi \vee \psi$ or $\vdash \phi$

$(C*)$ $\psi \in K * \phi$ iff either $\neg\phi \leq \neg\phi \vee \psi$ or $\vdash \neg\phi$

Theorem: If an ordering $\leq$ satisfies (EE1) – (EE5), then the contraction function which is uniquely determined by $(C \dot{-})$ satisfies (K$\dot{-}$1) – (K$\dot{-}$8) as well as condition $(C \leq)$.

Theorem: If a contraction function $\dot{-}$ satisfies (K$\dot{-}$1) – (K$\dot{-}$8), then the ordering that is uniquely determined by $(C \leq)$ satisfies (EE1) – (EE8) as well as condition $(C \dot{-})$.

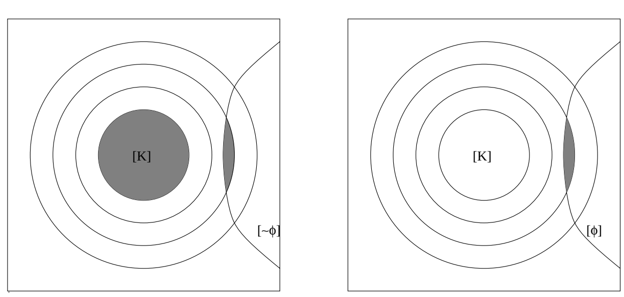
Grove's Spheres

Ordering over "worlds" (maximally consistent sets)

Motivated by Lewis' sphere semantics for counterfactuals

$[K]$ set of worlds consistent with K

System of spheres centred on $[K]$



Systems of Spheres

Definition: Let $\mathcal{S}$ be any collection of subsets of $\mathcal{M}_{\mathcal{L}}$. We call $\mathcal{S}$ a system of spheres, centred on $X \subseteq \mathcal{M}_{\mathcal{L}}$, if it satisfies the following conditions:

($\mathcal{S}$1) $\mathcal{S}$ is totally ordered by $\subseteq$; that is, if $U,\ V \in \mathcal{S}$, then $U \subseteq V$ or $V \subseteq U$

($\mathcal{S}$2) X is the $\subseteq$-minimum of $\mathcal{S}$

($\mathcal{S}$3) $\mathcal{M}_{\mathcal{L}}$ is the $\subseteq$-maximum of $\mathcal{S}$

($\mathcal{S}$4) If $\phi \in \mathcal{L}$ and $\not\vdash \neg\phi$, then there is a smallest sphere in $\mathcal{S}$ intersecting $[\phi]$ (i.e., there is a sphere $U \in \mathcal{S}$ such that $U \cap [\phi] \neq \emptyset$, and $V \cap [\phi] \neq \emptyset$ implies $U \subseteq V$ for all $V \in \mathcal{S}$)

Belief Change via SOSs

$c_{\mathcal{S}}(\phi)$ — the smallest sphere interesecting $[\phi]$

$f_{\mathcal{S}}(\phi) = c_{\mathcal{S}}(\phi) \cap [\phi]$ — innermost ϕ-worlds

Theorem: Let $\mathcal{S}$ be any system of spheres in $\mathcal{M}_{\mathcal{L}}$ centred on $[K]$ for some theory $K \in \mathcal{K}$. If one defines, for any $\phi \in \mathcal{L}$, $K * \phi$ to be $th(f_{\mathcal{S}}(\phi))$, then the axioms (K*1) – (K*8) are satisifed.

Theorem: Let $* : \mathcal{K} \times \mathcal{L} \to \mathcal{K}$ be any function satisfying axioms (K*1) – (K*8). Then for any (fixed) theory K there is a system of spheres on $\mathcal{M}_{\mathcal{L}}$, $\mathcal{S}$ say, centred on $[K]$ and satisfying $K * \phi = th(f_{\mathcal{S}}(\phi))$ for all $\phi \in \mathcal{L}$.

