Abductive Expansion: The Application of Abductive Inference to the Process of Belief Change

Abstract

Unlike the AGM framework of belief revision, where new information is added to the current belief state as it is (while preserving logical consistency), we model the following more natural proposal: *agents seek a justification (explanation) of the received information before "accepting" it and what is incorporated into the current belief state is actually the received information together with the justification that the agent comes up with.* We model this proposal using the process of *abductive inference.* The resulting process is called *abductive expansion* and is characterized by a set of intuitive postulates. A construction is offered which may be seen as a dual of the AGM construction for belief change based on a selection function. A representation theorem is proved to show the correspondence between the postulates and the construction. It is shown that traditional AGM expansion is a special case of abductive expansion.

Keywords: Abductive reasoning, Belief revision, Common sense reasoning.

1 Introduction

Recently, the area of belief revision has commanded increasing attention. The general aim of this enterprise is to model how an agent's beliefs change when presented with new information. Belief change is commonly represented as a transition from an initial belief (epistemic) state K to an updated belief state K'_α with respect to some new information (an *epistemic input*) α. Belief states (K, K'_α) are commonly represented by consistent sets of formulae while the epistemic input (α) is usually represented by a single formula.

In the belief revision framework developed by Alchourrón, Gärdenfors and Makinson [1][3] (henceforth referred to as the AGM framework) three types of change or transition are identified:

expansion (K^+_α) introduction of a new belief α to K without retracting any previous beliefs;

contraction (K^-_α) removal of a belief α from a belief state K;

revision (K^*_α) introduction of a new belief α which may involve removal of existing beliefs from the belief state K.

In the situation outlined above, the resulting belief state K'_α would be identified with one of K^+_α, K^-_α or K^*_α. An important consideration, central to the AGM framework, is the principle of minimal change — the new belief state should differ minimally (within reasonable limits) from the original belief state in achieving the desired change[1].

The other topic with which we are concerned in this paper is that of abductive reasoning [2][7]. Abductive reasoning (or simply abduction) is a form of logical inference that attempts to derive "explanations" for a body of data. Often we are interested in determining a best "explanation" or abduction from among the alternatives. It has been argued that abduction provides a logic of scientific discovery [9]. In fact, it is sometimes referred to as "inference to the best explanation" [6].

An abductive inference proceeds by "proposing" or generating hypotheses which would account for or "explain" a group of facts (often considered observations). For example, suppose we know that someone suffering from measles develops a red rash on their skin. If you notice that someone has a rash on their skin you might hypothesise that they are suffering from measles for this would account for your observation. There may of course be other explanations; the person in question may be suffering from some form of allergy, for example.

The aim of this paper is to investigate certain aspects of the role of abductive reasoning within the process of belief revision. Our motivation stems from the following claim: often when we modify our belief state upon receipt of new information we do not just simply add the new information as it stands but, rather, first seek some type of justification and proceed by adding this justification to our belief state (and thus also obtaining belief in the new information).

To continue our example, if someone comes to you with a rash on their skin you are more likely to add to your beliefs the fact that they are suffering from measles (or maybe some more plausible alternative) rather than simply adding the fact they have a red rash (especially if you are a doctor). In this

[1] If belief states are represented using sets of formulae then minimality is usually taken with respect to set inclusion.

paper we shall develop a framework incorporating the notions of abductive reasoning and belief revision in an attempt to model this claim.

A very similar approach has been advocated by Kakas and Mancarella [8] who apply it to database updates. They argue that updating in this way may lead to the explicit representation of information that would otherwise be implicit and that could be lost if the database is only revised with respect to the original information. Furthermore, if a database is structured in a particular way then it may help to "transform" new information into a form reflecting this structure.

In the following section we briefly outline the nature of the language and the logic assumed in this paper. Section 3 presents a brief logical exposition of the abductive process to be used (further details may be found in another report by the authors). A brief introduction to the AGM framework for belief revision is provided in Section 4 to set the tone for the ensuing discussion. In Section 5 we present a number of intuitive postulates to characterise the process of abductive expansion. A construction in terms of selection functions, mimicking one commonly investigated for AGM contraction and revision, is also provided together with relevant representation theorems.

2 Preliminaries

We shall consider a fixed language $\mathcal{L}$ consisting of the standard logical connectives $\neg$, $\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$ and the symbol $\bot$ (falsum). We shall adopt the convention that lower case Greek letters $\alpha, \beta, \gamma \ldots$ denote formulae of $\mathcal{L}$ while upper case Roman letters $A, B, C, \ldots$ and upper case Greek letters $\Delta, \Gamma, \ldots$ denote sets of formulae.

