Belief Change and Nonmonotonic Reasoning

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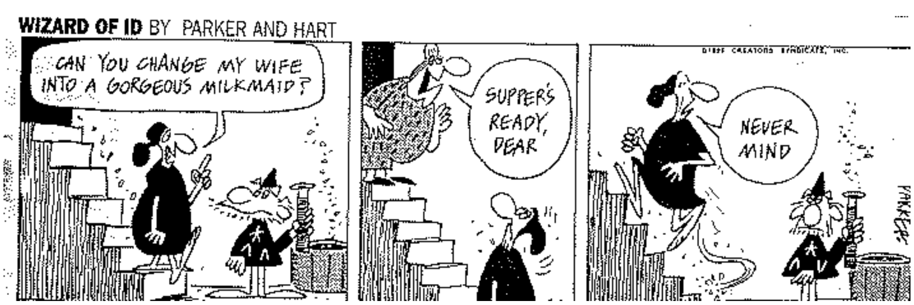


Overview

Nonmonotonic reasoning

Reiter's default logic

KLM approach to nonmonotonic consequence

Belief change and nonmonotonic consequence

Ramsey test

Conditionals

Gärdenfors triviality result

Summary

Nonmonotonicity

Classical logic satisfies the following property

Monotonicity: If $\Delta \subseteq \Gamma$, then $Cn(\Delta) \subseteq Cn(\Gamma)$
(equivalently, $\Gamma \vdash \phi$ implies $\Gamma \cup \Delta \vdash \phi$)

However, we often draw conclusions based on 'what is normally the case' or 'true by default'

More information can lead us to retract previous conclusions

Example

Suppose I tell you ‘Tweety is a bird’

You might conclude ‘Tweety flies’

I then tell you ‘Tweety is an emu’

You conclude ‘Tweety does not fly’

$bird(Tweety) \mid\!\sim flies(Tweety)$

$bird(Tweety) \wedge emu(Tweety) \mid\!\sim \neg flies(Tweety)$

Reiter’s Default Logic (1980)

Add default rules of the form $\frac{\alpha:\beta}{\gamma}$

“If α can be proven and it is consistent to assume β, then conclude γ”

Often consider *normal* default rules $\frac{\alpha:\beta}{\beta}$

Example: $\frac{bird(x):(flies(x)}{flies(x)}$

Default theory $\langle D,\ W\rangle$
  D – set of defaults; W – set of facts

Extension of default theory contains as many default conclusions as possible and must be consistent

Examples

$W = \{\}$; $D = \{\frac{:p}{\neg p}\}$ – no extensions

$W = \{p \vee r\}$; $D = \{\frac{p:q}{q},\ \frac{r:q}{q}\}$ – one extension $(p \vee r)$

$W = \{emu(Tweety),\ emu(x) \rightarrow bird(x)\}$ $\frac{bird(x):(flies(x)}{flies(x)}$ – one extension

What if we add $\frac{emu(x):\neg flies(x)}{\neg flies(x)}$

(See also Poole 1988 for default logic)

Nonmonotonic Consequence

Abstract study of nonmonotonic consequence relation $\mid\!\sim$ in terms of general properties (KLM 91)

Some common properties include:

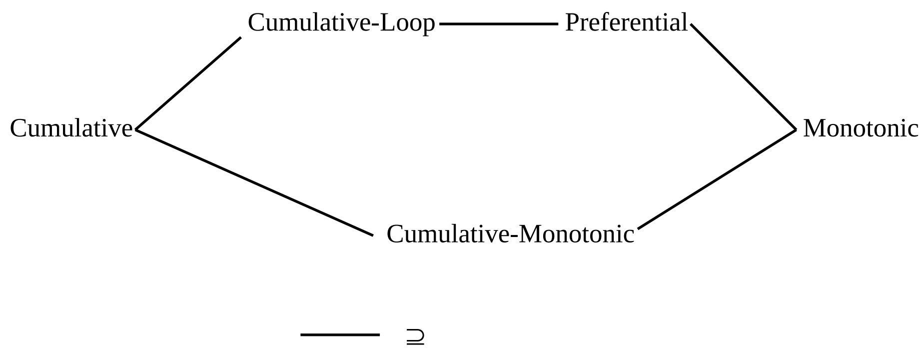
Supraclassicality If $\phi \vdash \psi$, then $\phi \mid\!\sim \psi$

