Experience and Trust — A Systems-Theoretic Approach

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Abstract. An influential model of agent trust and experience is that of Jonker and Treur [Jonker and Treur 99]. In that model an agent uses its experience of the interactions of another agent to assess its trustworthiness. We showed that key properties of that model are subsumed by classical mathematical systems theory. Using the latter theory we also clarify the issue of when two experience sequences may be regarded as equivalent. An intuitive feature of the Jonker and Treur model is that experience sequence orderings are respected by functions that map such sequences to trust orderings. We raise a question about another intuitive property — that of continuity of these functions, viz. that they map experience sequences that resemble each other to trust values that also resemble each other. Using fundamental results in the relationship between partial orders and topologies we also showed that these two intutive properties are essentially equivalent.

1 INTRODUCTION

In electronic internet trading systems like eBay an agent can rank other agents based on its assessment of the behavior of those agents in transactions. For an agent A observing another agent B over time, such sequential assessments may be said to form A's *experience sequence* of B, and result in its judgement of the trustworthiness of B. In an influential model of agent trust due to [Jonker and Treur 99], agents assess the quality of their interactions and map such experience sequences into a *trust* space. They required the experience sequence and trust spaces to be at least partially ordered, and the mapping to be order-preserving. They established properties of their model, including condtions for the updating of trust ranks that depend only on the existing rank and a new assessment of experience. In our paper we showed that the update and a number of other properties are in fact subsumed by classical mathematical systems theory. Space limitations restrict to merely outline our results, but a fuller version is in [Foo and Renz 07].

2 SYSTEMS-THEORETIC IMPLICATIONS

We took the work [Jonker and Treur 99] as a starting point accepting in particular the discrete time framework (modelled as the natural numbers) for all functions. A sequel to that work is that by Treur [Treur 07] on properties of states arising from it. We used systems theory to (i) connect established propositions with their work, (ii) showed constraints on trust structure imposed by experience structure, (iii) suggested a way to topologize these and other derivative structures, and showed that it is equivalent to the desirable order-preservation property of the maps between them.

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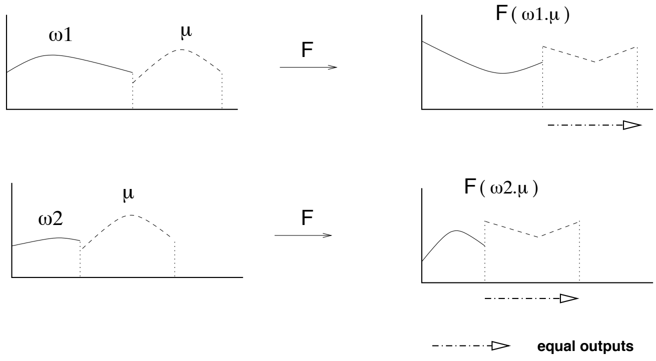

Figure 1. Nerode Equivalence

Conceptually the system we consider is a black box that accepts experience sequences as inputs and produces trust sequences as outputs. A basic result from systems theory (see [Padulo and Arbib 74] and [Zeigler, et.al. 2000]) guarantees that this black box can be endowed with a *state space* of trust values iff the input-output function F representing it is *causal*, i.e, for any point k in time the trust output at k depend only on the *initial segment* of the input sequence. This subsumes a key result of [Jonker and Treur 99]. Denoting the initial segment space of experience sequences by $\bar{\Omega}$, we then showed that the canonical state space is in fact a quotient space of $\bar{\Omega}$, with the quotient arising from an equivalence relation known in systems theory as the Nerode equivalence, denoted here by $\equiv_N$. Indeed, it follows that the trust space can be most succinctly identified with $\bar{\Omega}/\equiv_N$. To explain $\equiv_N$ we first make $\bar{\Omega}$ a semigroup using the concatenation (denoted by $\circ$) of segments as the binary operation. Next, we use the input-output function F to induce a function $\bar{F}$ that maps $\bar{\Omega}$ to corresponding length output segments. For any two input segments ω_1 and ω_2 we defined $\omega_1 \equiv_N \omega_2$ if for any arbitrary segment μ, $\bar{F}(\omega_1 \circ \mu)$ agrees with $\bar{F}(\omega_2 \circ \mu)$ from the respective times when μ is appended. This is only well-defined if F is causal. See figure 1 for intuition.

It is then intuitive that ω_1 and ω_2 cannot be distinguished once their end points are reached. Thus, $\bar{\Omega}/\equiv_N$ qualifies as a state of the system. It is a corollary of that result, known as the *State Realization Theorem*, that there is an update function δ from inputs and current state to the next state as follows: $\delta([\omega], e) = [\omega \circ \hat{e}]$ where $\hat{e}$ is the unit length segment with value e. Figure 2 illustrates the main points. In the figure γ is the map that "reads" the trust and outputs it into the trust value space V_{out}, and η is the map induced by the combination of γ and the state update function δ. Also, F' is the earlier defined map $\bar{F}$ restricted to the (value at the) end of its input segment.