A *consequence operator* is a function $Cn : 2^{\mathcal{L}} \rightarrow 2^{\mathcal{L}}$ with the following properties:

(i) $A \subseteq Cn(A)$ (inclusion)
(ii) If $A \subseteq B$, then $Cn(A) \subseteq Cn(B)$ (monotonicity)
(iii) $Cn(A) = Cn(Cn(A))$ (iteration)

Furthermore, we assume that Cn satisfies the following conditions:

(iv) If α can be derived from A by classical truth-functional logic, then $\alpha \in Cn(A)$ (supraclassicality)
(v) $\beta \in Cn(A \cup \{\alpha\})$ if and only if $(\alpha \rightarrow \beta) \in Cn(A)$ (deduction)
(vi) If $\alpha \in Cn(A)$, then $\alpha \in Cn(A')$ for some finite subset $A' \subseteq A$ (compactness)

A *theory* is a set of propositions closed under Cn. A set of propositions A is *consistent* if and only if for no proposition α do we have $\alpha \wedge \neg\alpha \in Cn(A)$[2]. We often write $A \vdash \alpha$ for $\alpha \in Cn(A)$.

We make no further assumptions regarding the nature of $\mathcal{L}$ or the underlying logic. In the examples presented however, we shall assume a language based upon a propositional logic.

3 Abduction

We shall begin with a fairly standard definition of abduction commonly found in the literature (see, for example, [7] [10]).

Definition 3.1 *An abduction of a formula α with respect to a domain theory Γ is a set of formulae E such that:*

(i) $\Gamma \cup E \vdash \alpha$;

(ii) $\Gamma \cup E$ *is consistent (i.e.,* $\Gamma \cup E \nvdash \bot$*).*

For example, suppose $\Gamma = \{measles \rightarrow rash,\ allergic_reaction \rightarrow rash,\ allergen \wedge allergy_sufferer \rightarrow allergic_reaction\ \}$ and $\alpha = rash$ then possible abductions include $measles$, $allergic_reaction$, $measles \wedge allergic_reaction$ and $allergen \wedge allergy_sufferer$.

Some definitions ([7] for example) consider abductions of a set of formulae Δ instead of a single formula α as we have done. We shall concentrate on abductions of a single formula here as this would seem a more fundamental consideration even though, in actual fact, the two approaches are quite similar.

The following observation is an immediate consequence of the compactness of Cn.

Observation 3.1 *If an abduction E of a formula α with respect to a domain theory Γ exists then a finite abduction $E' \subseteq E$ (where E' is understood to be finite) of α with respect to Γ exists.*

Moreover, the following result is easy to prove.

Observation 3.2 *If a finite abduction E of α with respect to Γ exists then it can be represented by a single formula β (i.e.,* $\Gamma \cup \{\beta\} \vdash \alpha$, $\Gamma \cup \{\beta\} \nvdash \bot$*).*

This motivates the following revised definition of abduction which we shall be using in the following sections:

[2] In the following the inconsistent theory will be denoted by $\mathcal{L}$ or $K_\bot$ (the inconsistent belief set).

Definition 3.2 *An abduction of a formula α with respect to a domain theory Γ is a formula β such that:*

(i) $\Gamma \cup \{\beta\} \vdash \alpha$*;*

(ii) $\Gamma \cup \{\beta\}$ *is consistent (i.e.,* $\Gamma \cup \{\beta\} \nvdash \bot$*).*

4 AGM Framework of Belief Change

The AGM postulates for expansion, contraction and revision attempt to characterise classes of functions for modelling the manner in which a rational agent alters its beliefs. Normally this framework represents belief states in terms of deductively closed sets of formulae called *belief sets* and epistemic inputs in terms of a formula α. We denote the set of all possible belief sets by **K**. The language used is similar to that presented in Section 2.

In this presentation, as we shall soon see, our main concern is with the process of expansion so, for completeness, we reproduce the six basic expansion postulates (for their motivation see [3]):

(K$^+$1)	K^+_α is a belief set	(closure)
(K$^+$2)	$\alpha \in K^+_\alpha$	(success)
(K$^+$3)	$K \subseteq K^+_\alpha$	(inclusion)
(K$^+$4)	If $\alpha \in K$, then $K^+_\alpha = K$	(vacuity)
(K$^+$5)	If $K \subseteq H$, then $K^+_\alpha \subseteq H^+_\alpha$	(monotonicity)
(K$^+$6)	For all belief sets K and all sentences α, K^+_α is the smallest belief set that satisfies (K$^+$1) — (K$^+$5)	(minimality)

Furthermore, the following important representation theorem [1] characterises the process of expansion.