Properties of th (Grove 1988)

$th : 2^{\mathcal{M}_\mathcal{L}} \rightarrow \mathcal{K}$

(i) $th([K]) = K$ for all belief sets (i.e., theories) K if the underlying logic is compact

(ii) $th(X) \neq K_\perp$ if and only if X is nonempty

(iii) For any sentence $\phi \in \mathcal{L}$ and $X \subseteq \mathcal{M}_\mathcal{L}$,
$th(X \cap [\phi]) = Cn(th(X) \cup \{\phi\})$

(iv) For $X,\ X' \subseteq \mathcal{M}_\mathcal{L}$, if $X \subseteq X'$, then $th(X') \subseteq th(X)$

(v) For $K,\ K' \in \mathcal{K}$, if $K \subseteq K'$, then $[K'] \subseteq [K]$

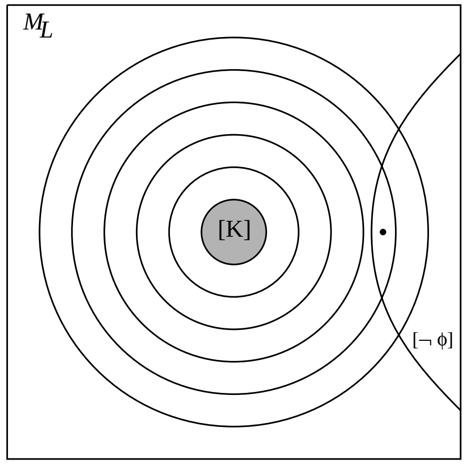
SOS $\Leftrightarrow$ EE

(Gärdenfors 1988)

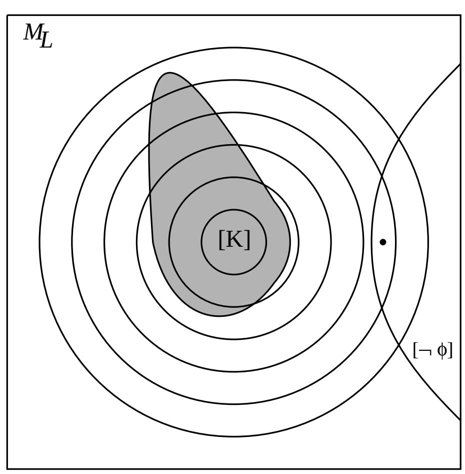
Can translate back and forth from a Systems of Spheres $\mathcal{S}$ and an epistemic entranchment relation $\leq$ using the following condition:

$\phi \leq \psi$ iff $c_\mathcal{S}(\neg\phi) \subseteq c_\mathcal{S}(\neg\psi)$

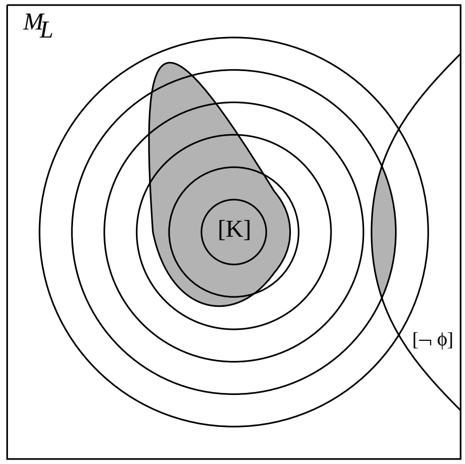
Maxichoice Contraction



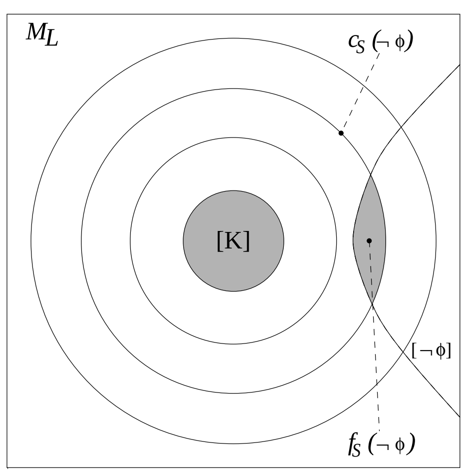
Saturatable set



Partial Meet of Saturatable Sets

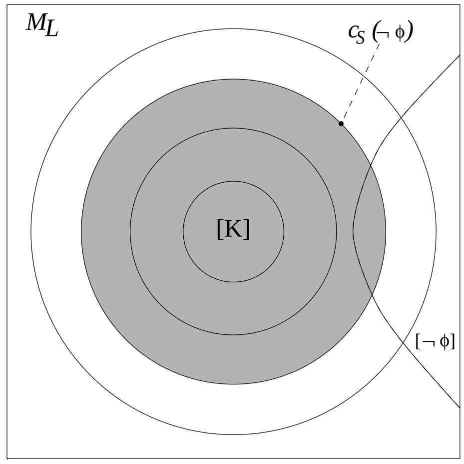


AGM Partial Meet Contraction

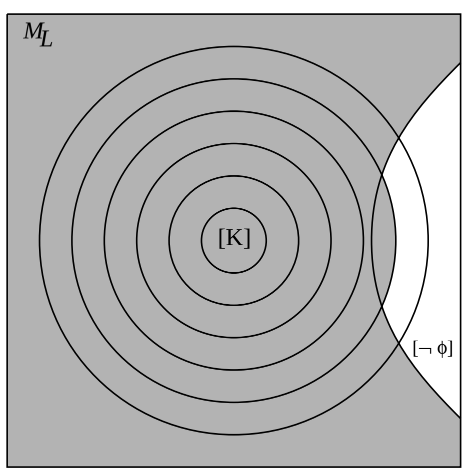


Severe Withdrawal (Rott & Pagnucco 1996)

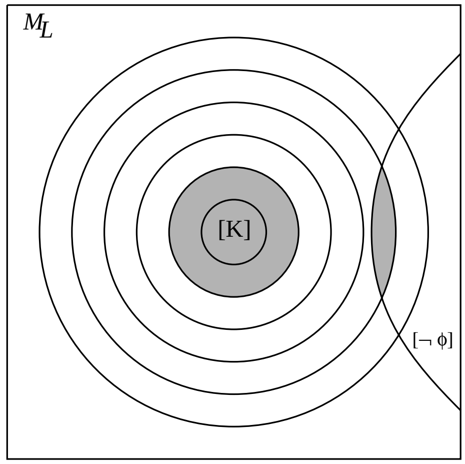
Also Levi 1996



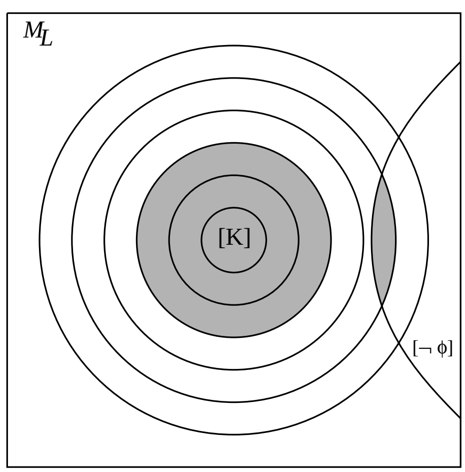
"Iron-fisted" withdrawal



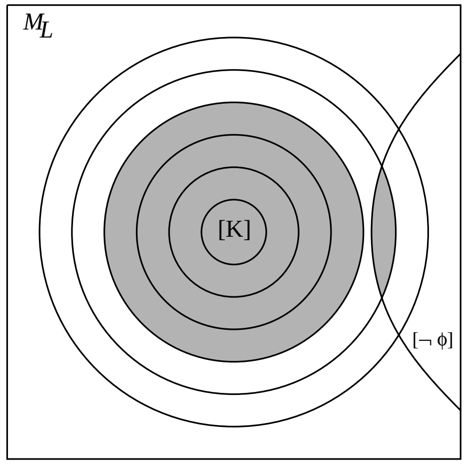
Levi Contraction - Damped Inf. Value 1



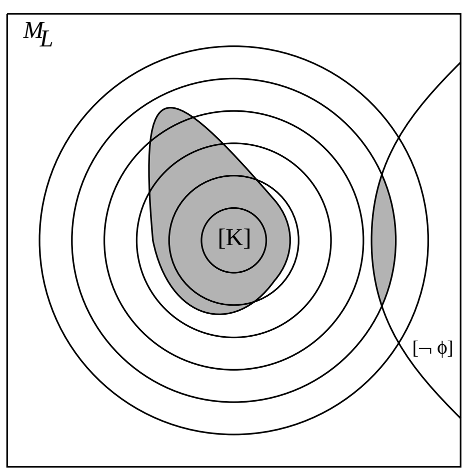
Cantwell “fallback-based”