Left Logical Equivalence If $\phi \leftrightarrow \psi$ and $\phi \mid\!\sim \chi$, then $\psi \mid\!\sim \chi$

Right Weakening If $\vdash \psi \rightarrow \chi$ and $\phi \mid\!\sim \psi$, then $\phi \mid\!\sim \chi$

And If $\phi \mid\!\sim \psi$ and $\phi \mid\!\sim \chi$, then $\phi \mid\!\sim \psi \wedge \chi$

KLM Systems



Belif Change and NMR

Can define $\phi \mid\!\sim_K \psi$ iff $\psi \in K * \phi$

(K*1) = If $\phi \mid\!\sim \psi_i$ for all $\psi_i \in K$ and $K \vdash \chi$, then $\phi \mid\!\sim \chi$ (Closure)
(K*2) = $\phi \mid\!\sim \phi$ (Reflexivity)
(K*3) = If $\phi \mid\!\sim \psi$, then $\top \mid\!\sim \phi \rightarrow \psi$ (Weak Conditionalisation)
(K*4) = If $\top \not\mid\!\sim \neg\phi$ and $\top \mid\!\sim \phi \rightarrow \psi$, then $\phi \mid\!\sim \psi$ (Weak Rational Monotony)
(K*5) = If $\phi \mid\!\sim \bot$, then $\phi \vdash \bot$ (Consistency Preservation)
(K*6) = LLE
(K*7) = If $\phi \wedge \psi \mid\!\sim \chi$, then $\phi \mid\!\sim \psi \rightarrow \chi$ (Conditionalisation)
(K*8) = If $\phi \not\mid\!\sim \neg\psi$ and $\phi \mid\!\sim \chi$, then $\phi \wedge \psi \mid\!\sim \chi$ (Rational Monotony)

Note that $\mid\!\sim$ is with respect to a particular K

Belief Change and Defaults

Can view normal defaults $\frac{\alpha:\beta}{\beta}$ as $\frac{\top:\alpha\rightarrow\beta}{\alpha\rightarrow\beta}$

Can encode such defaults in epistemic entrenchment as
$\alpha \rightarrow \neg\beta < \alpha \rightarrow \beta$

'Given α, prefer β to $\neg\beta$'

$bird(Tweety) \rightarrow \neg flies(Tweety) < bird(Tweety) \rightarrow flies(Tweety)$

Conditionals

Subjunctive conditionals (verb perfect)
'If Oswald hadn't killed Kennedy, someone else would have'

Indicative conditionals
'If Oswald didn't kill Kennedy, someone else did'

Is it possible to give truth conditions to conditionals?

Ramsey Test

One suggestion as to how to determine truth of conditionals

'An agent should accept the condition $\phi \Rightarrow \psi$ if ψ is accepted after a (hypothetical) minimal change to its stock of beliefs so as to incorporate ϕ'

Gärdenfors investigated the introduction of a conditional $\Rightarrow$ into the language for specifying belief sets

Gärdenfors Triviality Result

Define conditional as follows: $\phi \Rightarrow \psi$ iff $\psi \in K * \phi$

A consequence of this is, when $\not\vdash \phi$, if $K \subseteq H$, then $K * \phi \subseteq H * \phi$

A language is *non-trivial* iff it has at least three pairwise disjoint sentences

Triviality result: There is no non-trivial language which includes conditional sentences as defined above for a $*$ satisfying the AGM postulates

Summary

Nonmonotonic reasoning attempts to capture commonsense reasoning

Nonmonotonic reasoning often deals with inferences based on defaults or 'what is usually the case'

Belief change and nonmonotonic reasoning two sides of the same coin?

Can introduce abstract study of nonmonotonic consequence relations in same way as we study classical consequence relations

Similar links exist with conditionals