Theorem 4.1 *The expansion function + satisfies (K$^+$1) — (K$^+$6) iff* $K^+_\alpha = Cn(K \cup \{\alpha\})$.

5 Abductive Expansion Framework

Our aim is to model the claim that agents often seek some justification or "explanation" for newly acquired information and add this justification to their belief state. We propose to model this process using the notion of abductive inference presented in Section 3. Instead of merely adding an epistemic input (newly acquired information) to a belief state using the principle of minimal change we shall present a framework where the agent adds an abduction of the epistemic input (α) to its belief state (K). Since we shall be modelling

belief states using belief sets, as in the AGM framework, the epistemic input will also appear in the new belief state (if possible).

From the definition of abduction given in Section 3 we note that an abduction must be consistent with the domain theory which in this case is represented by the initial belief state K. Therefore, since the formula we shall be attempting to add to our belief state is consistent with it, then we really only need consider the process of expansion. We shall refer to this process as the abductive expansion of K with respect to α and denote it $K^{\oplus}_{\alpha}$. We begin by presenting a series of postulates to characterise this process.

5.1 Postulates for Abductive Expansion

An abductive expansion function $\oplus$ is assumed to be a function from pairs of beliefs sets and formulae to belief sets ($\oplus : \mathbf{K} \times \mathcal{L} \rightarrow \mathbf{K}$). It determines what belief state an agent would be in given that its current belief state is K and it has acquired new information α. This is captured by the following postulate:

($\mathrm{K}^{\oplus}1$) $K^{\oplus}_{\alpha}$ is a belief set (closure)

If the newly acquired information does not conflict with what is currently believed then any explanation or justification accepted should be for this new information. That is, the new information should be accepted in the resulting belief state. Hence the following postulate:

($\mathrm{K}^{\oplus}2$) If $\neg\alpha \notin K$, then $\alpha \in K^{\oplus}_{\alpha}$ (limited success)

During the process of abductive expansion an agent is augmenting its currently held beliefs. In the limiting case where the new infomation conflicts with the currently held beliefs no action is taken in order to maintain consistency. In this way, priority is given to established beliefs over new information (a skeptical agent!). In the conclusion we discuss how such cases may be handled. Abductive expansion cannot result in the retraction of currently held beliefs thus leading to the following postulate:

($\mathrm{K}^{\oplus}3$) $K \subseteq K^{\oplus}_{\alpha}$ (inclusion)

On occasion it may not be possible to find an abduction (i.e., a consistent "explanation") for the epistemic input. This occurs when the input does not agree with currently held beliefs. In such a case, a rational agent would refuse to accept the new information[3] as it would lead to inconsistency (the situation is actually more complex and may be more appropriately handled by abductive revision which is briefly discussed in the conclusion).

[3]Note the departure from AGM expansion where inconsistent information may be added to the current state of belief although it leads to the inconsistent belief set $K_{\perp}$. We have

($K^{\oplus}4$) If $\neg\alpha \in K$, then $K^{\oplus}_{\alpha} = K$ (vacuity)[4]

Whenever it is possible to find an explanation for new information the resulting belief state should remain consistent. That is, the explanation process should not introduce any inconsistencies. This will be the case if the expanded belief state remains consistent with the new information.

($K^{\oplus}5$) If $\neg\alpha \notin K$, then $\neg\alpha \notin K^{\oplus}_{\alpha}$ (consistency)

For reasons that will soon become apparent, we shall refer to ($K^{\oplus}1$) — ($K^{\oplus}5$) as the basic abductive expansion postulates. Another postulate that may be useful is:

($K^{\oplus}6$) If $\vdash \alpha \leftrightarrow \beta$, then $K^{\oplus}_{\alpha} = K^{\oplus}_{\beta}$ (preservation)

This postulate expresses the fact that it is the content of the input rather than its linguistic formulation that is important in determining the change in belief state[5]. In effect, it guarantees a well-behaved abductive inference procedure in the sense that it will not act randomly in selecting which is the "best" abduction.

The following theorem tells us that postulates ($K^{\oplus}1$)—($K^{\oplus}5$) capture the notion of expanding our belief state by an abduction[6] for epistemic input α with respect to the current belief state K which is what we desire.