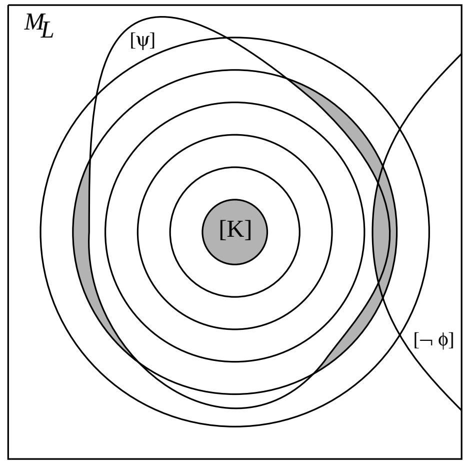
Meyer *et al.* systematic withdrawal 1997



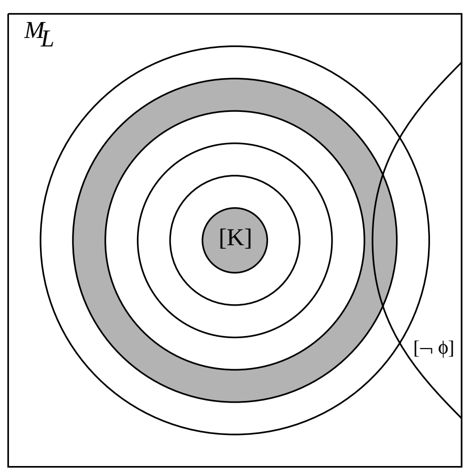
Lindström and Rabinowicz (Interpolation) 1991



Fermé and Rodriguez semi-contraction 1997



Nayak (Personal Communication)



	$[K \dot{-} \phi]$	... within $[\neg\phi]$	... within $[\phi]$
1.	AGM (trans. rel.) partial meet	$[\neg\phi] \cap S_\phi$	$[K]$
2.	Severe withdrawal	$[\neg\phi] \cap S_\phi$	$[\phi] \cap S_\phi$
3.	AGM maxichoice	single $\neg\phi$-world	$[K]$
4.	Saturatable set)	single $\neg\phi$-world	some X s.th. $[K] \subseteq X \subseteq [\phi]$
5.	Partial meet of saturatable sets	$[\neg\phi] \cap S_\phi$	some X s.th. $[K] \subseteq X \subseteq [\phi]$
6.	Iron-fisted withdrawal	$[\neg\phi] \cap S_\phi$	$[\phi]$
7.	Levi – damped type 1	$[\neg\phi] \cap S_\phi$	$[\phi] \cap S_2$
8.	Cantwell fallback-based	$[\neg\phi] \cap S_\phi$	$[\phi] \cap S_i$ for some $i \in \{1, \ldots, n\}$
9.	Systematic withdrawal	$[\neg\phi] \cap S_\phi$	$[\phi] \cap S_{\phi-1}$
10.	Lindström and Rabinowicz	$[\neg\phi] \cap S_\phi$	some X s.th. $[K] \subseteq X \subseteq [\phi] \cap S_\phi$
11.	Semi-contraction	$[\neg\phi] \cap S_\phi$	some X s.th. $[K] \subseteq X \subseteq [\phi] \cap S_\phi$
12.	Nayak [p.c.]	$[\neg\phi] \cap S_\phi$	$[\phi] \cap (S_\phi - S_{\phi-1})$

Summary

Construction in terms of ordering over formulae

Construction in terms of ordering over worlds

Other serious possiblities for belief removal operations