It can be shown that the state realization above, call it R, is in a strong sense the most economical among all possible state representations. Formally, it is said that this realization is *canonical* in that

$$\begin{tikzcd}
\bar{\Omega} \arrow[rr, "F'"] \arrow[dr, "\psi"'] & & V_{\mathbf{out}} \\
& \Omega / \equiv \arrow[ur, "\gamma"'] &
\end{tikzcd}$$

$$\bar{\Omega} / \equiv \; \times \; \bar{\Omega} \xrightarrow{\eta} \bar{\Lambda}$$

Figure 2. State Realization – The Key Ideas

if there is another realization R' that reproduces the same $\bar{F}$, then there is a unique homomorphism that maps R' to R. In particular a typical assumption (see e.g., Treur [Treur 07]) that input (trust, etc.) sequences and system states are both viable primitives in formalizing temporal dynamics is subject to this canonical constraint.

3 EXPERIENCE AND TRUST ORDERINGS

One example ordering considered by [Jonker and Treur 99] was *worst* $<$ *bad* $<$ *neutral* $<$ *good* $<$ *best* for experience values. They then used these to partially order, say experience sequences. We may as well identify these sequences with $\bar{\Omega}$ above, and the trust space T with $\bar{\Omega}/\equiv_N$, and it can be partially ordered by, say, $\sqsubset_T$. The order-preservation postulate of [Jonker and Treur 99] then translates in systems theory to the quotient map ψ from $\bar{\Omega}$ to T (= $\bar{\Omega}/\equiv_N$) defined by $\psi(\omega) = [\omega]_{\equiv_N}$ to be also order-preserving. That is an intuitive requirement — good experiences should lead to good trust.

If a measure of "nearness" is placed on experience sequences and trust values we may also desire the property that ψ maps near sequences to near trust. The formalization of this is the *continuity* of ψ. The most abstract way to do this is via *topologies* for both $\bar{\Omega}$ and $\bar{\Omega}/\equiv_N$. Fortunately, there is already much classical machinery [Kelley 55] to do this. We switch notation to the near synonyms of $\bar{\Omega}$ (calling it E) and $\bar{\Omega}/\equiv_N$ (calling it T) for brevity. If a topology τ_E is given to E, then since ψ is the quotient map a natural topology τ_T is induced by ψ that makes it both continuous and open.

We then showed that under a simple topology — the Alexandrov topology (see [Arenas 99] or [Wiki Alexandrov]) — the two requirements above, viz., order-presevation and continuity, are equivalent. We now outline how this was done.

There is a close connection between partial orders — in fact preorders will do — and *topologies* on a space. Given a partial order $\sqsubseteq$ on a space S, the Alexandrov topology defined by it has as open sets the so-called *up-sets*, viz., subsets θ such that $x \in \theta$ and $x \sqsubseteq z$ implies $z \in \theta$. of up-sets. Conversely, given a topology τ on a set S, the *specialization pre-order* $\leq$ is defined by $x \leq y$ iff y is in every open set that contains x. It is easily seen that $\leq$ so defined is indeed a pre-order. If we had started with some partial order $\sqsubseteq$ and used it to define the Alexandrov topology as before, it is natural to ask what is the specialization order that arises from that topology. The answer is that we get back $\sqsubseteq$, and although there are other topologies (e.g. the Scott topology [Abramsky and Jung 94] or [Stoy 77]) that have this "reversal" property the Alexandrov topology is the finest one. In this way the partial order $\sqsubseteq_E$ defines the Alexandrov topology on the input segment space $\bar{\Omega}$ (which in our context is identified with the space of experience sequences E) and is induced by it.

Any topology that is placed on the trust space T will induce a specialization pre-order (partial orders are special cases). So what is a suitable topology for it? If we identify T with the range of ψ, i.e., $\bar{\Omega}/\equiv$, then T is the quotient space of $\bar{\Omega}$. $\bar{\Omega}/\equiv$ can thus be given the quotient topology.

Experience values in the real interval $[-1, 1]$ rather than finite or even discrete values may alter the character of the results and observations because the experience and trust spaces can now be infinite and continuous. Continous values lend themselves to measurements of nearness using the metrics well-known in functional analysis, and it is an obvious question whether the nexus of order-preservation between E and T and continuity of the map ψ still hold. Unfortunately, if the space is Hausdorff (which is the most familiar one), its corresponding Alexandrov topology reduces to the discrete topology which is trivial for convergence. Therefore the requirements for order-preservation and continuity are distinct.

4 CONCLUSION

We used classical mathematical systems theory to underpin the foundations of an influential model of agent trust and experience. It was shown that many of the properties of that model follow from results in systems theory. Moreover, the latter provides deep insights into the structural interaction between experience and trust sequences, in particular what it means to say that *trust is condensed experience*. An intuitive feature of that model is that experience sequence orderings are respected by functions that map such sequences to trust orderings. We raised a question about another intuitive property — that of continuity of these functions, viz. that they map experience sequences that resemble each other to trust values that also resemble each other. Using fundamental results in the relationship between partial orders and topologies we showed that these two intutive properties are essentially equivalent.

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