Theorem 5.1 *The expansion function $\oplus$ satisfies ($K^{\oplus}1$) — ($K^{\oplus}5$) iff*

$$K^{\oplus}_{\alpha} = \begin{cases} Cn(K \cup \{\beta\}) & \text{for some } \beta \in \mathcal{L} \text{ such that:} \\ & \quad \text{(i) } \alpha \in Cn(K \cup \{\beta\}) \\ & \quad \text{(ii) } Cn(K \cup \{\beta\}) \neq \mathcal{L} \\ K & \text{if no such } \beta \text{ exists}[7] \end{cases}$$

chosen to place our emphasis on maintaining consistency which, seems to be in agreement with the spirit of abduction, rather than guaranteeing the incorporation of the epistemic input (as is also evidenced by ($K^{\oplus}2$)).

[4]This condition may be too naive. In future we hope to combine this account of expansion with an appropriate account of contraction to generate abductive revision via the Levi identity (see Section 6). The resulting abductive revision function will not possess this oddity.

[5]See also remarks in [3] p.55 regarding this property of contraction and revision.

[6]Note that conditions (i) and (ii) correspond exactly to those in Definition 3.2.

Note that only ($K^{\oplus}1$) — ($K^{\oplus}5$) are required to model the notion that we are trying to capture. Of course, since ($K^{\oplus}1$) — ($K^{\oplus}5$) satisfy the given definition of $K_{\alpha}^{\oplus}$ then ($K^{\oplus}1$) — ($K^{\oplus}6$) will also satisfy it. We prefer to consider the six postulates presented since, if only the first five are satisfied, then the abductive inference procedure may behave non-deterministically.

5.2 Constructions

In the AGM framework for belief revision there are three major constructions investigated for the processes of contraction and revision: selection functions [1][3], Grove's system of spheres [5] and epistemic entrenchment [4]. In this paper, due to limitations on space, we shall only investigate a construction based on the first of these and briefly mention constructions based on the other two, deferring a proper investigation to a future work. The construction provided will present analogous selection functions to that investigated in the AGM framework.

5.2.1 Abductive Expansion Functions

In the case where α, the epistemic input, is consistent with our belief set K the resulting (abductively) expanded belief set will be a superset of K containing α. Therefore, a choice has to be made from among the supersets of K that imply α. However, it is easily noted that every such superset can be represented as the intersection of a set of maximally consistent supersets of K that imply α. Accordingly we define:

Definition 5.1 *(Maximally Consistent Superset that Implies α)*
A set K' is a maximally consistent superset (of a set K) that implies α *if and only if*

(i) $K \subseteq K'$

(ii) $\alpha \in Cn(K')$[8]

(iii) $Cn(K') \neq K_{\perp}$

[7] An alternative idea is to make $K_{\alpha}^{\oplus} = K_{\perp}$ if no β satisfying the stated criteria exists. This would give us a closer correspondence to AGM expansion in the case where $\neg\alpha \in K$. However, as we have mentioned, we have decided to place more emphasis on maintaining logical consistency rather than guaranteeing the inclusion of the epistemic input. This would also alter ($K^{\oplus}4$) to *If* $\neg\alpha \in K$, *then* $K_{\alpha}^{\oplus} = K_{\perp}$ and guarantee success ($K^{\oplus}2$): $\alpha \in K_{\alpha}^{\oplus}$.

[8] This could in fact be replaced by $\alpha \in K'$ since K' is maximal and will therefore be closed (see Observation 5.2) but since we are not explicitly assuming K' to be closed we have chosen to express it in this manner.

(iv) There is no $K'' \supset K$ satisfying (i), (ii) and (iii)[9]

We use $K \top \alpha$[10] to denote the set of all the belief sets K' that are maximal consistent supersets of K that imply α. Note that in the case where $\neg\alpha \in K$ we obtain $K \top \alpha = \emptyset$.

It is straightforward enough to show the following observation:

Observation 5.2 *Any $K' \in K \top \alpha$ is a belief set.*

Various constructions can now be provided using the notion of a *selection function* γ. The simplest idea would be for the selection function to choose one of the maximal consistent subsets implying α.

5.2.2 Maxichoice Abductive Expansion

If $K^{\oplus}_{\alpha}$ is determined by a selection function γ that selects a single element $\gamma(K \top \alpha)$ from $K \top \alpha$ for any K and any α whenever $K \top \alpha \neq \emptyset$ then we call $\oplus$ a maxichoice abductive expansion function. It can be defined in the following manner:

Definition 5.2 *(Def Max)*

$$K^{\oplus}_{\alpha} = \begin{cases} \gamma(K \top \alpha) & \textit{whenever } K \top \alpha \textit{ is nonempty} \\ K & \textit{otherwise} \end{cases}$$

We can show that this type of construction satisfies the postulates presented in Section 5.1.

Lemma 5.3 *Any maxichoice abductive expansion function satisfies ($K^{\oplus}1$) — ($K^{\oplus}6$).*

However, this type of abductive expansion would correspond to an abductive inference procedure that always hypotheses as much as possible (provided consistency is maintained) in order to account for the new information. If $\oplus$ is a maxichoice abductive expansion function, then given any $\beta \in \mathcal{L}$ the agent, in belief state $K^{\oplus}_{\alpha}$, will have an opinion on whether β is true or false. Continuing the example introduced earlier in this paper, one possibility is $measles, rash, allergic_reaction, allergen, allergy_sufferer \in K^{\oplus}_{rash}$ even though $measles, rash, allergic_reaction, allergen, allergy_sufferer \notin K$. This approach would seem a bit too extreme. Instead of selecting just one element from $K \top \alpha$ we might consider selecting them all and take their intersection.

[9] That is, K' can be identified with a $K' \cup \{\alpha\}$ *"world"*.

[10] This is an anologue of the AGM notation $K \perp \alpha$ for maximal subsets of K not implying α used in constructions for contraction and revision. Moreover, the maxichoice, full meet and partial meet constructions to be introduced shortly have their contraction/revision analogues in the AGM framework [3].

5.2.3 Full Meet Abductive Expansion

If $K^{\oplus}_{\alpha}$ is determined by taking the intersection of all elements in $K \top \alpha$ (assume a selection function γ such that $\gamma(K \top \alpha) = K \top \alpha$) for any K and any α whenever $K \top \alpha \neq \emptyset$ then we call $\oplus$ a full meet abductive expansion function.

Definition 5.3 *(Def Meet)*

$$K^{\oplus}_{\alpha} = \begin{cases} \bigcap(K \top \alpha) & \textit{whenever } K \top \alpha \textit{ is nonempty} \\ K & \textit{otherwise} \end{cases}$$

Observation 5.4 $\bigcap(K \top \alpha)$ *is a belief set.*

This construction also satisfies the postulates for abductive expansion.

Lemma 5.5 *Any full meet abductive expansion function satisfies* ($K^{\oplus}1$) — ($K^{\oplus}6$).

Using this type of abductive expansion function would result in a belief set containing all the common elements in $K \top \alpha$. But this consists only of K, α and their consequences. In other words, the abductive procedure always selects the "trivial" explanation α for the input α. This implements the principle of minimal change and therefore coincides with AGM expansion as witnessed by the following theorem.

Theorem 5.6 *Let $\oplus$ be an abductive expansion function. For every belief set K, $\oplus$ is a full meet abductive expansion* iff *$\oplus$ satisfies postulates ($K^{+}1$) — ($K^{+}6$) for AGM expansion over K and $\neg\alpha \notin K$.*

As noted in the theorem, full meet expansion and AGM expansion coincide only if $\neg\alpha \notin K$ for if $\neg\alpha \in K$ then $K^{\oplus}_{\alpha} = K$ (by ($K^{\oplus}4$)) while in AGM expansion $K^{+}_{\alpha} = K_{\bot}$. AGM expansion is then, in a certain sense, a limiting case of abductive expansion. In our example $rash \in K^{\oplus}_{rash}$ but $measles, allergic_reaction, allergen, allergy_sufferer \notin K^{\oplus}_{rash}$. This type of expansion is not in accord with our proposal and therefore not desirable as a construction.

5.2.4 Partial Meet Abductive Expansion

We have seen that selection functions which choose a single element from $K \top \alpha$ and that which selects all elements from $K \top \alpha$ give undesirable results. In fact, Lemmas 5.3 and 5.5 do not in general reverse so these two constructions do not provide an exact characterisation of the notion we are attempting to capture. A compromise solution is to adopt a selection function γ that selects a suitable subset of $K \top \alpha$ (i.e., $\gamma(K \top \alpha) \subseteq K \top \alpha$) and to

take the intersection of these elements. Intuitively, γ selects all the "best" elements from $K \top \alpha$. Obviously maxichoice and full meet expansion are special cases of this type of expansion. If $K^{\oplus}_{\alpha}$ is determined in this manner we call $\oplus$ a partial meet abductive expansion function.

Definition 5.4 *(Def Part)*

$$K^{\oplus}_{\alpha} = \begin{cases} \bigcap \gamma(K \top \alpha) & \textit{whenever } K \top \alpha \textit{ is nonempty} \\ K & \textit{otherwise} \end{cases}$$

Observation 5.7 $\bigcap \gamma(K \top \alpha)$ *is a belief set.*

The following theorem shows that this construction exactly characterises the postulates from Section 5.1 as desired.

Theorem 5.8 *Let $\oplus$ be an abductive expansion function. For every belief set K, $\oplus$ is a partial meet abductive expansion* if and only if $\oplus$ *satisfies postulates ($K^{\oplus}1$) — ($K^{\oplus}6$) for abductive expansion over K.*

By altering γ we, in effect, alter the selection mechanism that determines the "best" abduction for α with respect to K and therefore alter the type of expansion being performed. For example, one type of selection function γ_1 may give rise to $\oplus_1$ such that $measles \in K^{\oplus_1}_{rash}$ and $allergen, allergy_sufferer \notin K^{\oplus_1}_{rash}$ while another, γ_2, gives rise to $\oplus_2$ such that $measles \notin K^{\oplus_2}_{rash}$ and $allergen, allergy_sufferer \in K^{\oplus_2}_{rash}$.

5.3 Alternative Constructions

Grove [5] presents a "sphere semantics" for AGM contraction and revision. He considers the set $\mathbf{M}_{\mathcal{L}}$ of all maximally consistent extensions of $\mathcal{L}$ which are the same as our maximally consistent sets (these may be thought of as possible worlds — a semantic notion). A belief set K is represented as a subset $[K]$ of $\mathbf{M}_{\mathcal{L}}$ with each element of $[K]$ containing the sentences in K. $[K]$ may be thought of as the set of possible worlds in which all members of K are true. A similar definition may be provided for any formula α to determine $[\alpha]$ the set of possible worlds in which α holds. The AGM expansion K^{+}_{α} can be identified with those worlds in $[K] \cap [\alpha]$. We can now specify a new system of spheres restricted to the worlds in $[K] \cap [\alpha]$ and use some mechanism for selecting the "best" worlds among them.

An epistemic entrenchment [4] is an ordering over the formulae of $\mathcal{L}$. It can be interpreted as specifying wich formulae we are willing to give up in performing a contraction or revision. For abductive expansion we can, in a similar fashion, adopt an ordering over the formulae of $\mathcal{L}$. This ordering would, in effect, determine the "best" abduction from among alternatives and tell us which formulae to include in the resulting belief state.

6 Conclusions and Future Work

We have presented a series of postulates to characterise the process of expanding a belief state by an abduction of the epistemic input rather than merely expanding by the epistemic input itself. This was advanced as a model for the claim that agents frequently do not simply expand their belief state by newly acquired information alone but first seek some sort of "explanation" for the information and expand their belief state with respect to this explanation.

Our claim effectively results in a partial rejection of the notion of minimal change (at least with respect to the process of adding beliefs to our current state) as it currently stands. Note that we are not advocating a return to a foundationalist approach to belief revision. Though our approach is not entirely unrelated to a foundationalist approach our aims are inconsistent with it.

Our preferred construction, partial meet abductive expansion, was shown to be characterised by the postulates ($\mathrm{K}^{\oplus}1$) — ($\mathrm{K}^{\oplus}6$). We have presented no specific selection criteria to discriminate between possible abductions but have preferred instead to present a general framework characterising the process of abductive expansion with the sole constraint that this process produce the same abduction for logically equivalent formulae. The imposition of specific selection criteria (for example, those studied in rational choice theory) would involve additional constraints and therefore more postulates. In terms of the construction provided, this would lead to specific selection functions or a class of particluar selection functions.

In future work it would be interesting to investigate the process of abductive revision $K^{\otimes}_{\alpha}$ of K with respect to α defined using the following variant of the Levi identity: $K^{\otimes}_{\alpha} = (K^{\ominus}_{\neg\alpha})^{\oplus}_{\alpha}$. In this case $K^{\ominus}_{\neg\alpha}$ represents some form of contraction (not necessarily AGM contraction) while $K^{\oplus}_{\alpha}$ represents abductive expansion (though it could be replaced by alternatives). $K^{\otimes}_{\alpha}$ represents the resulting revsion. We can then obtain a class of revision functions by altering the functions associated with $\oplus$ and $\ominus$. Abductive revision would then allow the incorporation of new information that does not agree with the current belief state.

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